Are there phase transitions in strong-field regimes?

ExHILP

September 15th, 2021

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LLNL-PRES-826584

This work was performed under the auspices of US DOE by LLNL under Contract DE-AC52-07NA27344.

Y.S. was supported by the Lawrence Fellowship through LLNL-LDRD under Project No. 19-ERD-038.

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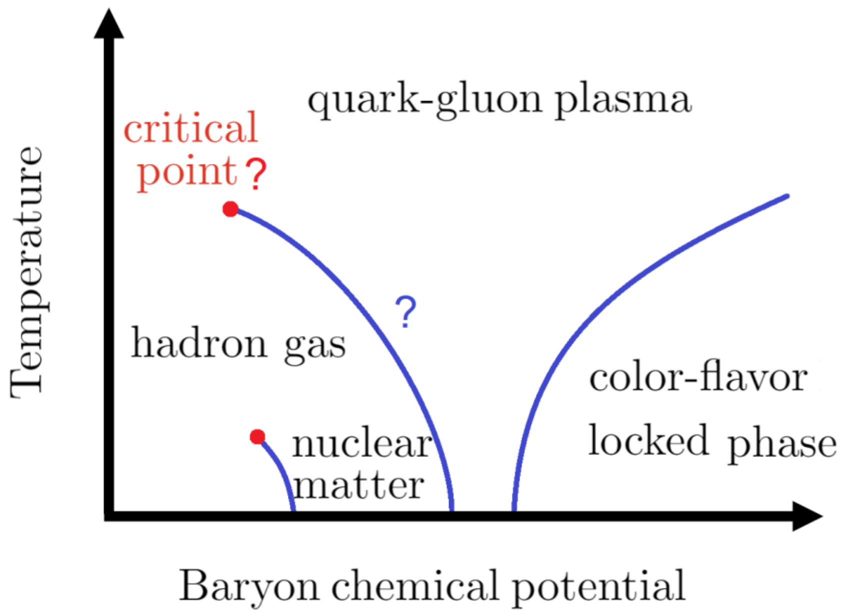
Strong-field QED has no known phase transition

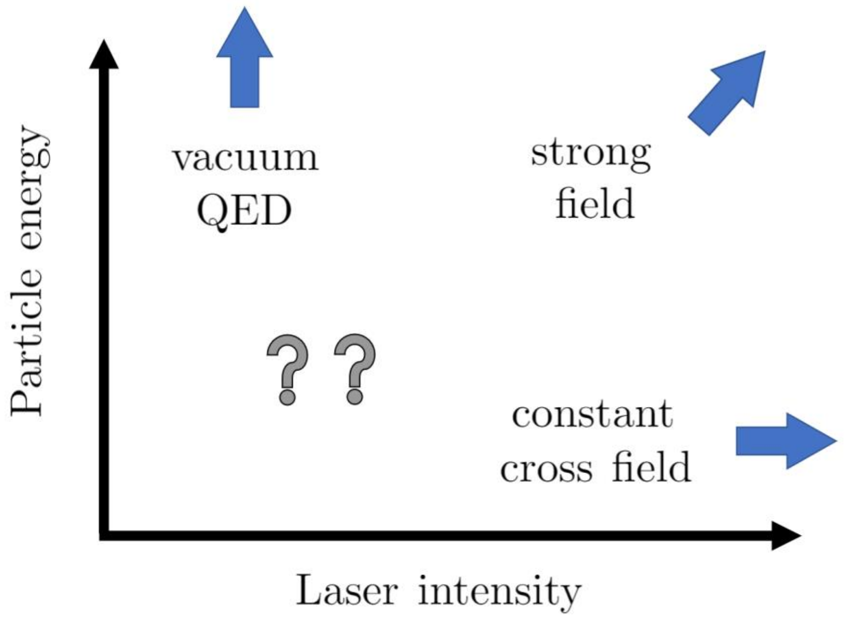
QCD phase diagram

- Commonly believed to have multiple phases
- Details largely uncertain: order of transitions, location of boundaries, existence of critical points
- Active research area: analytical theories, lattice simulations, heavy ion collision experiments, astrophysical observations…



SF-QED phase diagram?

- Observables depend on parameters smoothly? No discontinuity?
- Normalized parameters
  - Laser (A_μ, k_μ) intensity: $a = eA/m$
  - Particle (p_μ) energy: $b = pk/m^2$
  - Field strength in particle frame: $\chi = ab$



Phase transition: discontinuous behavior of order parameters

Ingredients of modern description of phase transitions

- Free energy ϵ is minimized by physically realized states
- Order parameter ψ is an observable that characterizes the states
- Control parameter τ specifies an adjustable condition

