Multiloop calculations in QED in a strong constant crossed field

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ExHILP 2021

- $\mathcal{L} = \bar{\psi}\left(i\gamma^\mu \partial_\mu + e\gamma^\mu A_\mu - m\right)\psi - \frac{1}{4} F^{\text{rad}}_{\mu\nu} F^{\mu\nu}_{\text{rad}} + e\bar{\psi}\gamma^\mu A^{\text{rad}}_\mu \psi$,
- $A_\mu = a_\mu \varphi$ — strong constant crossed field (CCF)

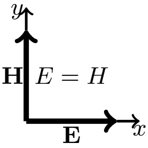
  $\varphi = (kx), \quad k^2 = (ak) = 0$

  $k^\mu = m\{1, 0, 0, 1\}$ — wave 4-vector,

  $a^\mu = \{0, E/m, 0, 0\}$,

  $F^{\mu\nu} = k^\mu a^\nu - a^\mu k^\nu$ — field strength tensor,

  $F^\star_{\mu\nu} = \dfrac{1}{2} \varepsilon_{\mu\nu\lambda\delta} F^{\lambda\delta}$



- Dirac equation in a CCF: $\left[(\gamma\hat{p}) + e(\gamma A) - m\right]\psi_p(x) = 0, \quad \hat{p}^\mu = i\partial^\mu$

- Solution: $\psi_{p,\sigma}(x) = E_p(x) u_{p,\sigma}, \quad (\not{p} - m) u_{p,\sigma} = 0$ — stable e^- states in presence of a CCF

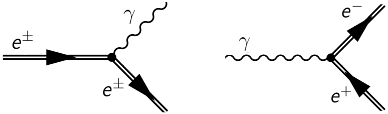
  $E_p(x) = e^{iS_p(x)} \Sigma_p(\varphi)$ — Ritus E_p-functions (matrices), orthogonal, full system;

  p^μ — e^- momentum

- Next: perturbative expansion over $\alpha = e^2/4\pi$

Elementary processes in a CCF

- $\chi = \frac{e}{m^3}\sqrt{pFFp} \sim \frac{\varepsilon}{m}\frac{E}{E_S} \sim \frac{E'}{E_S}$ — quantum dynamical parameter
- LO processes:

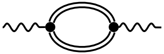


Nikishov, Ritus 1964

Dominant at $\chi \ll 1$ (classical radiation), $\chi \gtrsim 1$ (quantum recoil and pair creation)

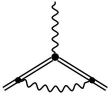
At $\chi \gg 1$ $\boxed{W_{\rm rad,cr} \propto \alpha\chi^{2/3}}$

- Loop corrections:



Narozhny 1969

Ritus 1970



Morozov 1981, Di Piazza 2020

At $\chi \gg 1$ also scale as $\boxed{g = \alpha\chi^{2/3}}$!!!

The Ritus-Narozhny conjecture

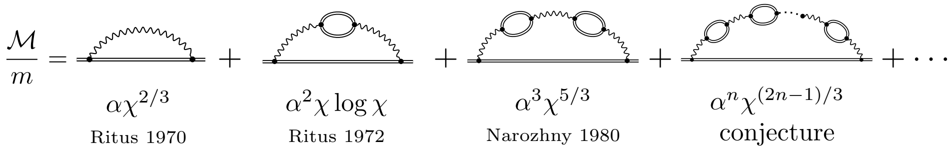
See also talks by A. Ilderton and A.M. Fedotov

Narozhny (1979, 1980) observed the same scaling up to the 3-loop level.

See full classification in A. Fedotov arXiv:1608.02261; AAM, S. Meuren and A. M. Fedotov PRD 102, 053005 (2020)

The conjecture:

1. Main contribution to $\mathcal{M}^{(n-loop)}$ at $\chi \gg 1$ — from polarization loop insertions
2. $\dfrac{\mathcal{M}^{(n+1)}}{\mathcal{M}^{(n)}} \sim \alpha\chi^{2/3}$ (except $n = 1$ or 2)
3. $\boxed{g = \alpha\chi^{2/3}}$ — effective parameter of perturbative expansion, $\chi \sim \dfrac{\varepsilon}{m}\dfrac{E}{E_S}$
4. $g \gtrsim 1$ — new *nonperturbative* regime, the main contributions should be *resummed*

