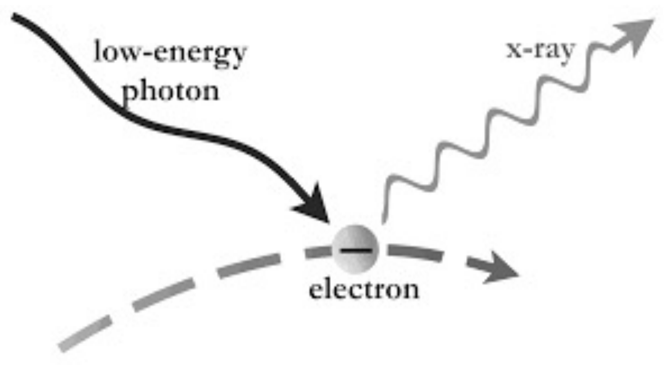
Imperial College London

Applications of Bayesian Inference in Radiation Reaction Experiments

E. E. Los, S. P. D Mangles

Definition: Recoil of accelerating charge upon emission of radiation.

Aim: Use Bayesian inference to compare different models of radiation reaction in a high field regime, where quantum effects are significant, against experimental data.



Apatruno.files.wordpress.com, 2019

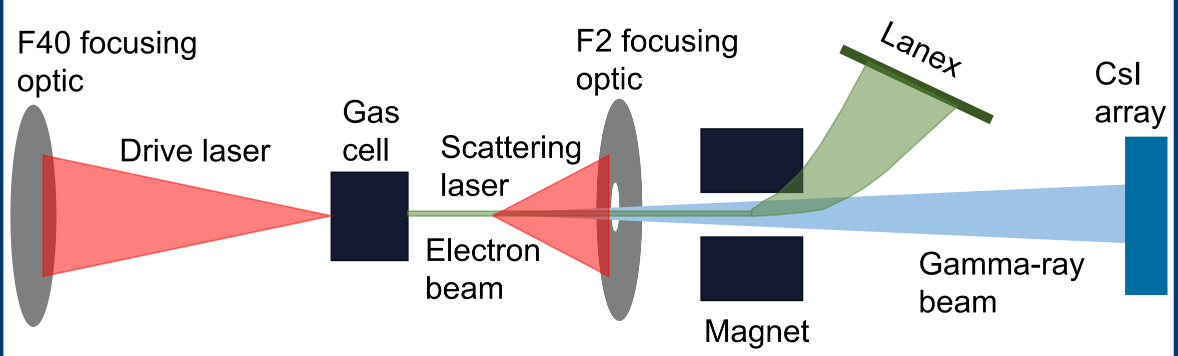
Motivation

High fields environments, e.g.:

- Next generation of lasers
- Astrophysics: e.g. the surfaces of quasars

Schwinger Field: $E_c \approx 1.32\times10^{18}$V/m.

Experiment



Radiation Reaction Models: Single Particle

Classical/Semiclassical

$$\frac{d\vec{p}}{dt} = \vec{F}_L + g^X P_{cl}(\eta)\hat{p}$$

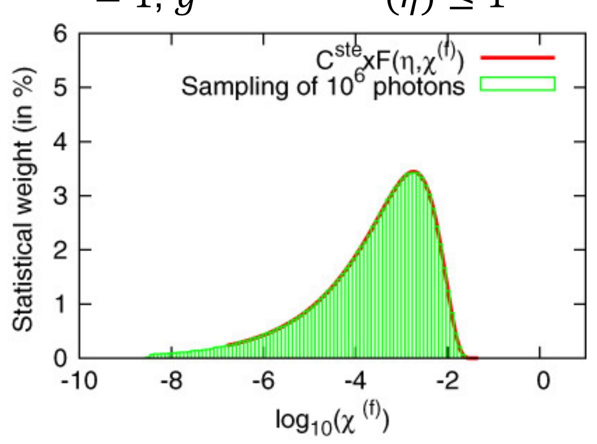
where $\eta \approx \frac{\gamma}{E_{crit}}\left|E_\perp + \vec{v}\times\vec{B}\right|$, $g^{classical} = 1$, $g^{semiclassical}(\eta) \le 1$ NB: $\gamma \gg 1$