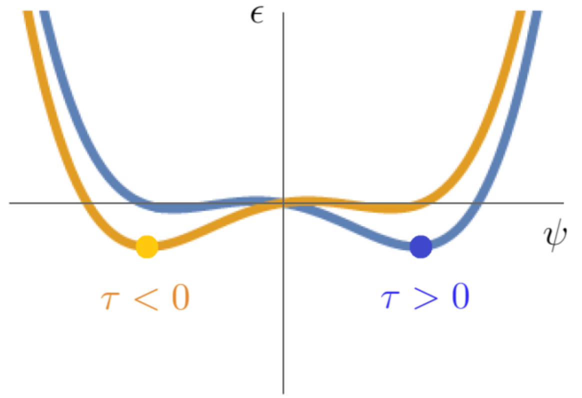
First-order phase transition

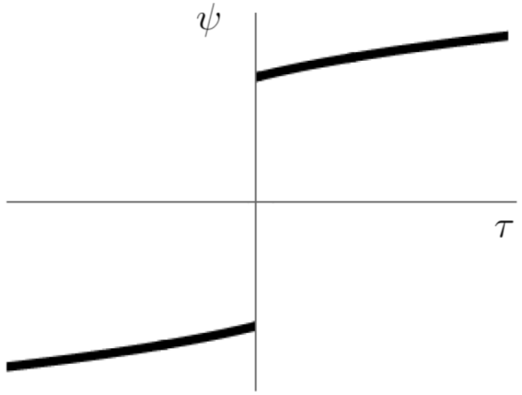
- $\psi(\tau)$ is a discontinuous function
- Example: solid → liquid → gas →plasma

  ϵ: Gibbs free energy

  ψ: entropy

  τ: temperature





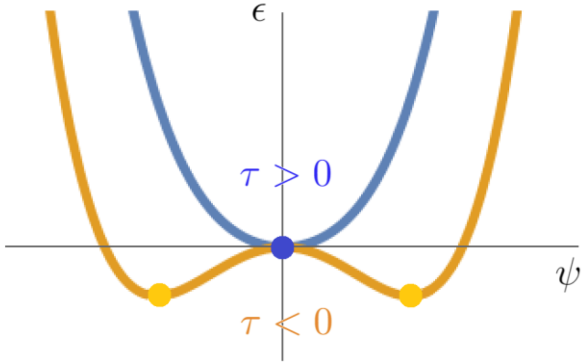
Second-order phase transition

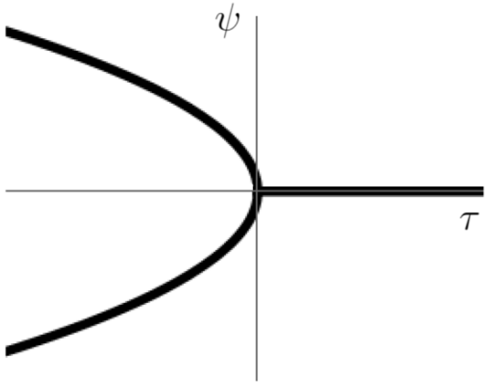
- $\psi(\tau)$ is continuous but $\psi'(\tau)$ is not
- e.g., ferromagnetism

  ϵ: Helmholtz free energy

  ψ: magnetization, τ: temperature
- e.g., superconductivity

  ϵ: energy, ψ: Cooper pair density, τ: B field

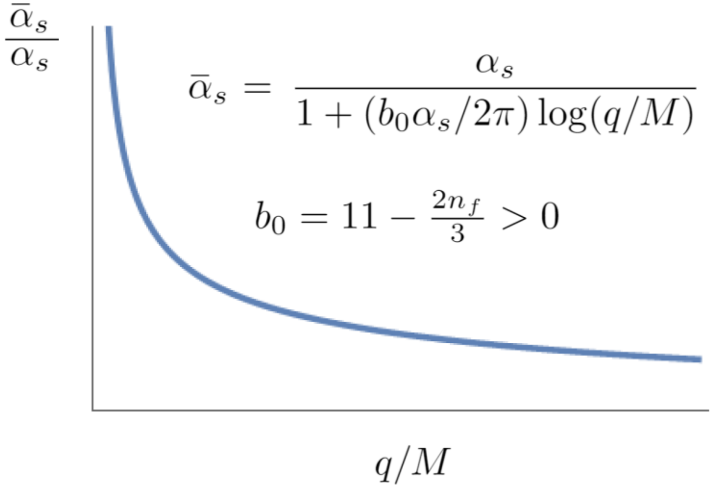




Phase transitions exist in nonabelian theories

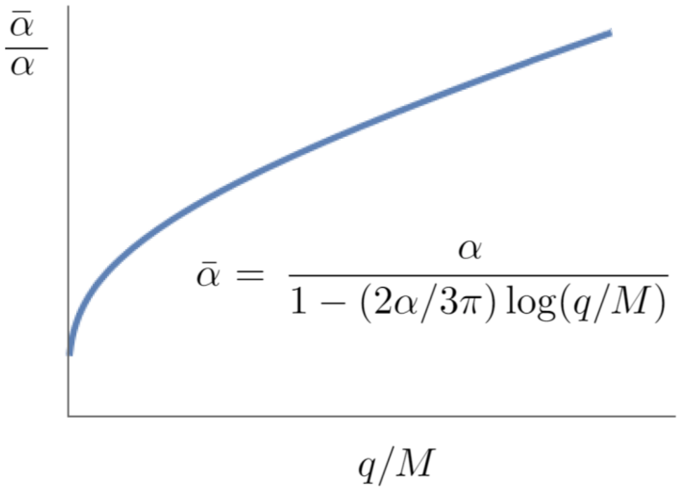
QCD

- Bound states obvious due to existence of hadrons (proton, neutron…) and mesons (pion, kaons…)
- Quark-gluon-plasma phase expected because of asymptotic freedom