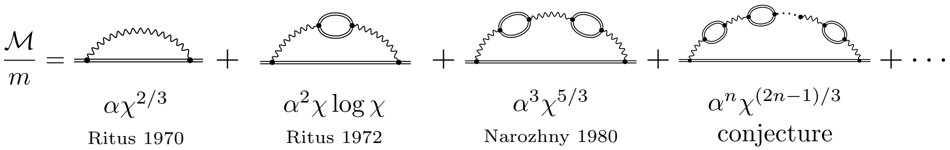


Current status of the RN conjecture

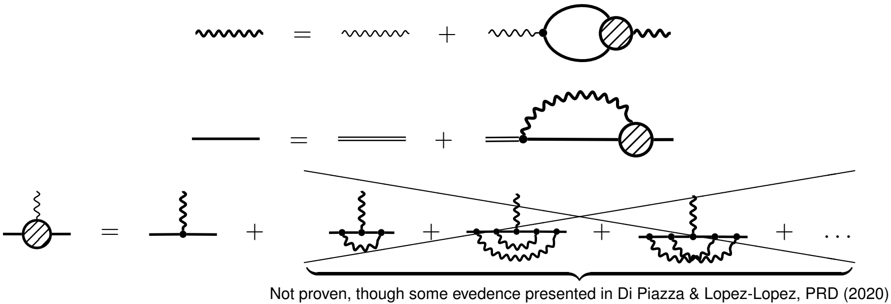
[AAM, S. Meuren and A. M. Fedotov PRD 102, 053005 (2020)]

For the electron elastic scattering amplitude:

1. Main contribution to $\mathcal{M}^{(n-loop)}$ at $\chi \gg 1$ — from polarization loop insertions
2. $\frac{\mathcal{M}^{(n+1)}}{\mathcal{M}^{(n)}} \sim \alpha\chi^{2/3}$ (except $n = 1$) for the bubble-type corrections
3. $\boxed{g = \alpha\chi^{2/3}}$ is the effective PT parameter, at least for the bubble-type corrections
4. $g \gtrsim 1$ the one-loop bubble-type corrections are resummed

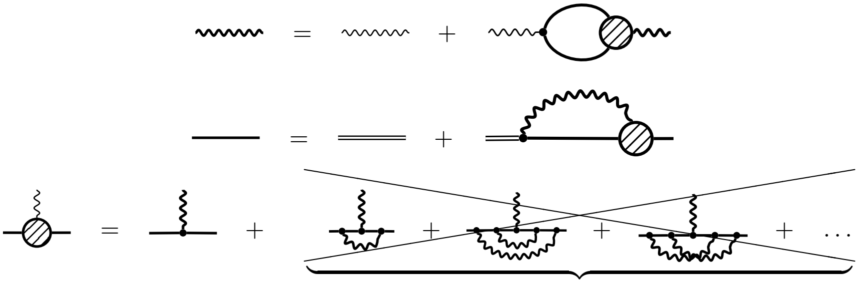


Consistent resummation: Dyson-Schwinger equations



- RN conjecture $\Longrightarrow$ DS equations become closed!
- In order to proceed, we need to:
  - (i) Define structures of the exact propagators
  - (ii) Find a proper gauge where the proper vertex $\Gamma^\mu \longrightarrow ie\gamma^\mu$
  - (iii) Calculate exact mass and polarization operators
  - (iv) Plug everything to the DS equations and try solving them

The first step towards Dyson-Schwinger equations



Not proven, though some evedence presented in Di Piazza & Lopez-Lopez, PRD (2020)

- RN conjecture $\Longrightarrow$ DS equations become closed!
- In this work we:
  - (i) Define structures of the exact propagators [Ritus 1972]
  - (ii) Find a proper gauge where the proper vertex $\Gamma^{\mu} \longrightarrow ie\gamma^{\mu}$
  - (iii) Calculate mass operator in the bubble-chain approximation and find the dominant contribution at $\chi \gg 1$
  - (iv) Plug everything to the DS equations and try solving them

The exact photon propagator

DS equations for the photon propagator

$$\left[l^2 g^{\mu\nu} - l^\mu l^\nu - \Pi^{\mu\nu}(l^2,\chi_l)\right] D_{\nu\lambda} = -i\delta^\mu_\lambda$$