Stochastic:

$$\frac{d\vec{p}}{dt} = \vec{F}_L,\ \frac{d\tau_e}{dt} = I(\eta,\chi)$$

$$\chi = \frac{\eta}{\gamma}\frac{h\nu}{2mc^2}.\ \text{When } \tau_e = \tau_e{}^f,$$

$$\vec{p}_{new} = \vec{p}_{old} - \frac{h\nu}{c}\hat{p}$$



Radiation Reaction Models: Charge Distribution

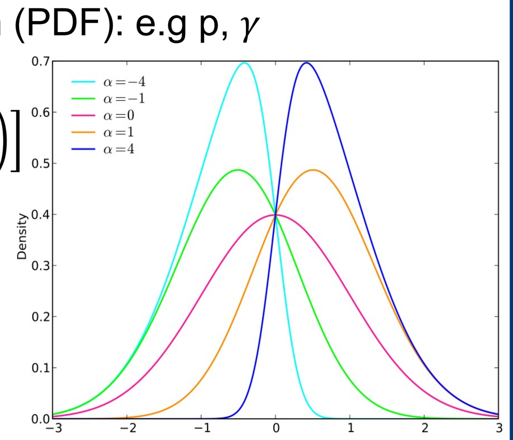
From kinetic theory: n^{th} moments $\langle g^n\rangle = \frac{1}{n_e}\int d^3\vec{p} f(t,\vec{x},\vec{p}) g^n$

$f(t,\vec{x},\vec{p})$ = probability density function (PDF): e.g p, γ

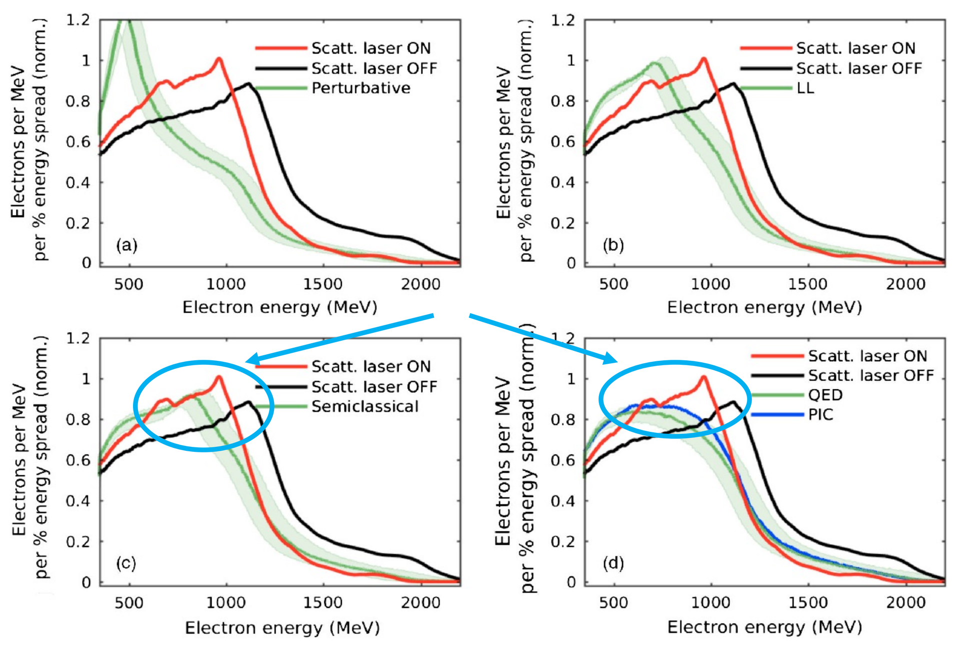
Gaussian PDF:

$$f(\gamma) = \frac{1}{\sqrt{2\pi}\omega} e^{-\frac{(\gamma-\zeta)^2}{2\omega^2}}\left[1+\text{erf}\left(\alpha\left(\frac{\gamma-\zeta}{\sqrt{2}\omega}\right)\right)\right]$$

$$\frac{d}{dt}\begin{cases}\langle\gamma\rangle \\ \sigma^2 = \langle(\gamma-\langle\gamma\rangle)^2\rangle \\ \mu_3 = \left\langle\left(\frac{\gamma-\langle\gamma\rangle}{\sigma}\right)^3\right\rangle\end{cases}$$



