Dynamically Assisted Tunneling

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Time-dependent Tunneling

S. Coleman: *"Every child knows..."*

$P \sim \exp\left[-2\int dx\,\sqrt{2m[V(x)-E]}/\hbar\right]$

Question: $V(x) \to V(t,x)$???

Here: $V(t,x) = V_0(x) + xV_1'(t)$

- pre-acceleration
- energy mixing
  $\to$ Franz-Keldysh effect
  $E \to E + \hbar\omega$ (Floquet ansatz)

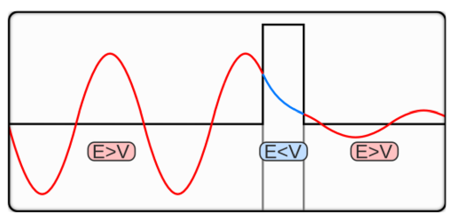
W. Franz, Z. Naturforsch. **13** A, 484 (1958);
L. V. Keldysh, Sov. Phys. JETP **34**, 788 (1958).

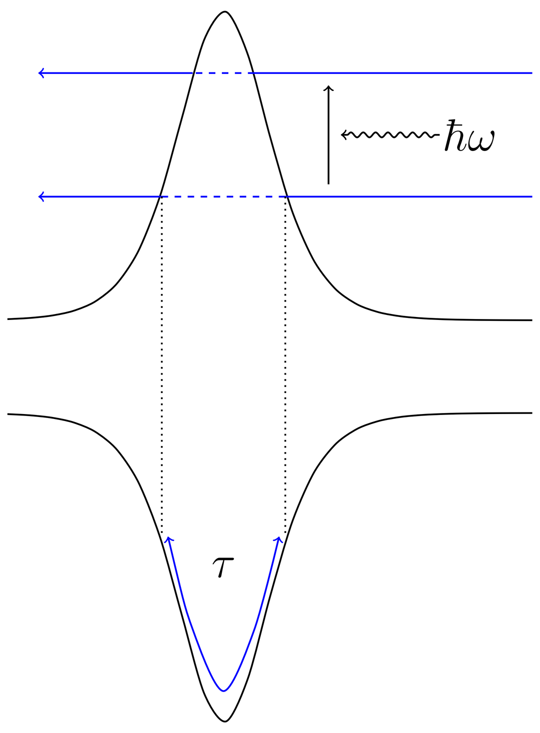
- potential deformation

Adiabatic versus non-adiabatic:
Büttiker-Landauer "traversal" time

$\mathfrak{T} = \sqrt{m}\int dx\,/\sqrt{2[V(x)-E]}$

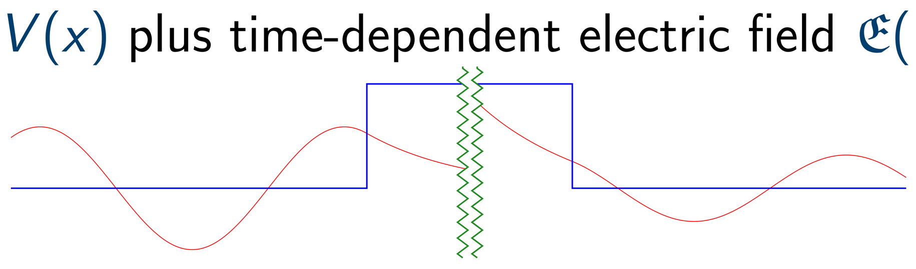
M. Büttiker and R. Landauer, PRL **49**, 1739 (1982).