QED

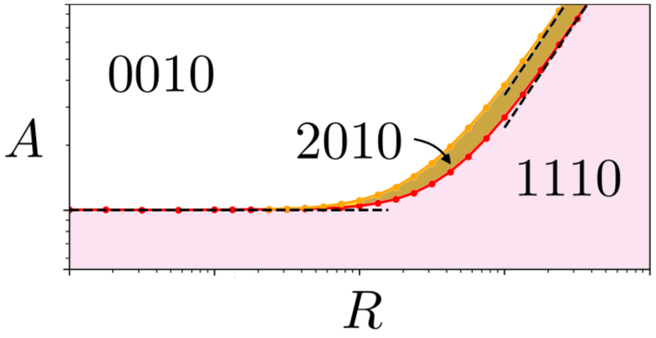
- Low-energy limit is quantum mechanics, describe e^- in materials, abundant phase transitions
- At higher energy, no more bound states, but stronger coupling. More phases?



ϕ^4

- Important toy model. Allows spontaneous symmetry breaking, phenomenological phase transitions
- **Not asymptotically free but additional phase transitions due to strong source fields!**

$$\bar{\lambda} = \frac{\lambda}{1 - (3\lambda/16\pi^2) \log(q/M)}$$



Analogies between strong-field QED and ϕ^4 model

QED with background fields

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu(\partial_\mu - ie\bar{A}_\mu) - m]\psi$$

$\bar{A}_\mu$: background fields generated by external current, e.g. laser pulses

- 1-loop running coupling: not asymptotically free

$$\bar{\alpha} = \frac{\alpha}{1 - (2\alpha/3\pi)\log(q/M)}$$

- Scales of the background fields
  - Size: wavelength of laser
  - Strength: intensity of laser

ϕ^4 with external sources

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4!}(\phi^2 - v^2)^2 - f\bar{\psi}\psi\phi$$

$S = f\langle\bar{\psi}\psi\rangle$: sources due to fermion distributions, e.g. neutron stars

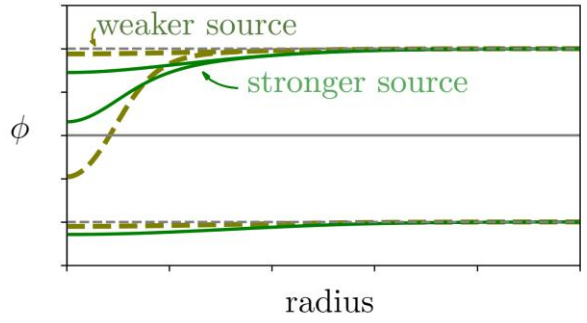
- 1-loop running coupling: not asymptotically free

$$\bar{\lambda} = \frac{\lambda}{1 - (3\lambda/16\pi^2)\log(q/M)}$$

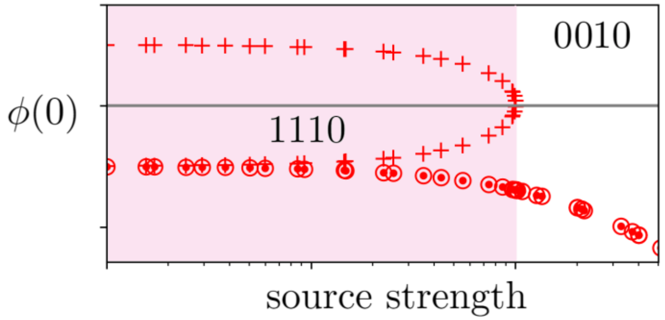
- Scales of the background fields
  - Size: radius of matter distribution
  - Strength: density of matter

ϕ^4 model: finite-energy potentials due to external sources

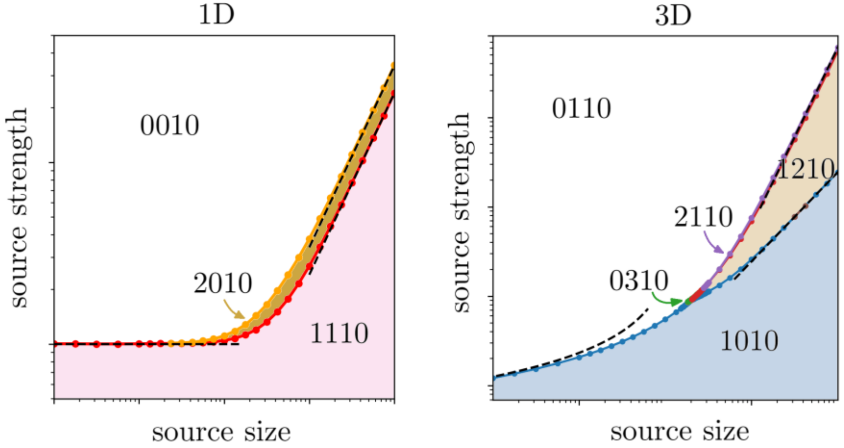
- Static ϕ^4 potentials are **not unique** for given $\phi(\infty)$!
  v.s. electrostatic potential due to external charge is unique