Experimental Observation of Radiation Reaction



(K. Poder et al., 2018)

Missing/inaccurate collision parameters, e.g. chirp?

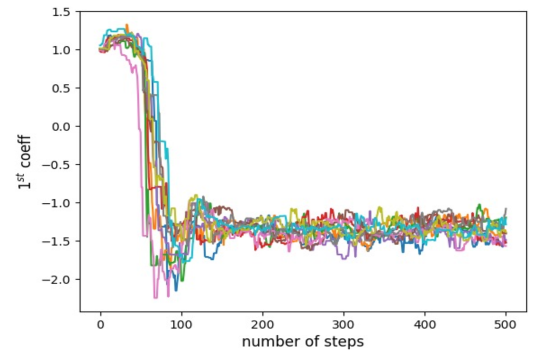
How (much) does knowledge of collision parameters affect ability to determine which radiation reaction model best explains data?

Inaccuracies in models of RR

Bayesian Inference

$$P(M|D) = \frac{P(D|M)P(M)}{P(D)}$$

M=model, D=data



Model Comparison

Bayesian information criterion, (BIC)=ln(n)k-2ln(likelihood), n=no. data points, k=no. fitted parameters. Model with BIC closest to -∞ wins.

BIC model2-BIC model1	Interpretation
0-2	Neither model favoured
2-6	M_1 favoured
6-10	M_1 strongly favoured

Modelling Radiation Reaction: Gaussian Decomposition

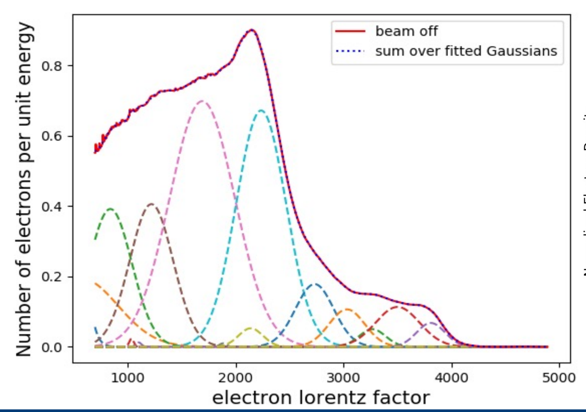
Every step of Markov Chain, need:

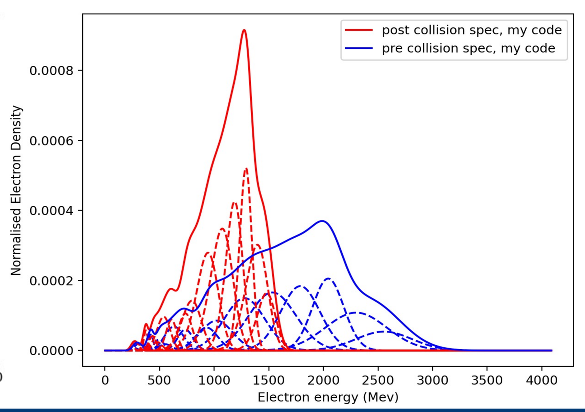
Input electron spectrum => output electron spectrum