We consider $\Pi^{\mu\nu}$ of general form

$$\Pi_{\mu\nu}(l) = \underbrace{\widehat{\Pi}(l^2,\chi_l)\left(l^2 g_{\mu\nu} - l_\mu l_\nu\right)}_{\propto \alpha \log \chi_l \text{ (at 1-loop)}} + \underbrace{\sum_{i=1}^{2} \Pi_i(l^2,\chi_l)\epsilon^{(i)}_\mu(l)\epsilon^{(i)}_\nu(l)}_{\text{dominant term at } \chi_l \gtrsim 1}$$

$$D^c_{\mu\nu}(l) = D_0(l^2,\chi_l)\left[g_{\mu\nu} - \cancel{(1-d_l)\frac{l_\mu l_\nu}{l^2}}\right] + \sum_{i=1}^{2} D_i(l^2,\chi_l)\epsilon^{(i)}_\mu(l)\epsilon^{(i)}_\nu(l),$$

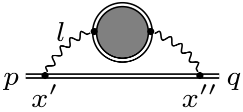
$$\epsilon^{(1)}_\mu(l) = \frac{eF_{\mu\nu}l^\nu}{m^3\chi_l}, \qquad \epsilon^{(2)}_\mu(l) = \frac{eF^\star_{\mu\nu}l^\nu}{m^3\chi_l}, \qquad \chi_l = \frac{\xi(kl)}{m^2}, \qquad \xi^2 = -e^2a^2/m^2$$

After renormalization [AAM, S. Meuren, AM Fedotov PRD 2020]

$$D_0(l^2,\chi_l) = \frac{-i}{l^2+i0}, \quad D_{1,2}(l^2,\chi_l) = \frac{i\Pi_{1,2}}{(l^2+i0)\,(l^2-\Pi_{1,2})}$$

We plug this into the electron mass operator in order to see how D^c affects its structure

Bubble-chain mass operator



$$-i\Sigma(q,p) = \Lambda^{2(D-4)} \int d^D x'\, d^D x''\, \bar{E}_q(x'')(ie\gamma^\mu) S_0^c(x'',x')(ie\gamma^\nu) E_p(x') D^c_{\mu\nu}(x'',x')$$

- E_p-representation
- Dimensional regularization
- x-representation appears to be more convenient for our calculations
- $S_0^c(x'',x')$ — LO electron propagator

$$S_0^c(x'',x') = e^{i(ax)\Phi} e^{-i\frac{\pi}{2}\frac{D-2}{2}} \frac{\Lambda^{4-D}}{(4\pi)^{D/2}} \int_0^\infty \frac{ds}{s^{D/2}} \exp\left\{-im^2 s - i\frac{x^2}{4s} + i\frac{s}{12}e^2 (Fx)^2\right\}$$
$$\times \left[m + \frac{(\gamma x)}{2s} - \frac{s}{3}e^2(\gamma F^2 x) + \frac{i}{2} mse(\sigma F) + \frac{i}{2}e(\gamma F^\star x)\gamma^5\right],$$

where $x = x'' - x'$, $X = (x' + x'')/2$, $\Phi = (kX)$, s — proper time

E_p-representation: $S_0^c(p) = \Lambda^{D-4} \int d^D x \bar{E}_p(x'') S_0^c(x'',x') E_p(x') = i\dfrac{(\gamma p)+m}{p^2 - m^2 + i0}$

Exact photon propagator in the proper time representation

Fourier-transform to $x = x'' - x'$

$$D^c_{\mu\nu}(x) = \frac{\Lambda^{4-D}}{(2\pi)^D} \int d^D l \, D^c_{\mu\nu}(l) e^{-ilx}$$