- Finite-energy potentials (FEP) are **countable**
  For smooth source, FEPs are of finite total energy
  Other configurations are of finite energy density but infinite total energy
  In large volume, FEPs are configurations that minimize energy
  Field value at origin $\phi(0)$ serves as an *order parameter*



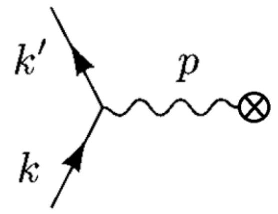
- External sources lead to 2^{nd} order **phase transitions**
  How many FEPs exist depends on source size and strength
  When source parameters vary, order parameter has discontinuities
- The phase diagrams **depend on dimensionality**
  In 1D, there are 3 phases in total
  In higher dimensions, 5 phases in total



arXiv:2107.04206

Classical potentials as static solutions of field equations

- Low-energy interactions described by classical potentials
  e.g. Coulomb potential, Yukawa potential



$$i\mathcal{M} \approx \frac{-ie^2}{|\mathbf{p}|^2}(2m\xi'^{\dagger}\xi)_p(2m\xi'^{\dagger}\xi)_k$$

$$\phi = \frac{e^2}{4\pi r}$$

- Potentials are static solutions of classical field equation
  ϕ: real scalar field, D: spatial dimension, $\mathcal{S}$: spherically symmetric source

$$\frac{\partial^2\phi}{\partial r^2} + \frac{D-1}{r}\frac{\partial\phi}{\partial r} = \frac{\partial U}{\partial\phi} + \mathcal{S} \qquad U = \frac{\lambda}{4!}(\phi^2 - v^2)^2$$

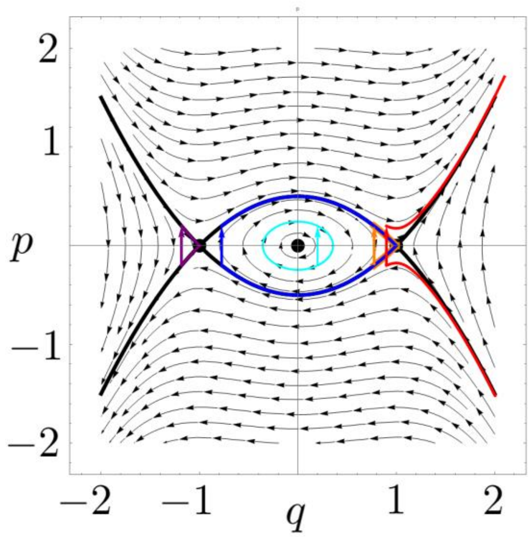
- Nondimensionalized equation resembles dynamical systems

$$q = \phi/v$$
$$\rho = rm$$
$$m = v\sqrt{\lambda/3}$$
$$p = dq/d\rho$$
$$\dot{p} = dp/d\rho$$

Gaussian source

$$R = am$$
$$A = \alpha m^{D-2}/v$$
$$S_0 = A/R^D\pi^{D/2}$$

$$\dot{p} + \frac{D-1}{\rho}p = \frac{1}{2}q(q^2-1) + S_0 e^{-\rho^2/R^2}$$



- Total energy of field configurations usually infinite in $\mathbb{R}^D$

$$E = \int d^D x[\tfrac{1}{2}(\partial_t\phi)^2 + \tfrac{1}{2}(\nabla\phi)^2 + U + \mathcal{S}\,\phi] = v^2 m^{2-D}\sigma_{D-1}H$$

$$H = \int_0^\infty d\rho\rho^{D-1}\mathcal{E}, \qquad \mathcal{E} = \frac{1}{2}p^2 + \frac{1}{8}(q^2-1)^2 + Sq$$

