Sauter-Schwinger effect for colliding laser pulses

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Colliding-pulse scenario

Electron-positron pair creation in space-time dependent field

$$\boldsymbol{A}(t,\boldsymbol{r}) = A_y(t,x)\boldsymbol{e}_y = [f(t-x) + f(t+x)]\,\boldsymbol{e}_y$$

Vacuum solution to Maxwell equations ✓ transversal field ✓

Parity symmetry $A_y(t,-x) = A_y(t,x)$

At symmerty axis $x = 0$ we have $B_z = \partial_x A_y = 0$ and $E_y \to \max$

$\to$ expect maximum contribution to pair creation $P_{e^+e^-}$

E.g., world-line instantons $(E \ll E_{\rm crit} = m^2/q \text{ and } \omega \ll m)$

$$P_{e^+e^-} \sim \exp\{-\mathcal{A}_{\rm inst}\}\,, \qquad m\frac{d^2x^\mu}{d\tau^2} = qF^{\mu\nu}\frac{dx_\nu}{d\tau}$$

Instanton stays in $x = 0$ plane

Same exponent $\mathcal{A}_{\rm inst}$ as for purely time-dependent field $A_y(t, x = 0)$

Pre-factor?

WKB approach

Klein-Fock-Gordon equation

$$\left[\left(\partial_\mu + iqA_\mu\right)\left(\partial^\mu + iqA^\mu\right) - m^2\right]\phi = 0$$

WKB ansatz $\phi(t,x,y,z) = \alpha(t,x)\exp\{iS(t,x,y,z)\} \to$ order $\mathcal{O}(m^2)$

$$\left(\partial_\mu S + qA_\mu\right)\left(\partial^\mu S + qA^\mu\right) = m^2$$

Eikonal $S(t,x,y,z) = k_y y + k_z z + s(t,x)$ (Hamilton-Jacobi)

$$\partial_t s = \sqrt{m^2 + (\partial_x s)^2 + (k_y + qA_y)^2 + k_z^2}$$

Parity symmetry $s(t,-x) = s(t,x)$ implies $\partial_x s(t,x=0) = 0$

At symmetry axis $x = 0$ same as for purely time-dependent field

Amplitude from sub-leading order $\mathcal{O}(m\omega)$, neglect $\Box\alpha = \mathcal{O}(\omega^2)$

$$(\partial^\mu s)\partial_\mu\alpha = -\frac{\alpha}{2}\Box s \;\to\; \partial_t\alpha|_{x=0} = -\frac{\alpha}{2\dot{s}}\Box s\Big|_{x=0}$$

Replace $\ddot{s} \to \Box s = \partial_t^2 s - \partial_x^2 s$

Focusing/de-focusing effects

Taylor expansion of eikonal (phase function)

$$s(t,x) = s_0(t) + \frac{x^2}{2}s_2(t) + \mathcal{O}(x^4)$$

Zeroth order $s_0(t)$ as in purely time-dependent field $A_y(t, x = 0)$

$$\partial_t s_0 = \left.\sqrt{m^2 + (k_y + qA_y)^2 + k_z^2}\right|_{x=0}$$

Evolution of curvature → divergence of Lorentz force $\partial_x F_x$

$$\partial_t s_2 = \left.\frac{s_2^2 + [k_y + qA_y]\, q\partial_x^2 A_y}{\sqrt{m^2 + [k_y + qA_y]^2 + k_z^2}}\right|_{x=0} \leadsto \partial_x(qv_y B_z)$$

→ impact of magnetic field: focusing or de-focusing

Non-linearity s_2^2 → caustics...

Particle creation

Introduce pseudo-vector $\boldsymbol{\varphi} = (\phi, \dot{\phi})^T$ $\rightarrow$ generalized WKB ansatz

$$\boldsymbol{\varphi} = \alpha \boldsymbol{u}_+ e^{+is} + \beta \boldsymbol{u}_- e^{-is}$$

Leading order $\mathcal{O}(m^2)$ $\rightarrow$ eikonal equation ✓

Sub-leading order $\mathcal{O}(m\omega)$ $\rightarrow$ evolution of Bogoliubov coefficients

Along symmerty axis $x = 0$ we find, neglecting $\mathcal{O}(\omega^2)$

$$2\dot{s}\dot{\alpha} + \alpha \Box s = \beta (\Box s) e^{-2is}$$
$$2\dot{s}\dot{\beta} + \beta \Box s = \alpha (\Box s) e^{+2is}$$

Up to replacement $\ddot{s} \rightarrow \Box s$ same as for purely time-dependent field

$\partial_t^2 s$ $\rightarrow$ electric field $\propto E_y = \dot{A}_y$ $\rightarrow$ pair creation (tear apart vacuum)

$\partial_x^2 s$ $\rightarrow$ magnetic field $\propto \partial_x B_z = \partial_x^2 A_y$ $\rightarrow$ focusing/de-focusing effects

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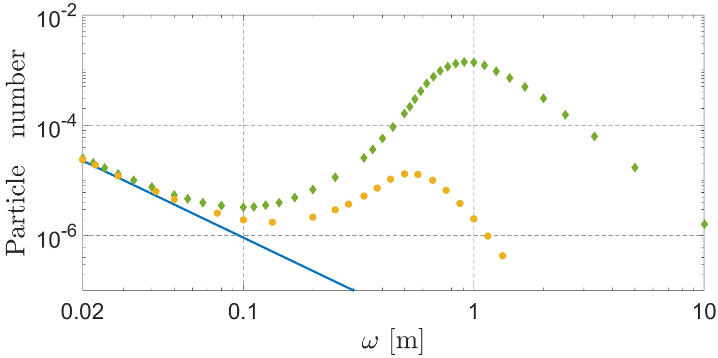
Numerical simulations

Pulses with $f(t) = t\exp\{-\omega^2 t^2\}E_{\text{crit}}/6$



Locally constant field approximation

Dirac-Heisenberg-Wigner approach

Spatially homogeneous field approximation

Improvement $\ddot{s} \to \Box s$

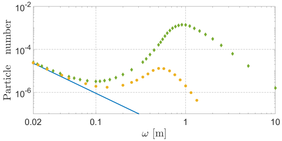
Summary & Outlook

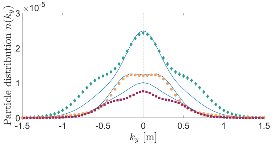
C. Kohlfürst, N. Ahmadiniaz, J. Oertel, R.S., arXiv:2107.08741

Head-on collision of equal plane-wave pulses $\to$ pair creation

$$\boldsymbol{A}(t,\boldsymbol{r}) = A_y(t,x)\boldsymbol{e}_y = [f(t-x)+f(t+x)]\boldsymbol{e}_y$$

- parity symmetry $A_y(t,-x) = A_y(t,x)$
- pair-creation exponent:
  same as for purely time-dependent field
- pre-factor: replacement $\ddot{s} \to \Box s$
  - $\partial_t^2 s \to$ electric field
  - $\partial_x^2 s \to$ magnetic field
    $\to$ focusing/de-focusing effects





Outlook

- pair-creation spectra
- pulse shape optimization