PIC, kinetic model <= **computationally expensive**

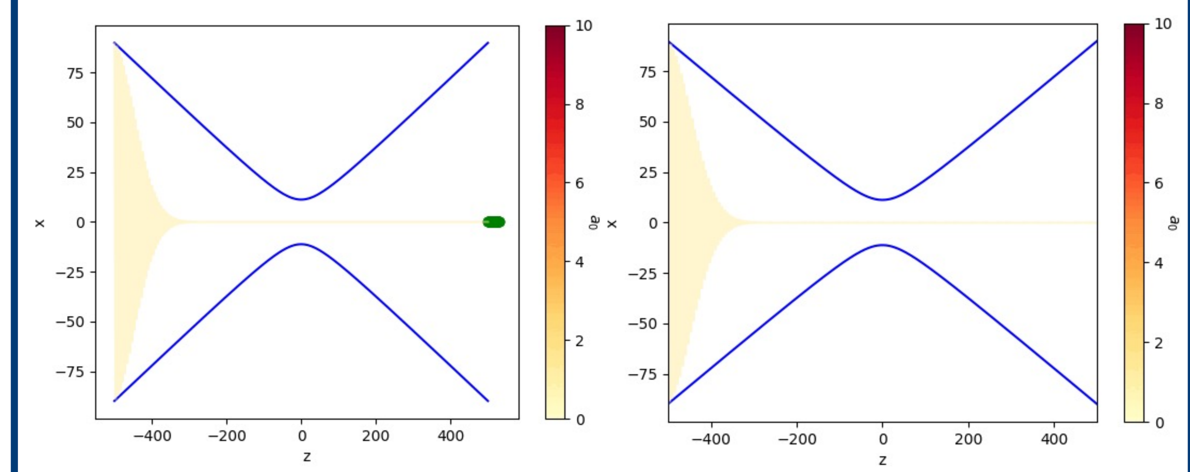
Solution: Gaussian decompositon

Interpolation tables: $\langle\gamma\rangle_{initial}$=> $\langle\gamma\rangle_{final}$, $\sigma_{initial}$=> σ_{final}



