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Electron-positron pair production by an ultra-intense laser pulse focused by RFM

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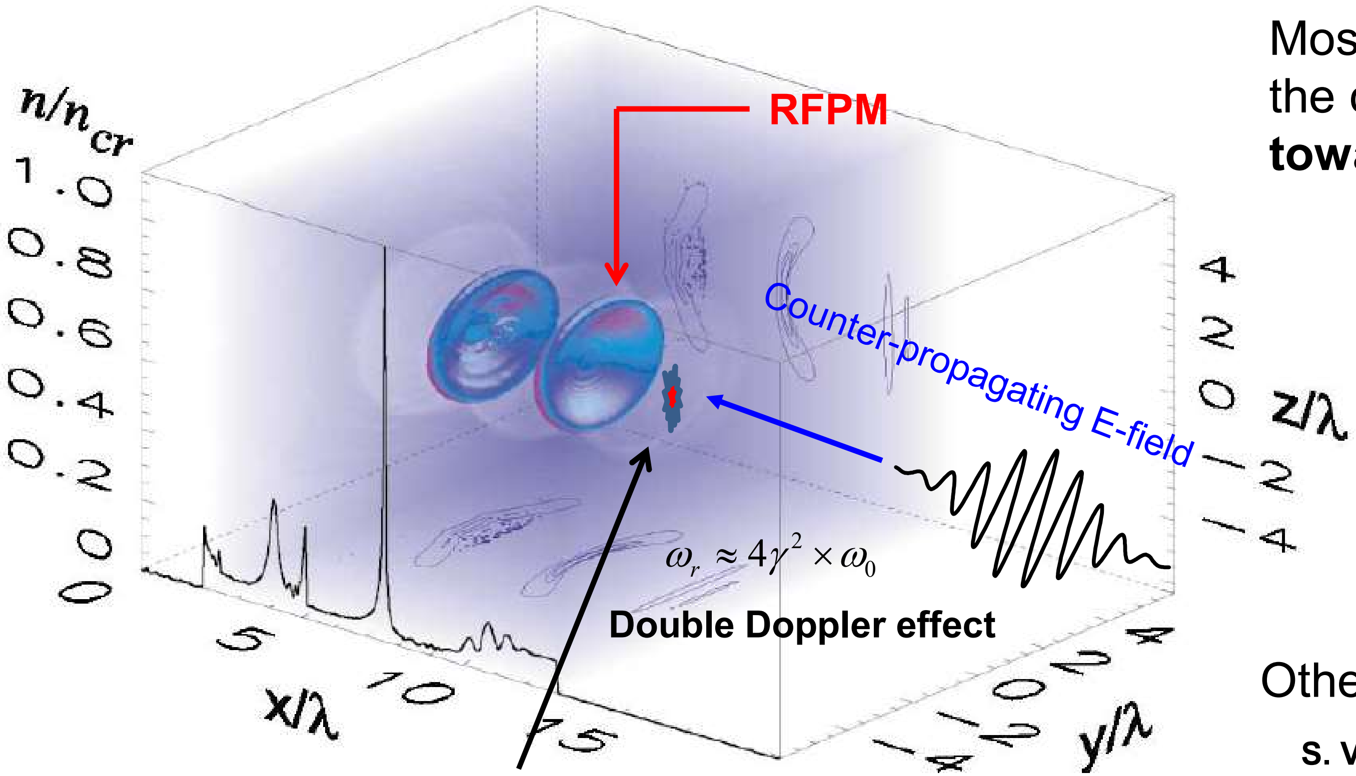
Relativistic-flying Parabolic Mirror

Relativistic-flying Parabolic Mirror (RFPM)



Ultra-strong relativistic laser focus (nm, attosecond)

S. V. Bulanov, et al., Phys. Rev. Lett. 91, 085001 (2003).

Most interesting feature is the capability of "**intensifying laser field toward Schwinger field**".