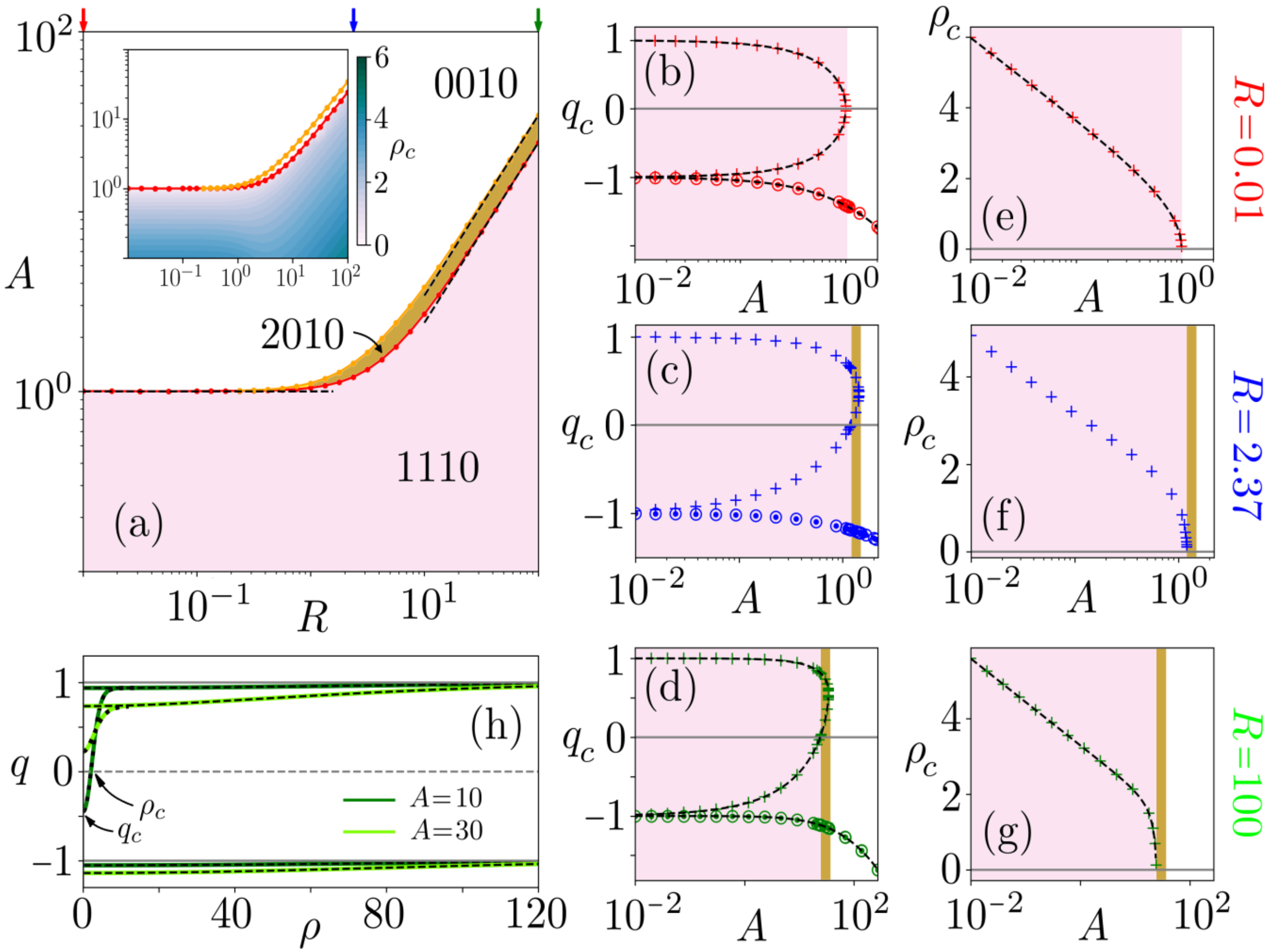
Energy density

1D phase diagram: 2nd order phase transitions

- **Finite energy potential (FEP)** is a static configuration that minimizes energy
- All FEPs have same boundary conditions ± 1
- FEPs have different **critical initial values** $q(0) = q_c$, serve as an order parameter
- Fermions become massless when FEPs cross zero $q(\rho_s) = 0$ at **light horizon (LH)**: order parameter
- Control parameters: source size R and strength A
- Three phases
  - 1110 phase: 3 FEPs, 1 with LH
  - 2010 phase: 3 FEPs, 0 with LH
  - 0010 phase: 1 FEP, 0 with LH
- Asymptotic phase boundaries

$$R \ll 1, \qquad A^{0010}_{1110} \simeq 1$$

$$R \gg 1, \qquad A^{2010}_{1110} \simeq \sqrt{\frac{\pi}{6}\frac{R}{3}}, \qquad A^{0010}_{2010} \simeq \sqrt{\frac{\pi}{3}\frac{R}{3}}$$