By carrying out $l_\perp^{D-2}$-integrals and passing to l^2 and proper time $t = \varphi/2(kl)$, we arrive at

$$\begin{aligned} D^c_{\mu\nu}(x) = e^{-i\frac{\pi}{2}\frac{D-4}{2}} \frac{\Lambda^{4-D}}{(4\pi)^{D/2+1}} \int_0^\infty \frac{dt}{t^{D/2}} \exp\left(-i\frac{x^2}{4t}\right) \Big\{ & \mathcal{J}_0(t,\chi_l) g_{\mu\nu} \\ & + \frac{\mathcal{J}_1(t,\chi_l)}{m^2\xi^2\varphi^2} e^2 \left[(Fx)_\mu (Fx)_\nu - 2it(F^2)_{\mu\nu}\right] \\ & + \frac{\mathcal{J}_2(t,\chi_l)}{m^2\xi^2\varphi^2} e^2 \left[(F^\star x)_\mu (F^\star x)_\nu - 2it(F^2)_{\mu\nu}\right] \Big\} \end{aligned}$$

where $\varphi = (kx)$, $\chi_l = \dfrac{\xi\varphi}{2m^2 t}$ and

$$\mathcal{J}_n(t,\chi_l) = -i \int_{-\infty}^\infty dl^2 \, D_n(l^2,\chi_l) e^{-il^2 t}, \quad n = 0,1,2$$

- $\mathcal{J}_n(t,\chi_l)$ smear causal $\theta(t)$-functions: $\mathcal{J}_n(t<0,\chi_l) = 0$
- $\mathcal{J}_n(t,\chi_l)$ contain all information about the pole structure of $D^c_{\mu\nu}$

Calculation of $\Sigma(q,p)$

The calculation is lengthy but doable, see details in `https://arxiv.org/abs/2109.00634`

We rely on computer algebra (FeynCalc), scripts are open-access (stay tuned for updates)
`https://github.com/ArsenyMironov/SFQED-Loops`

Main properties

- Diagonality in E_p-repr: $\Sigma(q,p) = \Lambda^{D-4}(2\pi)^D \delta^{(D)}(q-p)\Sigma(p,F)$
- $\Sigma \propto \alpha m \int_0^\infty \frac{ds}{s^{D/2}} \int_0^\infty \frac{dt}{t^{D/2}}\, \Gamma\, e^{i\Theta}$, where $\Gamma = \sum_{n=0}^{2} \left[\mathcal{J}_n(t,\chi_l)\Gamma_n + (\mathcal{J}_n)'_{\chi_l}\tilde{\Gamma}_n\right]$ — γ-matrix factor
- Passing to variables (u,σ): $s = \frac{1+u}{m^2\chi^{2/3}u^{1/3}}\sigma, \quad t = t(u,\sigma) = \frac{s(u,\sigma)}{u}$
- Phase factor: $\Theta = -\frac{\sigma^3}{3} - z\sigma$, where $z = \left(\frac{u}{\chi}\right)^{2/3}\left[1 - \frac{1}{u}\left(\frac{p^2}{m^2} - 1\right)\right]$
- Field-free on-shell renormalization scheme: $\Sigma(\gamma p = m, F=0) = \frac{d}{d(\gamma p)}\Sigma(\gamma p = m, F=0) = 0$

Structure of Σ

The final result is spanned by 5 γ-matrix structures

$$\Sigma(p,F) = \sum_{n=0}^{2} \Big[m S_n(p^2,\chi) + (\gamma p) V_n^{(1)}(p^2,\chi) + \frac{e^2(\gamma F^2 p)}{m^4\chi^2} V_n^{(2)}(p^2,\chi)$$
$$+ \frac{e(\sigma F)}{m\chi} T_n(p^2,\chi) + \frac{e(\gamma F^\star p)\gamma^5}{m^2\chi} A_n(p^2,\chi) \Big]$$

- All the structures but (γp) are $\mathcal{O}(m)$
- The factors $S_n(p^2,\chi)$ etc are scalars. They define the asymptotic properties at $\chi \gg 1$
- $n = 0$: coincides with the one-loop mass operator [Ritus 1970]
- $n = 1, 2$: nontrivial bubble-chain contribution

The final result is spanned by 5 γ-matrix structures

$$\Sigma(p,F) = \sum_{n=0}^{2}\Bigg[mS_n(p^2,\chi) + (\gamma p)V_n^{(1)}(p^2,\chi) + \frac{e^2(\gamma F^2 p)}{m^4\chi^2}V_n^{(2)}(p^2,\chi) + \frac{e(\sigma F)}{m\chi}T_n(p^2,\chi) + \frac{e(\gamma F^\star p)\gamma^5}{m^2\chi}A_n(p^2,\chi)\Bigg]$$