Schwinger Field (~1.32×10^{16} V/cm): critical field in QED which generate e^+e^- pairs from vacuum

Other interesting features are described in

S. V. Bulanov, et al., Physics Uspekhi (2013).
S. V. Bulanov, et al., Plasma Sources Sci. Tech. (2016).
J. K. Koga, et al., Plasma Phys. Control. Fusion (2018).
P. Valenta, et al., Phys. Plasmas (2020).
T. Zh. Esirkepov, et al., Phys. Plasmas (2020).
J. Mu, et al., Phys. Rev. E (2020).

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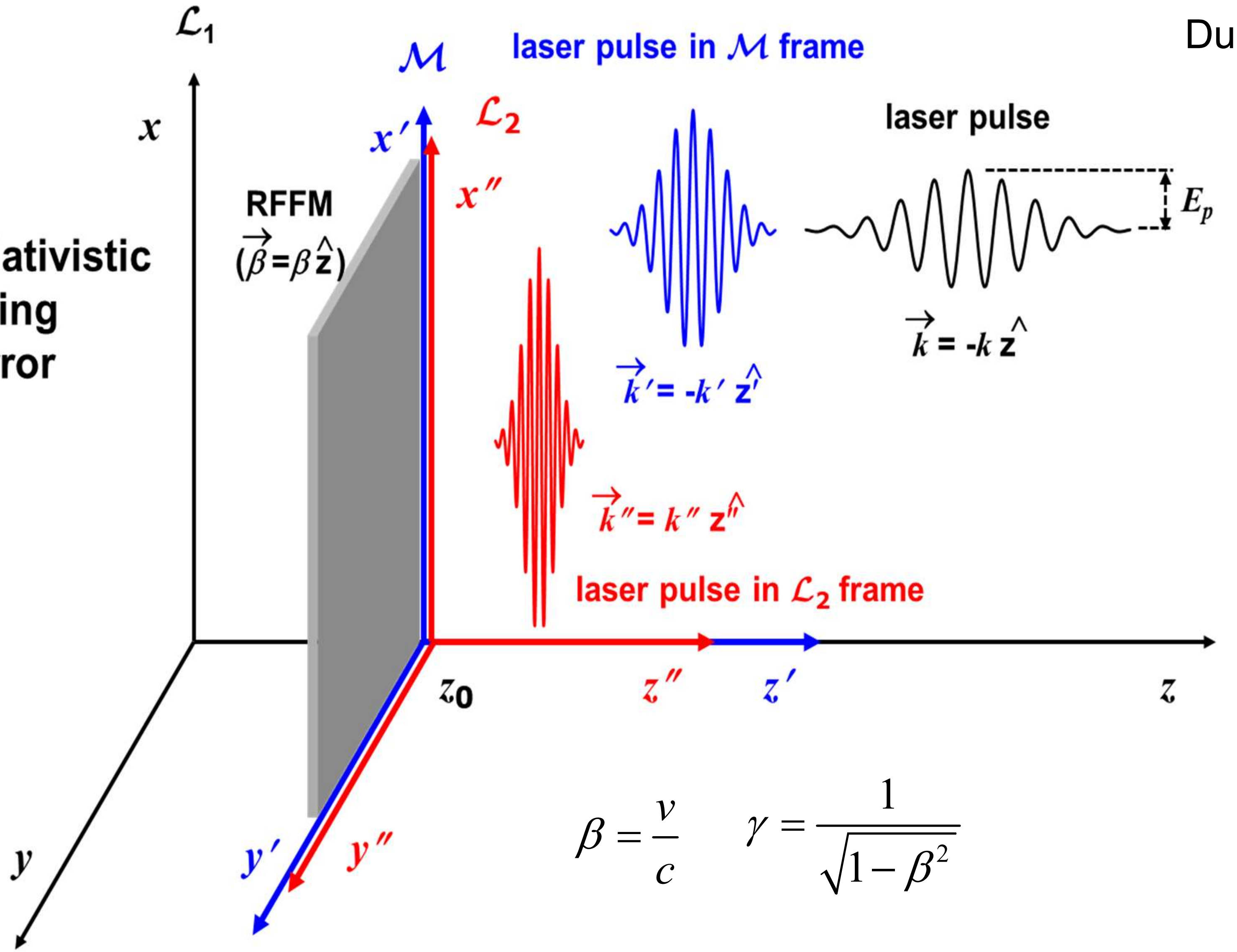
Basic Characteristics of RFM