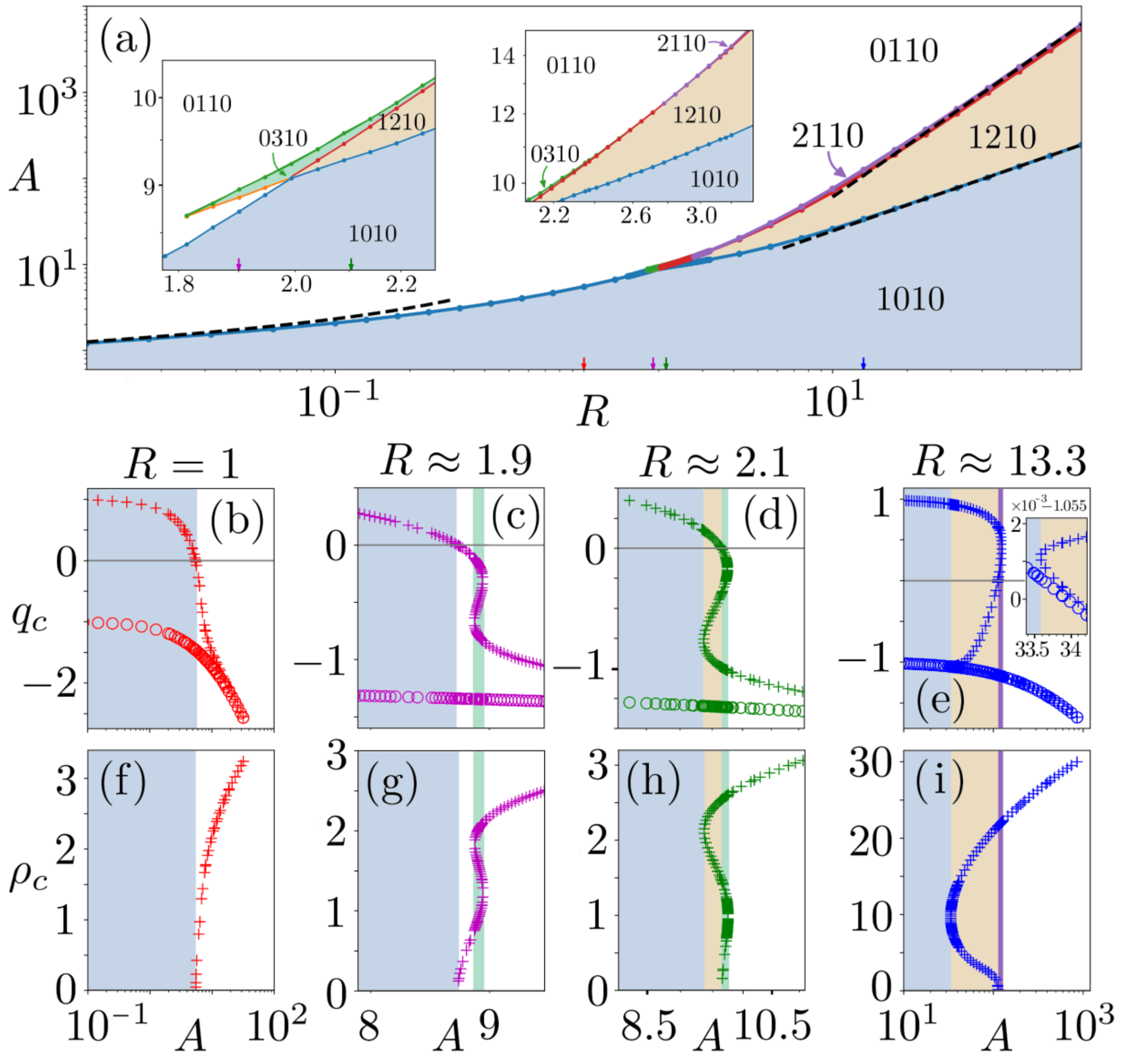


2D phase diagram: additional phases

- Without external source, no nontrivial field configuration (Derrick's theorem)
- External source balances singular term $1/r$, leading to multiple FEPs. e.g. pierced Yukawa potential, hopping potential, adiabatic potential…
- Five phases
  - 1010 phase: 2 FEPs, 0 with LH
  - 0110 phase: 2 FEPs, 1 with LH
  - 2110 phase: 4 FEPs, 1 with LH
  - 1210 phase: 4 FEPs, 2 with LH
  - 0310 phase: 4 FEPs, 3 with LH
- Phase diagram has 1 double point, 2 quadruple points, no critical point
- Asymptotic phase boundaries

$$A^{0110}_{1010}(D=2) \simeq \frac{2\pi}{\ln(2/R)-\gamma/2}, \qquad A^{1210}_{1010} \simeq \frac{D-1}{3}\sqrt{2e\pi^D}R^{D-1}$$

$$A^{0110}_{1010}(D=3) \simeq \frac{2\pi R}{1/\sqrt{\pi}-R/2}, \qquad A^{2110}_{1210} \lesssim A^{0110}_{2110} \simeq \frac{\pi^{D/2}}{3\sqrt{3}}R^D$$