The scalar functions (just to get an impression...), $\mathcal{J}_i = \mathcal{J}_i\left(t(u,\sigma),\chi_l\right)$, $\chi_l = u\chi/(1+u)$

$$S_{1,2}(p^2,\chi) = \frac{i\alpha}{8\pi^2}\int_0^\infty \frac{du\,u}{(1+u)^2}\int_0^\infty \frac{d\sigma}{\sigma}\mathcal{J}_{1,2}\,e^{-i\frac{\sigma^3}{3}-iz\sigma},$$

$$V_{1,2}^{(1)}(p^2,\chi) = -\frac{i\alpha}{8\pi^2}\int_0^\infty \frac{du}{(1+u)^3}\int_0^\infty \frac{d\sigma}{\sigma}\mathcal{J}_{1,2}\,e^{-i\frac{\sigma^3}{3}-iz\sigma},$$

$$V_{1,2}^{(2)}(p^2,\chi) = \frac{i\alpha}{16\pi^2}\int_0^\infty \frac{du}{(1+u)^2}\int_0^\infty d\sigma\left\{2\left(\frac{u^2+2u+2}{1+u}\pm 1\right)\left(\frac{\chi}{u}\right)^{2/3}\sigma\mathcal{J}_{1,2} + \left[\left(1+\frac{2}{u}-\frac{u^2+u+2}{u(1+u)}\frac{p^2}{m^2}\right)\mathcal{J}_{1,2} + \frac{u^2+2u+2}{u^2}\tilde{\mathcal{J}}_{1,2}\right]\frac{1}{\sigma}\right\}e^{-i\frac{\sigma^3}{3}-iz\sigma},$$

$$T_{1,2}(p^2,\chi) = \frac{\alpha}{16\pi^2}\int_0^\infty \frac{du}{1+u}\int_0^\infty d\sigma\left(\frac{1}{1+u}\pm 1\right)\left(\frac{\chi}{u}\right)^{1/3}\mathcal{J}_{1,2}e^{-i\frac{\sigma^3}{3}-iz\sigma},$$

$$A_{1,2}(p^2,\chi) = -\frac{\alpha}{8\pi^2}\int_0^\infty \frac{du}{(1+u)^2}\int_0^\infty d\sigma\left(\frac{1}{1+u}\pm 1\right)\left(\frac{\chi}{u}\right)^{1/3}\mathcal{J}_{1,2}e^{-i\frac{\sigma^3}{3}-iz\sigma}.$$

Bubble-chain electron propagator

DS equation for an electron in E_p-repr:

$$-iD(p,F)S^c(p,F) = -i\left[(\gamma p) - m - \Sigma(p,F)\right]S^c(p,F) = 1$$

Solution in a general form [Ritus 1972]:

$$S^c(p,F) = i\left[mS - (\gamma p)V^{(1)} - \frac{e^2(\gamma F^2 p)}{m^4\chi^2}V^{(2)} - \frac{e(\sigma F)}{m\chi}T + \frac{e(\gamma F^\star p)\gamma^5}{m^2\chi}A\right]\sum_{\pm}\frac{1 \pm (\gamma n)\gamma^5}{2D_\pm},$$

$$D_\pm = m^2S^2 - p^2V^{(1)\,2} + m^2\left(A^2 - 2V^{(1)}V^{(2)}\right) \pm 2m^2\left(SA - 2TV^{(1)}\right)$$

where $n_\mu = e(F^\star p)_\mu/m^3\chi$, and $D_\pm = D_\pm(p^2,\chi)$ factorize $\det D(p,F) = D_+D_-$, and

$$S = -1 - \sum_{n=0}^{2}S_n,\ V^{(1)} = 1 - \sum_{n=0}^{2}V_n^{(1)},\ V^{(2)} = -\sum_{n=0}^{2}V_n^{(2)},\ T = -\sum_{n=0}^{2}T_n,\ A = -\sum_{n=0}^{2}A_n.$$

Has infinite number of physical poles $D_\pm(p^2,\chi) = 0$

The same structures as in Σ! Asymptotic properties are defined by $S_n(p^2,\chi)$ etc

Bubble-chain scattering amplitude

$$T_s(p) = -\mathcal{M}(\chi)/(2p^0), \quad \mathcal{M}(\chi) \equiv \bar{u}_{p,\lambda}\Sigma(p,F)|_{p^2=m^2}u_{p,\lambda}$$