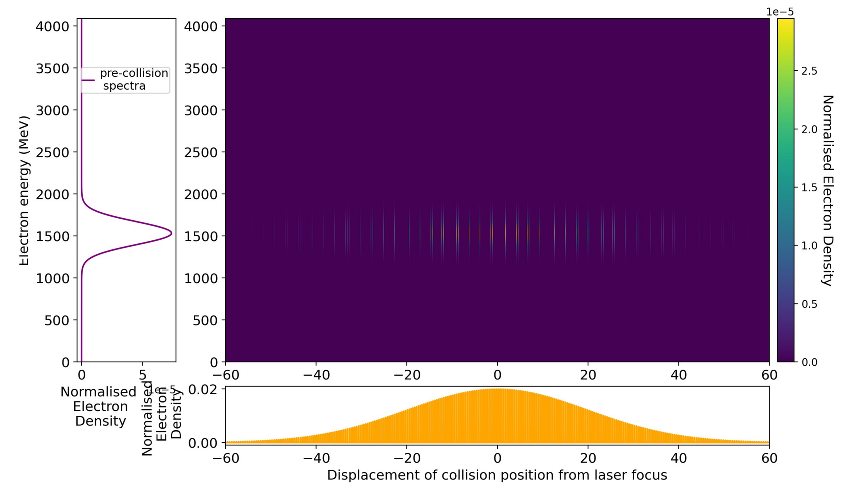


Effect of electron spatial distributions on the post-collision electron spectra

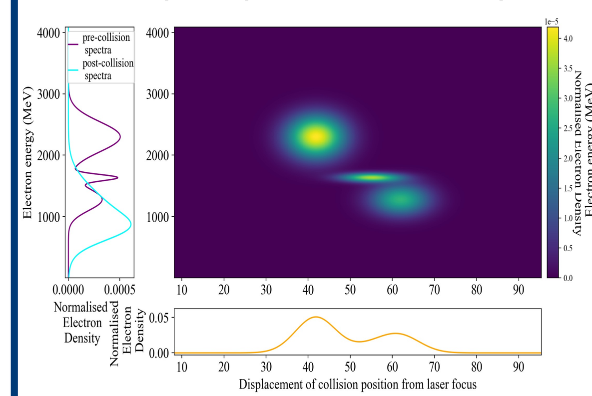


Electron energy spectrum decomposition & projection along longitudinal axis

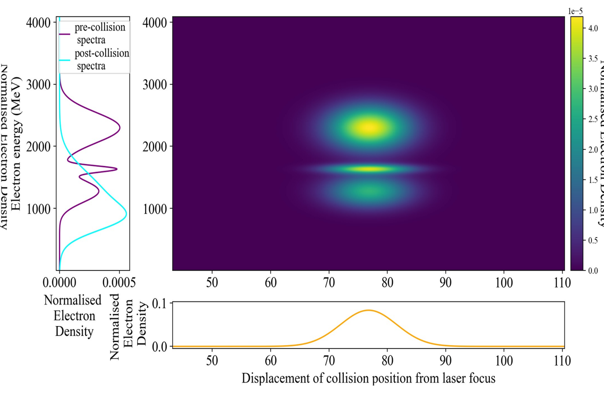
For electron spectra modelled by multiple gaussians, apply this method to every gaussian



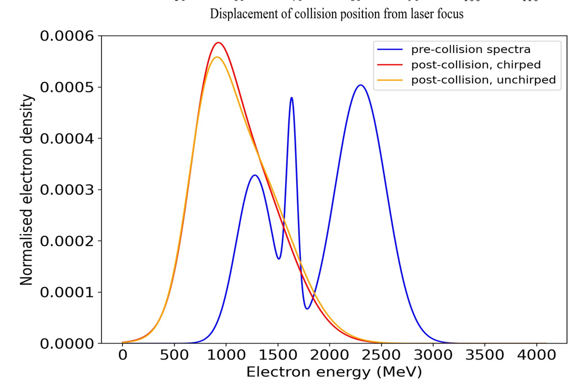
Example spectra with chirp



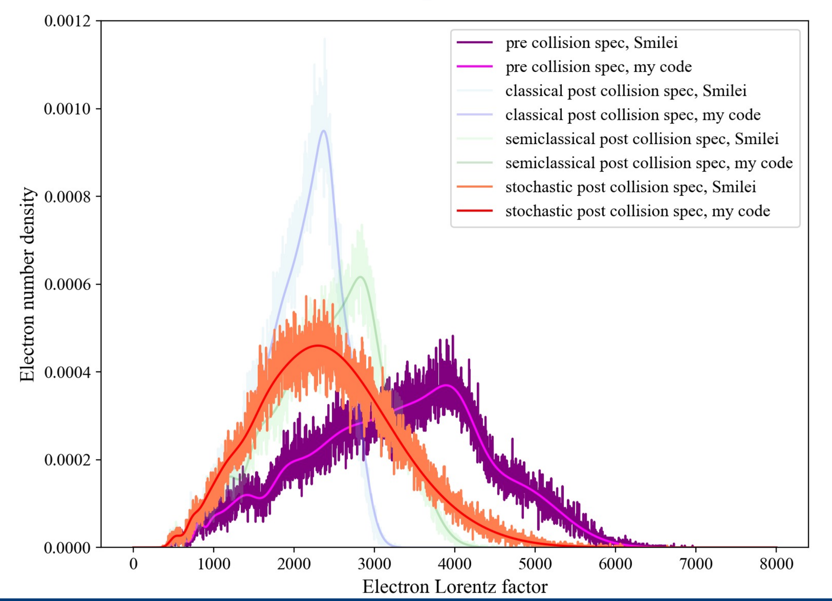
Example spectra without chirp



Comparison of post-collision spectra for chirped and un-chirped electron beams.