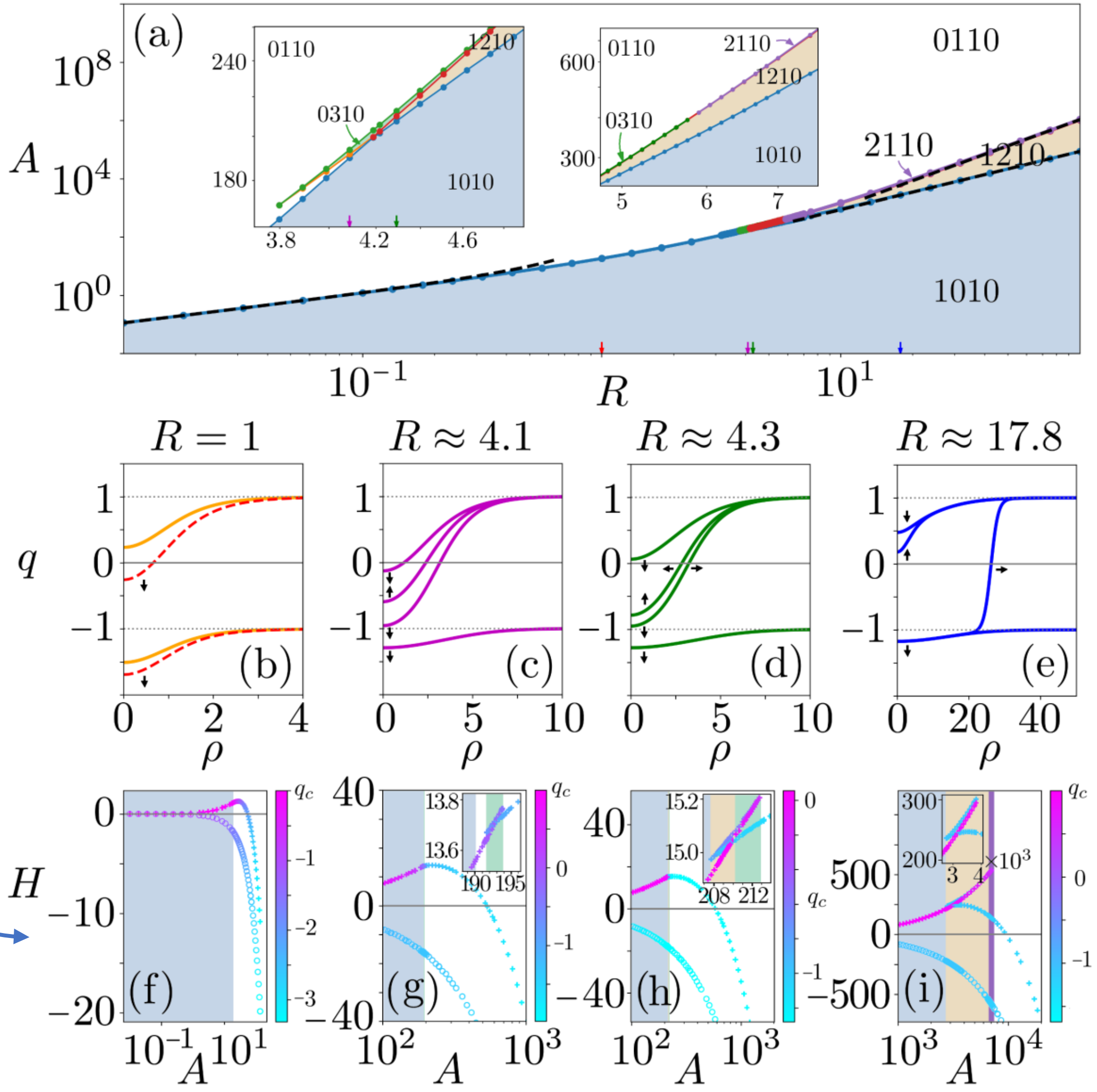
3D phase diagram: qualitatively similar

- Two FEP are created or annihilated, **add/remove pair of energy levels,** at 0110-0310, 2110-0110 transitions
- A FEP crosses $q(\rho_s) = 0$, **add/remove a light horizon** (LH), at 1010-0110, 1210-0310, 1210-2110 transitions
- Asymptotic LH radius in 1210 phase

$$\rho_+ \simeq R\sqrt{\ln\frac{3RS_0}{D-1}}\ , \qquad \rho_- \simeq \frac{D-1}{3S_0}$$

LH is a **nonperturbative** phenomena, depends on $1/\lambda$
LH **potentially macroscopic** $\gg$ Compton wavelength

- Energy ordering of FEPs switches when A, R change e.g. 1210 phase in 3D, normalized total energy is H
  - $S_0 R > 0.57$, hopping potential is 1st excited state
  - $S_0 R < 0.57$, hopping potential is 2nd excited state



Physical implications of source-induced phase transitions

Example: Standard-Model Higgs with matter distributions as sources in 3D

- In Standard Model, Higgs field originates from SU(2) doublet with ϕ^4 nonlinearity. Spontaneous symmetry breaking leads to vacuum expectation value (VEV), endow elementary particles with masses
- Most matter distributions are in the perturbative phase, causing little distortions to the Higgs VEV
- Compact sources (heavy fermions, neutron stars, white dwarfs…) are potentially in nonperturbative phases with strong Higgs VEV distortions, light horizon radius comparable to source sizes