Relativistic
Flying
Mirror



$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Due to the double Doppler effect

Wavenumber

$$k'' = \frac{1+\beta}{1-\beta} k \approx 4\gamma^2 \cdot k$$

Pulse duration

$$\tau'' = \frac{1-\beta}{1+\beta} \tau \approx \frac{1}{4\gamma^2} \cdot \tau$$

Field strength

$$E_p'' = \frac{1+\beta}{1-\beta} E_p \approx 4\gamma^2 \cdot E_p$$

Ti:S laser pulse (w/ γ=12.7)

800 nm ⟶ 1.24 nm

30 fs ⟶ 47 as

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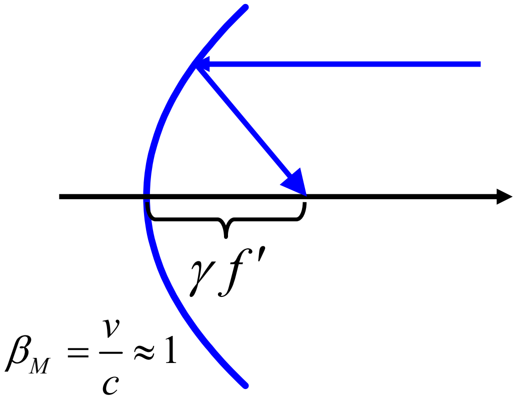
Parabolic shapes in Laboratory and Boosted srames

Due to the Lorentz contraction,

in laboratory frame

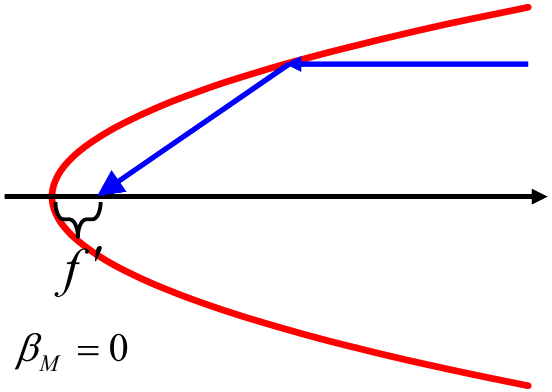


Equation for parabolic surface

$$z = \frac{x^2 + y^2}{4\gamma f'} - \gamma f' + \frac{\gamma^2 - 1}{\gamma} f' + \beta ct.$$

Focal length: $\gamma f'$

in boosted (Moving) frame



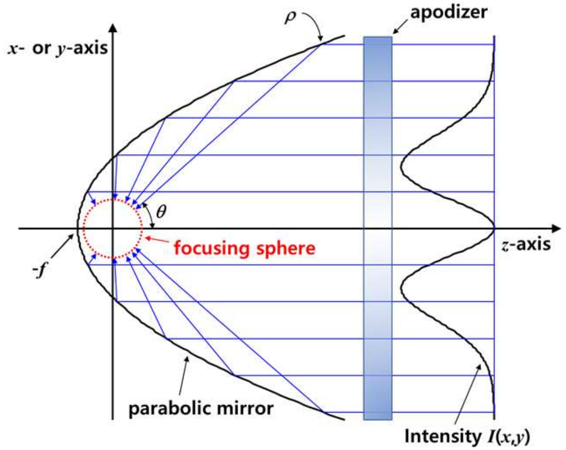
Equation for parabolic surface

$$z' = \frac{(x')^2 + (y')^2}{4 f'} - f',$$

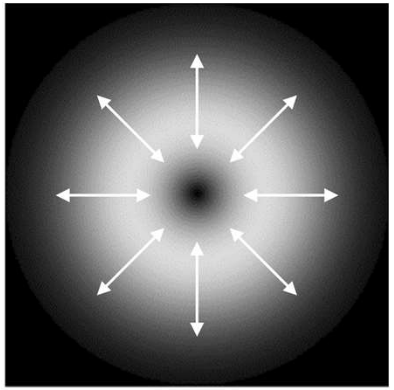
Focal length: f' $\quad F_N = \frac{f'}{D} \ll 1$