Applying the rules

$$\bar{u}_p(\gamma p)u_p = 2m, \quad \bar{u}_p\, e^2(\gamma F^2 p)\, u_p = 2m^6\chi_p^2, \quad \bar{u}_p\, e(\sigma F)\, u_p = 4s^\mu e F^*_{\mu\nu} p^\nu,$$

we obtain

$$\mathcal{M}(\chi) = \sum_{n=0}^{2} \Big\{ 2m^2 \left[S_n(m^2,\chi) + V_n^{(1)}(m^2,\chi) + V_n^{(2)}(m^2,\chi) \right] + \frac{2e(sF^\star p)}{m\chi} \left[2T_n(m^2,\chi) + A_n(m^2,\chi) \right] \Big\}.$$

$\mathcal{M} = \mathcal{M}^{(0)} + \delta\mathcal{M}_1 + \delta\mathcal{M}_2$ as in [AAM, S. Meuren, AM Fedotov PRD 2020]

Again, the asymptotic properties are defined by $S_n(p^2 = m^2, \chi)$ etc

$$S^c(p,F) = i\left[mS - (\gamma p)V^{(1)} - \frac{e^2(\gamma F^2 p)}{m^4\chi^2}V^{(2)} - \frac{e(\sigma F)}{m\chi}T + \frac{e(\gamma F^\star p)\gamma^5}{m^2\chi}A\right]\sum_{\pm}\frac{1\pm(\gamma n)\gamma^5}{2D_\pm},$$

Consider $n = 1, 2$. Let us find the leading contribution at $\chi \gg 1$

- $T_{1,2}$, $A_{1,2} \propto \chi^{1/3}$, $V^{(2)}_{1,2} \propto \chi^{2/3} \Longrightarrow \boxed{T,\, A \ll V^{(2)}}$
- $S_{1,2} \sim V^{(1)}_{1,2}$

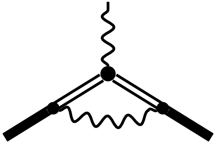
$$V^{(1)}_{1,2} \sim \frac{\alpha\Pi_{1,2}(0,\chi)}{m^2\chi^{2/3}}\int_0^1 d\sigma e^{-i\sigma^3/3 + i\sigma\nu/m^2\chi^{2/3}}, \quad \nu = p^2 - m^2$$

- At $\nu \ll m^2\chi^{2/3}$: $V^{(1)}_{1,2} \sim \alpha\Pi_{1,2}(0,\chi)/m^2\chi^{2/3} \ll \alpha \ll V^{(2)}$
- At $\nu \gg m^2\chi^{2/3}$: $V^{(1)}_{1,2} \sim \alpha\Pi_{1,2}(0,\chi)/\nu, \quad (\gamma p)V^{(1)}_{1,2} \sim \alpha\Pi_{1,2}(0,\chi)$
  but one term in $V^{(2)}_{1,2}$: $\tilde{V}^{(2)}_{1,2} \sim \frac{\alpha}{m^2}\chi\Pi_{1,2}(0,1) \Longrightarrow \boxed{S,\, V^{(1)} \ll V^{(2)}}$
- $V^{(2)}$ dominates in the bubble-chain approximation

$$\Sigma(p,F) \propto \frac{e^2(\gamma F^2 p)}{m^4\chi^2}V^{(2)}, \quad S^c(p,F) \propto \frac{e^2(\gamma F^2 p)}{m^4\chi^2 D_\pm}V^{(2)}, \qquad \underbrace{\mathcal{M}(\chi) \approx 2m^2V^{(2)}}_{\text{coincides with [AAM, SM, AMF PRD 2020]}}$$

Back to the Ritus-Narozhny conjecture

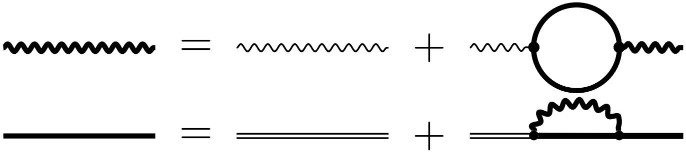
Suppose we insert 1-loop correction into a vertex connecting two bubble-chain S^c and $\chi \gg 1$