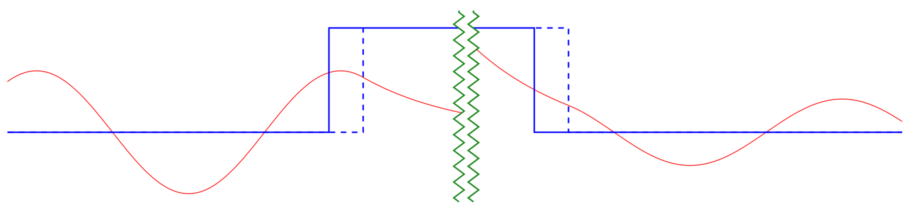


Dynamically Assisted Tunneling

Potential barrier $V(x)$ plus time-dependent electric field $\mathfrak{E}(t)$



Kramers-Henneberger displacement $m\ddot{\chi}(t) = q\mathfrak{E}(t)$



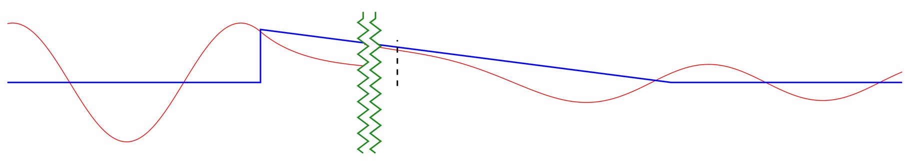
Energy mixing $\chi(t + i\mathfrak{T})$ displacement ("pushing out") $\chi(t)$

$$\psi_{\rm tra}(E) \approx \psi_E^0 \int \frac{dt}{2\pi}\, e^{i(E-E_{\rm in})t - \sqrt{2mV_0}[\chi(t+i\mathfrak{T})-\chi(t)]}$$

Low-energy + opaque-barrier approximation

Triangular Barrier $\to$ Quantum Ratchets

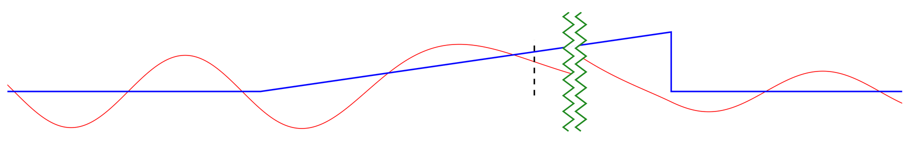
Steep incidence



Mainly energy mixing at front end

$$\psi_{\rm tra}(E) \approx \psi_E^0 \int \frac{dt}{2\pi}\, e^{i(E-E_{\rm in})t - \sqrt{2mV_0}\chi(t+i\mathfrak{T})}$$

Gradual incidence

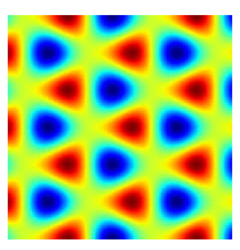


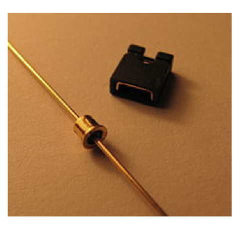
Mainly displacement at rear end

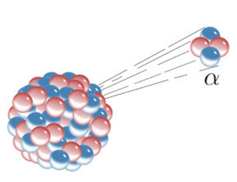
$$\psi_{\rm tra}(E) \approx \psi_E^0 \int \frac{dt}{2\pi}\, e^{i(E-E_{\rm in})t + \sqrt{2mV_0}\chi(t)}$$

$\to$ quantum ratchets

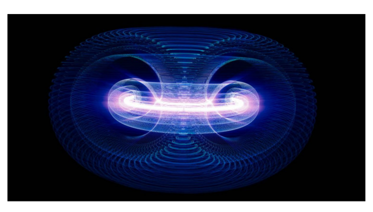
Rough Scaling Analysis

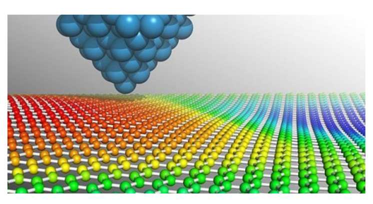


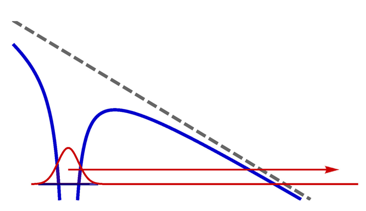




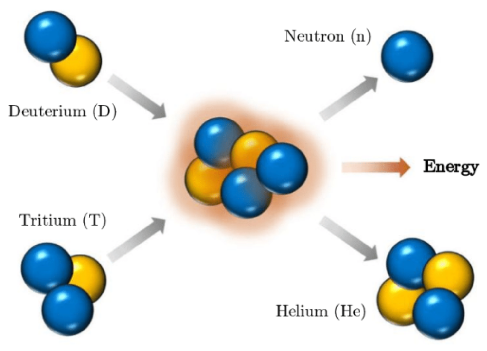
Length	System	Energy	Field Strength
μm	optical lattices	peV	n.a.
nm	solids	meV	10^5 V/m
	atoms	eV	10^{10} V/m
pm	nuclear fusion	keV	10^{16} V/m
fm	α-decay	MeV	10^{18} V/m