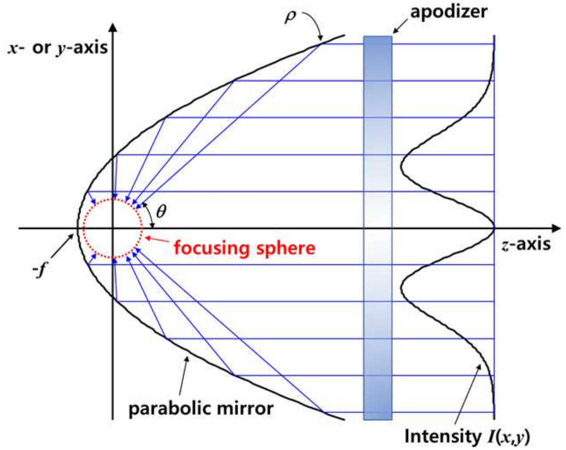
(Spherical focusing)

4π-spherical focusing of RP and AP laser pulses

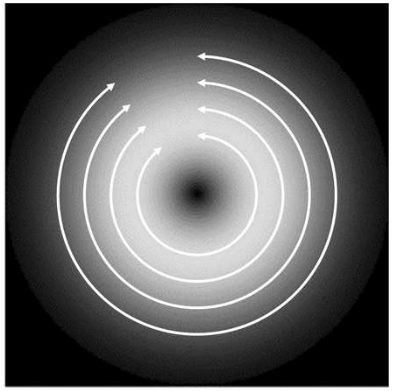


Radially-Polarized (RP) Laguerre Gaussian (LG) beam

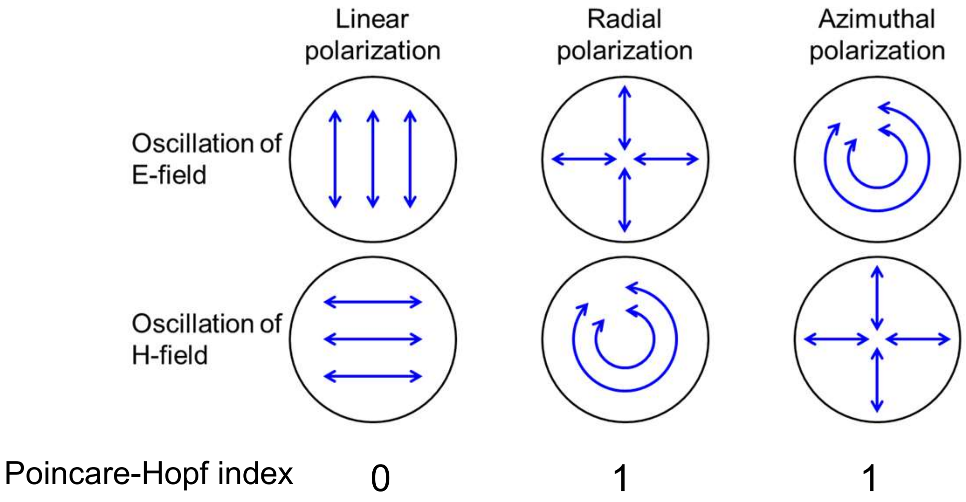


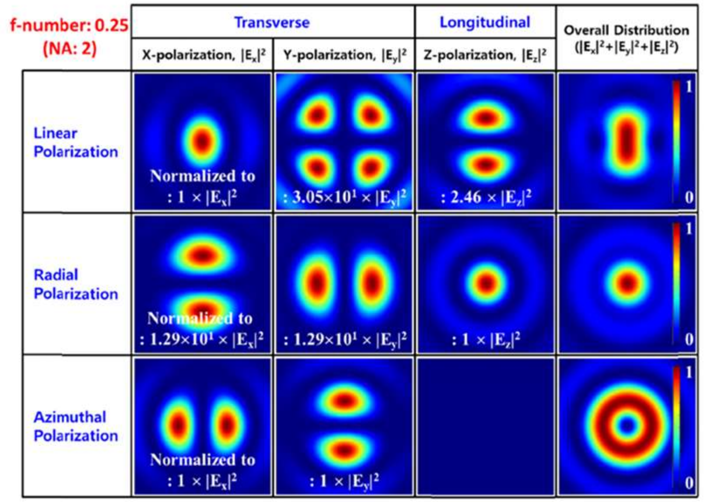


Azimuthally-Polarized (AP) Laguerre Gaussian (LG) beam



Polarization configuration





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How to calculate EM field in Lab. frame

Incident laser pulse in $\mathcal{L}$ frame

↓

Lorentz Transformation $\mathcal{L} \rightarrow \mathcal{M}$

↓

Incident laser pulse in $\mathcal{M}$ frame

↓

Focused laser field in $\mathcal{M}$ frame
- Diffraction integral in space (4π-spherical focusing)
- Fourier Transform in frequency

↓

Lorentz Transformation $\mathcal{M} \rightarrow \mathcal{L}$

↓

Focused laser field in $\mathcal{L}$ frame

Assumption: Perfectly reflecting surface

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Calculation of Focused Field

Focused laser field in $\mathcal{M}$ frame (**monochromatic**)

E-field:

$$\vec{E}'_f\left(x'^{\mu};\omega'\right)=\hat{\theta}' i E'_{0,f}\left(\omega'\right)a\left(\rho',\theta';\omega'\right)\exp\left(-i\omega' t'\right)=\vec{E}'_{\perp,f}+\vec{E}'_{\parallel,f}, \quad \longrightarrow \quad \begin{bmatrix} E'_{f,x'} \\ E'_{f,y'} \\ E'_{f,z'} \end{bmatrix}=iE'_{0,f}\left(\omega'\right)a\left(\rho',\theta';\omega'\right)\exp\left(-i\omega' t'\right)\begin{bmatrix} \cos\theta'\cos\phi' \\ \cos\theta'\sin\phi' \\ -\sin\theta' \end{bmatrix},$$

B-field:

$$\vec{H}'_f\left(x'^{\mu};\omega'\right)=-\hat{\phi}' H'_{0,f}\left(\omega'\right)b\left(\rho',\theta';\omega'\right)\exp\left(-i\omega' t'\right)=\vec{H}'_{\parallel,f}. \quad \longrightarrow \quad \begin{bmatrix} H'_{f,x'} \\ H'_{f,y'} \\ H'_{f,z'} \end{bmatrix}=-H'_{0,f}\left(\omega'\right)b\left(\rho',\theta';\omega'\right)\exp\left(-i\omega' t'\right)\begin{bmatrix} \sin\phi' \\ -\cos\phi' \\ 0 \end{bmatrix}.$$

Lorentz transformation ($\mathcal{M}\rightarrow\mathcal{L}$) for the field

$$\begin{bmatrix} E''_{f,x}\left(\omega'\right) \\ E''_{f,y}\left(\omega'\right) \end{bmatrix}=\begin{bmatrix} \gamma\left(E'_{f,x}+\beta B'_y\right) \\ \gamma\left(E'_{f,y}-\beta B'_x\right) \end{bmatrix}=\gamma E'_{0,f}\left(\omega'\right)\exp\left(-i\omega' t'\right)\left[ia\left(\rho',\theta';\omega'\right)\cos\theta'+\beta b\left(\rho',\theta';\omega'\right)\right]\begin{bmatrix} \cos\phi' \\ \sin\phi' \end{bmatrix},$$ **(perpendicular polarization comp.)**

$$E''_{f,z}=E'_{f,z}=-iE'_{0,f}\left(\omega'\right)a\left(\rho',\theta';\omega'\right)\exp\left(-i\omega' t'\right)\sin\theta',$$ **(parallel polarization comp.)**