- Leading contribution $=$ LO term in Γ^μ $\times$ LO term in S^c

  If this is true, vertex insertion will enhance the total amplitude by $g = \alpha\chi^{2/3}$
- Γ^μ: dominant $\mathcal{O}(g)$ contribution is $\propto (\gamma k)k^\mu$ [Morozov 1981, Di Piazza PRD 2020]
- S^c: dominant contribution is $\propto (\gamma F^2 p)V^{(2)} = -a^2(\gamma k)(kp)V^{(2)}$
- HOWEVER: $(\gamma k)k^\mu \times (-a^2)(\gamma k)(kp)V^{(2)} \propto (\gamma k)^2 = 0$
- Therefore, the LO nonvanishing contribution should be enhanced by a factor weaker than $\alpha\chi^{2/3}$!
- This supposition is in favour of the RN conjecture and the bubble-chain approximation (yet to be confirmed by a full-length calculation)

Summary and outlook



- To resum radiative corrections in the nonperturbative regime $g \gtrsim 1$ consistently, we need to:
  - (i) $+$ Define structures of the exact propagators
  - (ii) ? Find a proper gauge where the proper vertex $\Gamma^\mu \longrightarrow ie\gamma^\mu$
  - (iii) $\pm$ Calculate exact mass and polarization operators
  - (iv) ? Plug everything to the DS equations and try solving them
- Next step: plug the bubble-chain electron propagator into Σ and $\Pi^{\mu\nu}$
- Technically, the calculations will be the same as presented here (for Σ). The structures will remain, scalar coefficients will change
- Scalar coefficients in S^c and D^c will be connected $\Longrightarrow$ bubble-chain DS equations

Manuscript with details: `https://arxiv.org/abs/2109.00634`
Open-access scripts: `https://github.com/ArsenyMironov/SFQED-Loops`

THANK YOU FOR YOUR ATTENTION!

Volkov solution, $(\not{\hat{p}} + e\not{A} - m)\,\psi_p = 0$

- CCF: $A^\mu = a^\mu \varphi, \quad \varphi = kx, \quad k^2 = ka = 0$

- Solution: $\boxed{\psi_{p,\sigma}(x) = E_p(x) u_{p,\sigma}}, \qquad (\not{p} - m) u_{p,\sigma} = 0$

$$E_p(x) = e^{iS_p}\Sigma_p \quad \text{– Ritus } E_p\text{-function}$$

$$\Sigma_p = 1 + \frac{e}{2(kp)}\not{k}\not{A} \quad \text{– spin factor (matrix 4x4)}$$

$$S_p(x) = -px + \frac{e(ap)}{2(kp)}\varphi^2 + \frac{e^2 a^2}{6(kp)}\varphi^3 \text{ — classical action}$$

- Properties of E_p-functions:

$$\int d^4x \overline{E}_p(x) E_q(x) = (2\pi)^4 \delta^{(4)}(p-q),$$

$$\int \frac{d^4p}{(2\pi)^4} E_p(x)\overline{E}_p(y) = \delta^{(4)}(x-y), \quad \overline{E}_p = \gamma^0 E_p^\dagger \gamma^0,$$

$$\hat{\not{\mathcal{P}}} E_p = E_p \not{p}, \quad \hat{\not{\mathcal{P}}} = \hat{\not{p}} - e\not{A}$$

$$\mathcal{J}_n(t,\chi_l) = -i\int_{-\infty}^{\infty} dl^2\, D_n(l^2,\chi_l)e^{-il^2t}, \quad n = 0,1,2$$

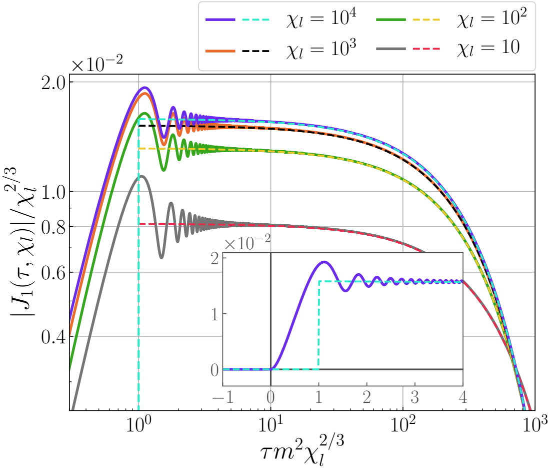
$$\tilde{\mathcal{J}}_{1,2}(t,\chi_l) = \frac{i\,[\mathcal{J}_n]'_t}{m^2} = -i\int_{-\infty}^{\infty} dl^2\,\frac{l^2}{m^2} D_{1,2}(l^2,\chi_l)e^{-il^2t}$$