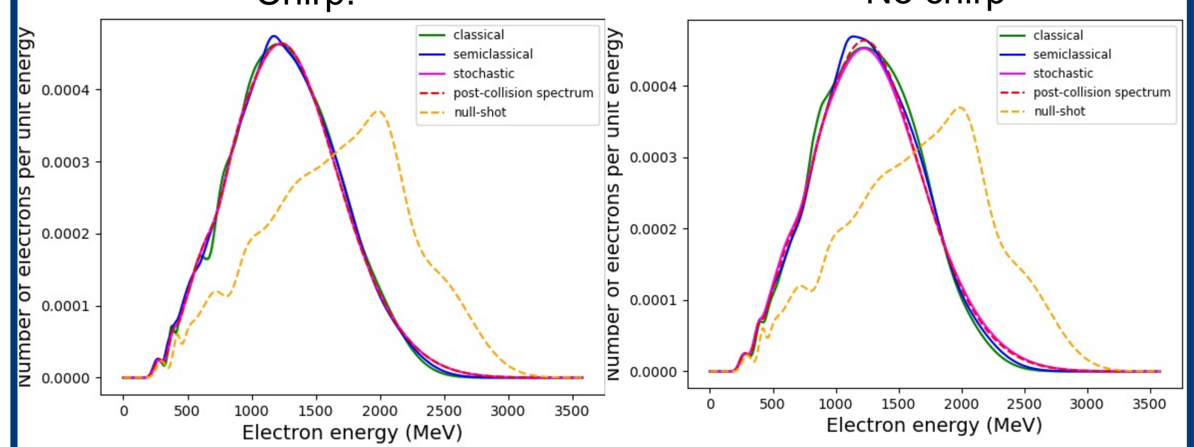


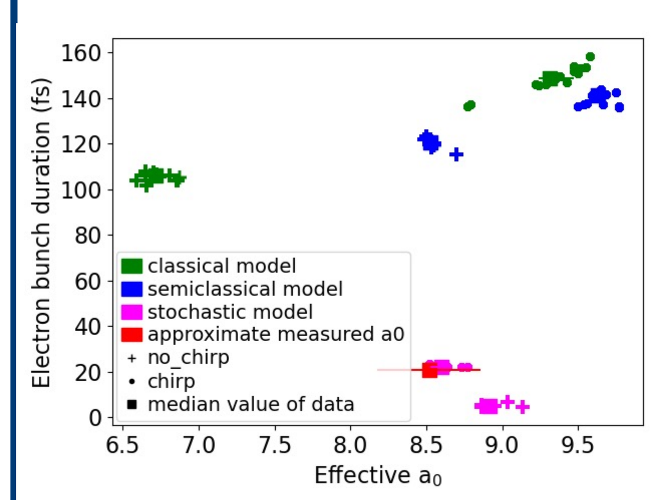
Benchmarks: my code vs Smilei

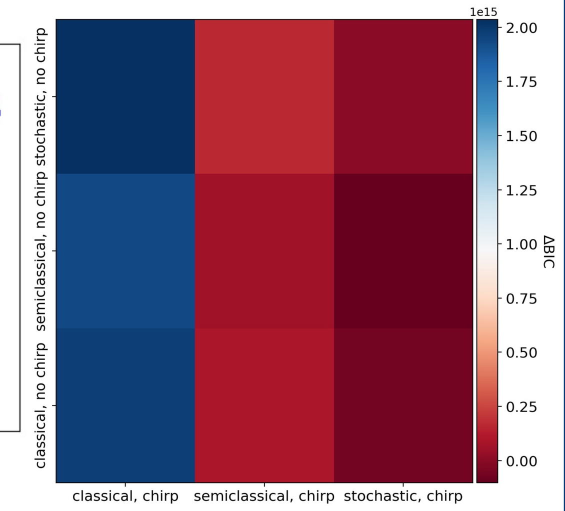


Results: retrieval of post-collision spectrum using linear model

Chirp: No chirp:



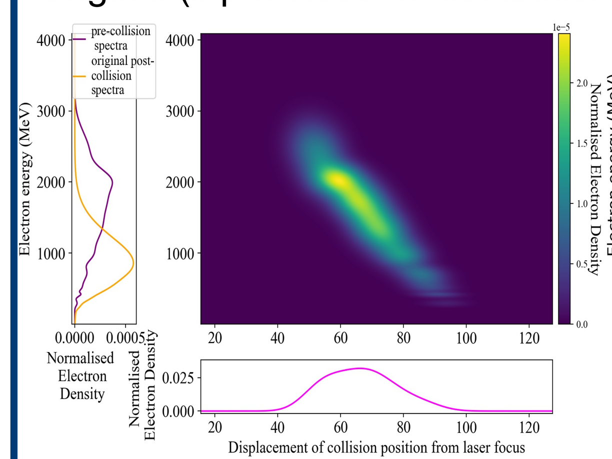


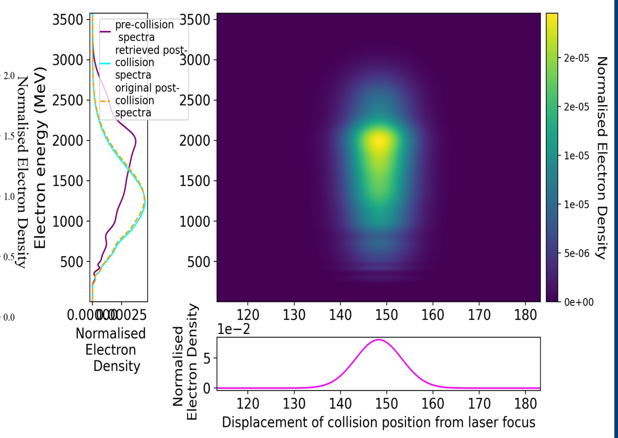


Stochastic model with chirp falls well within 1 std of true parameters, stochastic model without chirp falls just within 1 std of true parameters

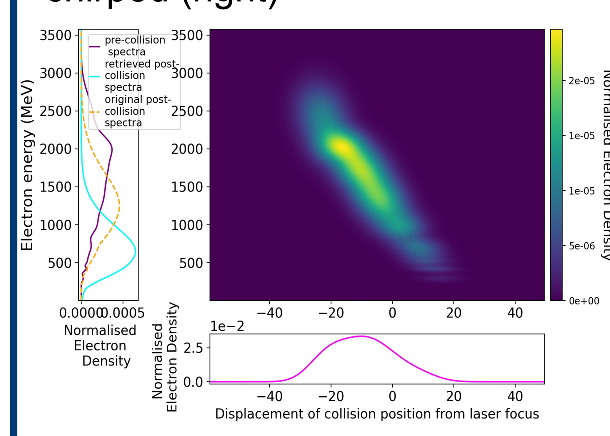
If ΔBIC< 0 (i.e. Red), model with chirp wins.
If ΔBIC> 0 (i.e. Blue), model without chirp wins.

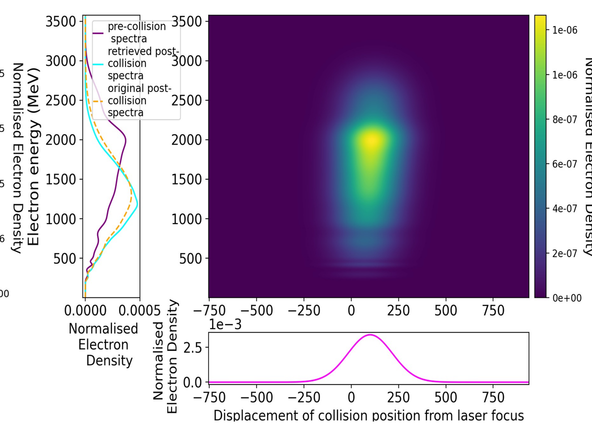
Original (input electron distribution) Retrieved: stochastic, no chirp





Retrieved spectrum: Stochastic, chirped (left), semiclassical, chirped (right)





Future Work:

Apply to experimental data

References:

R. Ruffini *et al* 2010, Phys. Rep. 487, 1

J. Sultana *et al* 2013, Astrophys. J. 779, 16.

I. C. E. Turcu *et al.* 2016, Romanian reports in Physics 68, S145