## Abstract

We theoretically and numerically demonstrate the generation of a relativistic modified Bessel-Gaussian beam (MBGB) via plasma-based beam braiding. It is realized by injecting several intense Gaussian pulses
with well- designed offsets and angles into an underdense plasma channel which acts as a laser-pulse combiner via refractive coupling. The MBGB propagates stably in the plasma channel with a well-controlled with well- designed offsets and angles into an underdense plasma channel which acts as a laser-pulse combiner via refractive coupling. The MBGB propagates stably in the plasma channel with a well-controlled
orbital angular momentum of large value exciting a twisted plasma wave. After leaving the plasma, it becomes unguided and survives in vacuum for at least hundreds of femtoseconds. This method is insensitive orbital angular momentum of large value, exciting a twisted plasma wave. After leaving the plasma, it becomes unguided and survives in vacuum for at east hundreds of femtoseconds. This method is insensitive
to the initial laser injection conditions and thus should be robust for experimental implementation. It provides an alternative approach in generating high-quality tunable intense optical vortex beams which are to the initial aser injection cond
desired for various applications.



FIG. 1 (a) Schematic of PBB scheme in a parabolic plasma channel with three Gaussian pulses (green beamlets). Each pulse undergoes a centroid oscillation around the channel center along the $z$ axis (green arrow) with a helical trajectory (orange line). A plasma wave is excited with a twisted structure (blue surface). Electrons can be trapped and accelerated to form the structured beam (yellow beamlets at the back). (b) Transverse slice at the position indicated by the gray frame in (a). One of the laser pulses (green spots) is indicated with its initial offset $r_{j, c 0}$ and centroid tilt $\psi_{j}$. The transverse density gradient of the plasma channel is indicated by the blue-white color. (c), (d) Numerical result of the initial field structure of the first two vortex submodes. of the MBGB, $|n|=0,1$. The white curves are the plots of the field along the $x$ axis at $y=0$.

## Centroid Oscillation

We are proposing a new approach in which a relativistic OVB is generated in plasma from focused gaussian beams and, at the same time, promises high beam-quality for in- situ applications in the plasma. It happens in underdense micro-scale parabolic plasma channels which have been widely used in lasma wakefield acceleration for stably guiding the intense laser pulses wel equally arranged along a circle and obliquely injected into the plasma channel simultaneously. A schematic of the method with initially three gaussian pulses is shown in Fig.1. since it experiences asymmetric transverse focusing force periodically, each of them undergoes a centroid oscillation in cylindrical coordinate $(r, \phi)$ as

$$
\tilde{a}_{j}(r, \phi, \tau)=a_{0, j} e^{-\frac{r^{2}+r_{j, c 0}^{2}-2 r r_{j, c 0} \cos \left(\Omega_{j, c} \tau+\phi+\psi_{j}\right)}{w_{j, 0}^{2}}}
$$

where the helical guiding imprints the transverse tilt $\psi_{j}$ on the pulse front. $a_{j, 0}$ is the normalized $j^{t h}$ laser strength and $\Omega_{j}$ is its centroid oscillation frequency depending on the laser focusing spotsize $w_{j, 0}$ and plasma density channel.

## Laser field in superposition

The superposition of all the injected pulses with the same frequency, initial spot size $w_{j, 0}=w_{0}$, and trans- verse offset $r_{j, c 0}=r_{c 0}$, but different $\psi_{j}=2 \pi j / l$, is written as

$$
\begin{align*}
\tilde{a}(r, \phi, \tau) & =\sum_{j=1}^{l} \tilde{a}_{j}(r, \phi, \tau)  \tag{2}\\
& =C_{0} l e^{-\frac{r^{2}}{w_{0}^{2}}} \sum_{n=-\infty}^{+\infty} I_{n l}\left(\frac{2 r_{c 0} r}{w_{0}^{2}}\right) e^{i n l\left(\Omega_{c} \tau+\phi+\psi_{0}\right)}
\end{align*}
$$

where $l$ is chosen such that $l<2 \pi r_{c 0} / w_{0}$ to avoid significant beam overlap and the accompanying pulse-pulse interactions. $C_{0}=a_{0} e^{r_{c o}^{2} / w_{0}^{2}}$, and $I_{n l}$ are the modified Bessel functions of the first kind. $\psi_{0}$ represents the initial reference phase which can be arbitrarily set to zero. The composed laser pulse describe index number nl. Each carries a well-defined OAM characterized by the Hilbert factor $e^{i n l \phi}$ except the mode $n=0$. The field structures of the first two modes are shown in FIG.1(c) and 1(d), where the intensity is dominated by the first mode $n=0$ and the vortex structure by the second mode $|n|=1$ depending on the number of subpulses. The total OAM carried can be calculated as
(3)

$$
L_{z}=\int M_{z} d^{3} \boldsymbol{r}=2 \pi C_{0}^{2} l^{4} \Omega_{c} \sigma_{t} \sum_{n} n^{2} \mathcal{A}_{n}
$$



FIG. 3. The generated structured wakefield shows an $l$-lobe structure which is inherited from the intensity structure of the MBGB. Plasma wakefield at $\tau=32$ (or $z^{\prime}=170 \mu \mathrm{~m}$ ). (a) Initial intensity structure of MBGB in of the background plasma density distribution in the $x z$ plane at $y=0$. Red and blue curves present the plot along the $z$ axis at $x=0$ and $x=15$, respectively. The dashed red ellipses indicate the region of the lobe structure of the plasma wave. (c) Slice of longitudinal field $E_{z}$. The solid black curve shows the plot of $E_{z}$ along the $z$ axis at $x=0$ and the dashed magenta line shows the numerical approximation in the first bubble region. (d) Slice of transverse focusing field $W_{r}$, which is capable of positively charged particle acceleration. (e) Density distribution of a twisted electron beam at $\tau=72$ in 3D. (f) Slice of an axial magnetic field after the MBGB. The field along the $z$ axis at $x=0$ (indicated by the dashed line) is plotted by the blue curve, with a maximum value close to 10 MG

## Reference

[1] Bifeng Lei et.al., PHYSICAL REVIEW A 104, L021501 (2021)