Approximation:

$$\mathcal{J}_{1,2}(t,\chi_l) \approx -2\pi i\theta\,(\mathrm{Re}\,t - t_{\text{eff}})\left[e^{-i\Pi_{1,2}(0,\chi_l)t} - 1\right],$$

$$\tilde{\mathcal{J}}_{1,2}(t,\chi_l) \approx -2\pi i\theta\,(\mathrm{Re}\,t - t_{\text{eff}})\,\frac{\Pi_{1,2}(0,\chi_l)}{m^2}e^{-i\Pi_{1,2}(0,\chi_l)}$$

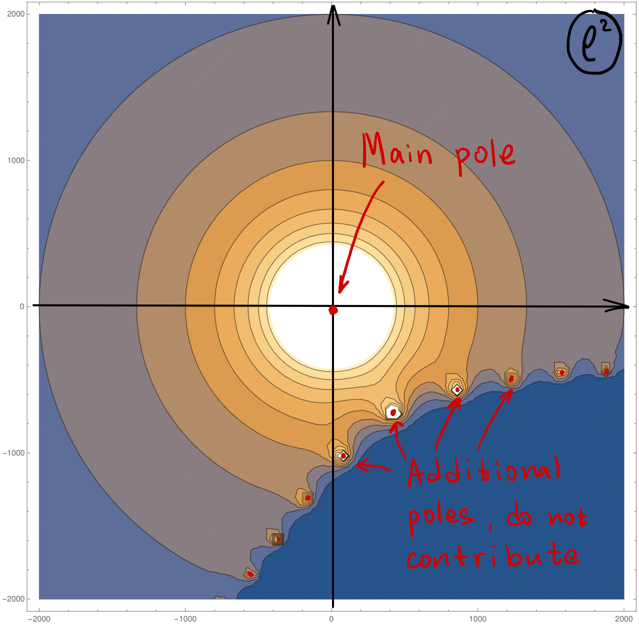
Valid at $\boxed{\chi_l \gtrsim 1}$ $(u\chi \gtrsim 1)$



Analytic properties of $\Pi^{\mu\nu}(l^2, \chi_l)$ in l^2

Plot: $\dfrac{1}{|l^2 - \Pi_1(l^2, \chi_l)|}$ in l^2-plane

- $\Pi_{1,2}$ are whole transcendent functions of l^2
- Main pole: $l^2 = -i0$
- Can be shown with an alternative approach by resummation of PT series, see [AAM, S. Meuren and A. M. Fedotov PRD 102, 053005 (2020)]



Bubble-chain electron propagator in proper time representation

$$S^c(x'',x') = \Lambda^{4-D} \int \frac{d^D p}{(2\pi)^D} E_p(x'') S^c(p,F) \bar{E}_p(x')$$

$$\begin{aligned} S^c(x'',x') =& e^{i(ax)\Phi} e^{-i\frac{\pi}{2}\frac{D-4}{2}} \frac{\Lambda^{4-D}}{(4\pi)^{D/2+1}} \int_0^\infty \frac{ds}{s^{D/2}} \exp\left\{-i\frac{x^2}{4s} + i\frac{s}{12}e^2 (Fx)^2\right\} \\ & \times \left[m\mathcal{S}(s,\varphi) + \frac{(\gamma x)}{2s}\mathcal{V}^{(1)}(s,\varphi) + \frac{e^2(\gamma F^2 x)}{m^2\xi^2\varphi^2}\mathcal{V}^{(2)}(s,\varphi) + \frac{e(\sigma F)}{m\xi\varphi}\mathcal{T}(s,\varphi) \right. \\ & \left. + \frac{e(\gamma F^\star x)\gamma^5}{\xi\varphi}\mathcal{A}(s,\varphi) + \frac{e(\gamma F^\star x)\gamma^5(\gamma x)}{2ms\xi\varphi}\mathcal{A}'(s,\varphi) \right] \end{aligned}$$