source prototype	size	R	mass	A	A/A_c	phase	r_c	$-E_0$	ΔE
hypothetical fermion	0.2 am [a]	0.1	10^3 GeV	10^0	10^0	0110	0.2 am [b]	10^3 GeV	10^3 GeV
top quark	1 am [a]	0.7	173 GeV	10^{-1}	10^{-2}	1010		10^2 GeV	10^2 GeV
^{12}C	3 fm	2×10^3	11.3 GeV	10^{-4}	10^{-11}	1010		10^{-1} GeV	10^{-1} GeV
^{197}Au	8 fm	5×10^3	185 GeV	10^{-3}	10^{-11}	1010		10^0 GeV	10^0 GeV
electron	0.4 pm [a]	2×10^5	0.511 MeV	10^{-6}	10^{-18}	1010		10^0 MeV	10^0 MeV
fusion stagnation	50 μm	3×10^{13}	1 mg	10^{16}	10^{-12}	1010		10^9 J	10^9 J
uranium ball	1 m	6×10^{17}	80 t	10^{27}	10^{-10}	1010		10^{20} J	10^{20} J
neutron star	10 km	10^{22}	$1.5M_\odot$	10^{52}	10^8	1210	40 μm, 50 km	10^{11} $L_\odot\cdot$yr	10^4 $L_\odot\cdot$yr
earth	6×10^3 km	4×10^{24}	6×10^{24} kg	10^{47}	10^{-3}	1010		10^5 $L_\odot\cdot$yr	10^6 $L_\odot\cdot$yr
white dwarf	10^4 km	10^{25}	$0.6M_\odot$	10^{52}	10^2	1210	70 km, 2×10^4 km	10^{11} $L_\odot\cdot$yr	10^{10} $L_\odot\cdot$yr
Jupiter	7×10^4 km	4×10^{25}	2×10^{27} kg	10^{49}	10^{-3}	1010		10^8 $L_\odot\cdot$yr	10^8 $L_\odot\cdot$yr
Sun	$R_\odot$	4×10^{26}	$M_\odot$	10^{52}	10^{-2}	1010		10^{11} $L_\odot\cdot$yr	10^{11} $L_\odot\cdot$yr
supergiant	$500R_\odot$	10^{29}	$15M_\odot$	10^{53}	10^{-6}	1010		10^{12} $L_\odot\cdot$yr	10^{12} $L_\odot\cdot$yr
dwarf galaxy halo [c]	25 kpc	10^{38}	$10^9 M_\odot$	10^{61}	10^{-17}	1010		10^{20} $L_\odot\cdot$yr	10^{20} $L_\odot\cdot$yr
Milky Way halo [c]	280 kpc	10^{40}	$10^{12} M_\odot$	10^{64}	10^{-16}	1010		10^{23} $L_\odot\cdot$yr	10^{23} $L_\odot\cdot$yr

A_c: source strength at phase boundaries

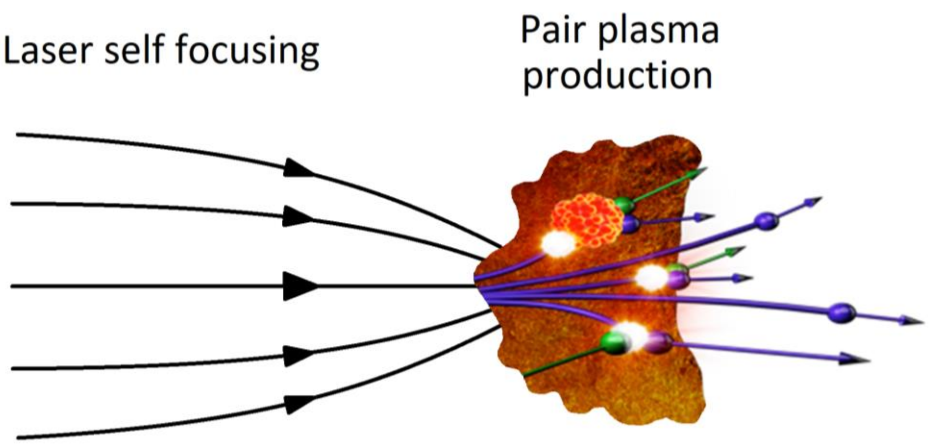
r_c: light horizon radius

E_0: energy of ground-state configuration

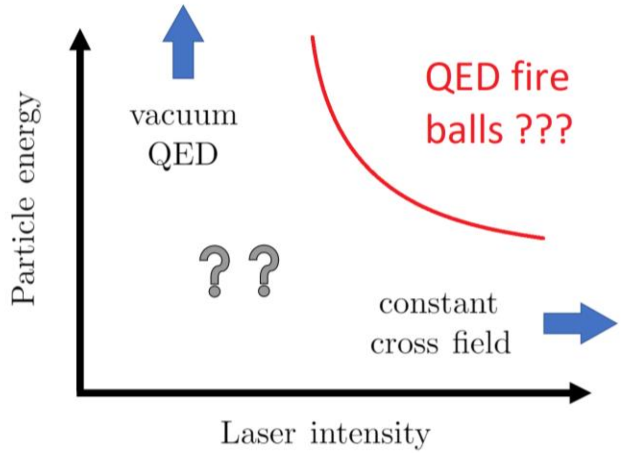
ΔE: gap between ground and 1st excited state

What about strong-field QED?

- From the ϕ^4 toy model, we see
  - **Asymptotic freedom is not essential**, abelian theories can have additional phase transitions in the presence of external sources
  - **Nonlinearity is essential** for phase transitions, which leads to multiple local minima of energy. Switching between minima is discontinuous.



- Speculation of possible phase transition in SF-QED
  - Intense lasers interact with seed particles to create a pair plasma via γ - e^+/e^- cascade
  - Laser self focus in pair plasma, creating stronger fields, run-away cascade until laser exhausts?
  - At low density, particles are free to fly apart
  - At high density, stronger couplings bind fire balls?



❖ Future work: Model feedback to laser fields, capture self-consistent nonlinear evolutions

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