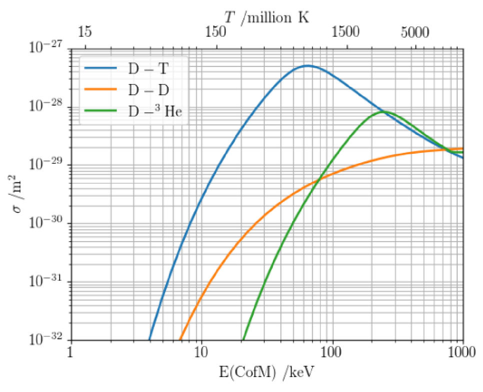


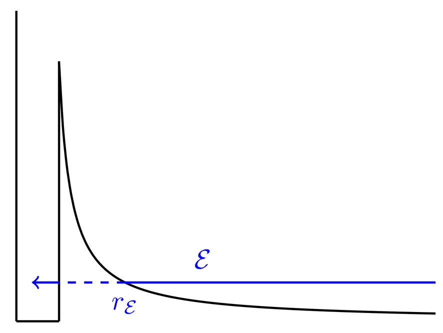




Dynamically Assisted Nuclear Fusion







F. Queisser and R. Schützhold, Phys. Rev. C **100**, 041601(R) (2019)

${}^2_1\mathrm{D} + {}^3_1\mathrm{T} \to {}^4_2\mathrm{He} + {}^1_0\mathrm{n} + 17.6\ \mathrm{MeV}$

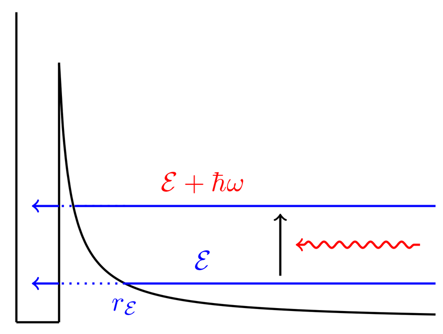
${}^1_1\mathrm{p} + {}^{11}_5\mathrm{B} \to 3 \times {}^4_2\mathrm{He} + 8.7\ \mathrm{MeV}$

Assistance by XFEL field/pulse

$A_x(t) = A_0/\cosh^2(\omega t)$

Field of α-particles?

C. Kohlfürst, F. Queisser and R. Schützhold, Phys. Rev. Research 3, 033153 (2021)



Analytical Model

Two-body Lagrangian with Coulomb (+nuclear) field and XFEL

$$L_{12} = \frac{m_1}{2}\dot{\boldsymbol{r}}_1^2 + \frac{m_2}{2}\dot{\boldsymbol{r}}_2^2 - V(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) + (q_1\dot{\boldsymbol{r}}_1 + q_2\dot{\boldsymbol{r}}_2)\cdot\boldsymbol{A}(t)$$

Center-of-mass and relative coordinates with reduced mass

$$L = \frac{m}{2}\dot{\boldsymbol{r}}^2 - V(|\boldsymbol{r}|) + q_{\rm eff}\dot{\boldsymbol{r}}\cdot\boldsymbol{A}(t)$$

Approximate scaling symmetry $\to$ dimension-less parameters

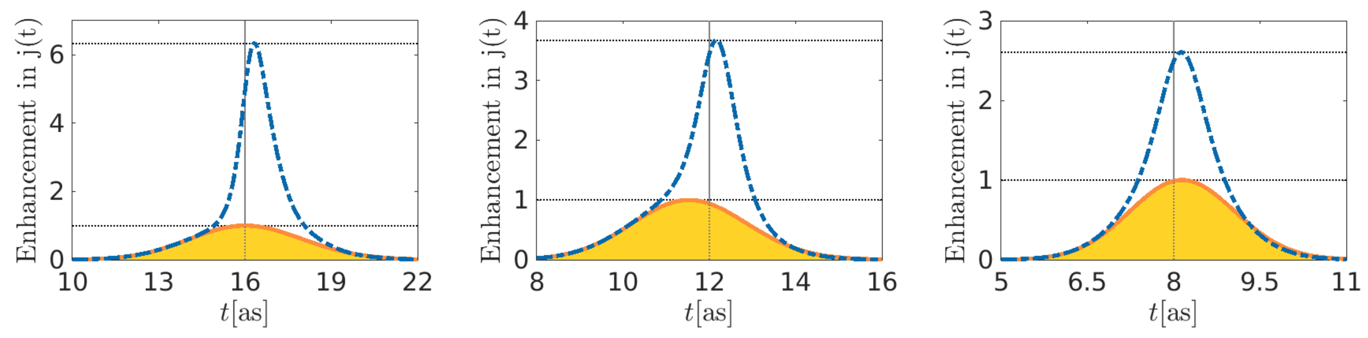
$$\eta = 2mEr_E^2 = \frac{2m}{E}\left(\frac{q_1q_2}{4\pi\epsilon_0}\right)^2, \quad \zeta = \frac{q_{\rm eff}A}{m\omega r_E} = \frac{q_{\rm eff}A}{mc}\frac{E}{\omega}\frac{4\pi\epsilon_0 c}{q_1q_2}$$

WKB tunneling exponent $\mathcal{P} \sim \exp\{-\pi\sqrt{\eta}\}$

Scaling $E_{\rm p+B} \leftrightarrow 20E_{\rm D+T}$

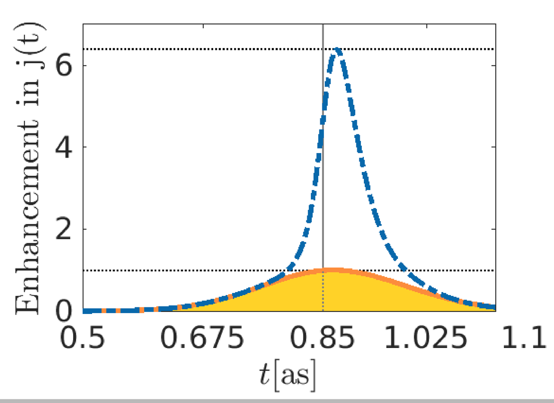
Numerical Simulations

1D-Schrödinger solver for D+T fusion with $\omega = 1$ keV and 10^{16} V/m



Initial kinetic energy: 2 keV, 4 keV and 8 keV

Comparison: p+B fusion with
$E = 38$ keV and pulse with
$\omega = 19$ keV and 28×10^{16} V/m
$\to$ scaling behavior



Summary & Outlook

Dynamically assisted tunneling

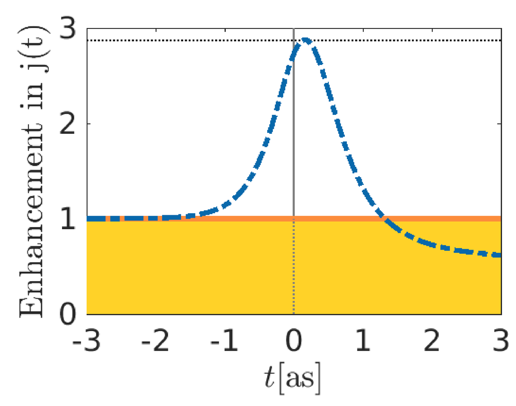
- pre-acceleration
- energy mixing (front end)
- potential deformation
- displacement (rear end)

Dynamically assisted nuclear fusion

- gradual incidence $\to$ displacement (?)
- tunneling exponent versus pre-factor
- $\omega = 1$ keV and 10^{16} V/m

Outlook

- field of α-particles?
- muon-assisted fusion



Dynamically assisted tunneling in the impulse regime
C. Kohlfürst, F. Queisser and R. Schützhold, Phys. Rev. Research 3, 033153 (2021)

Dynamically assisted nuclear fusion
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