Fourier transformation for the field of laser pulse

$$\vec{E}''_f\left(x'^{\mu};t'\right)=\int_{-\infty}^{\infty}d\omega'\vec{E}''_f\left(x'^{\mu};\omega'\right)\exp\left(i\omega' t'\right)=\gamma\begin{bmatrix} \left\{i\cos\theta' I_a\left(x'^{\mu};t'\right)+\beta I_b\left(x'^{\mu};t'\right)\right\}\cos\phi' \\ \left\{i\cos\theta' I_a\left(x'^{\mu};t'\right)+\beta I_b\left(x'^{\mu};t'\right)\right\}\sin\phi' \\ -i\left(1/\gamma\right)\sin\theta' I_a\left(x'^{\mu};t'\right) \end{bmatrix},$$

$$I_a\left(x'^{\mu};t'\right)=\int_{-\infty}^{\infty}d\omega' E'_{0,f}\left(\omega'\right)\exp\left[-\frac{\left(\omega'-\omega'_C\right)^2}{\left(\Delta\omega'\right)^2}\right]a\left(\rho',\theta';\omega'\right)\exp\left(i\omega' t'\right),$$

$$I_b\left(x'^{\mu};t'\right)=\int_{-\infty}^{\infty}d\omega' E'_{0,f}\left(\omega'\right)\exp\left[-\frac{\left(\omega'-\omega'_C\right)^2}{\left(\Delta\omega'\right)^2}\right]b\left(\rho',\theta';\omega'\right)\exp\left(i\omega' t'\right).$$

Field expressions of the focused laser intensity

For a radially-polarized incident laser pulse

Electric field vector

$$\vec{E}''_f(\rho,\theta;t) = 2\pi^2\gamma^3\sqrt{\frac{3\pi}{c\varepsilon_0}}\left(\frac{w_0}{\lambda_0}\right)\sqrt{\mathcal{I}_p}\begin{bmatrix} -\left\{ j_1\left(\frac{\omega'_C}{c}R\right)\cos(\omega'_0 T)\Upsilon_2 - j_0\left(\frac{\omega'_C}{c}R\right)\sin(\omega'_0 T)\Upsilon_1 \right\}\sin\phi \\ \left\{ j_1\left(\frac{\omega'_C}{c}R\right)\cos(\omega'_0 T)\Upsilon_2 - j_0\left(\frac{\omega'_C}{c}R\right)\sin(\omega'_C T)\Upsilon_1 \right\}\cos\phi \\ 0 \end{bmatrix}$$

Magnetic field vector

$$\vec{B}''_f(\rho,\theta;t) = \frac{2\pi^2\gamma^3}{c}\sqrt{\frac{3\pi}{c\varepsilon_0}}\left(\frac{w_0}{\lambda_0}\right)\sqrt{\mathcal{I}_p}\begin{bmatrix} \left\{ -j_0\left(\frac{\omega'_0}{c}R\right)\sin(\omega'_0 T)\Upsilon_1 + j_1\left(\frac{\omega'_0}{c}R\right)\cos(\omega'_0 T)\Upsilon_2 \right\}\cos\phi \\ \left\{ -j_0\left(\frac{\omega'_0}{c}R\right)\sin(\omega'_0 T)\Upsilon_1 + j_1\left(\frac{\omega'_0}{c}R\right)\cos(\omega'_0 T)\Upsilon_2 \right\}\sin\phi \\ (1/\gamma)\, j_0\left(\frac{\omega'_0}{c}R\right) \end{bmatrix}$$

Field strength ~ $\gamma^3\left(\frac{w_0}{\lambda_0}\right)\mathcal{I}_p^{0.5}$

Geometrical factor:
beam radius-wavelength ratio

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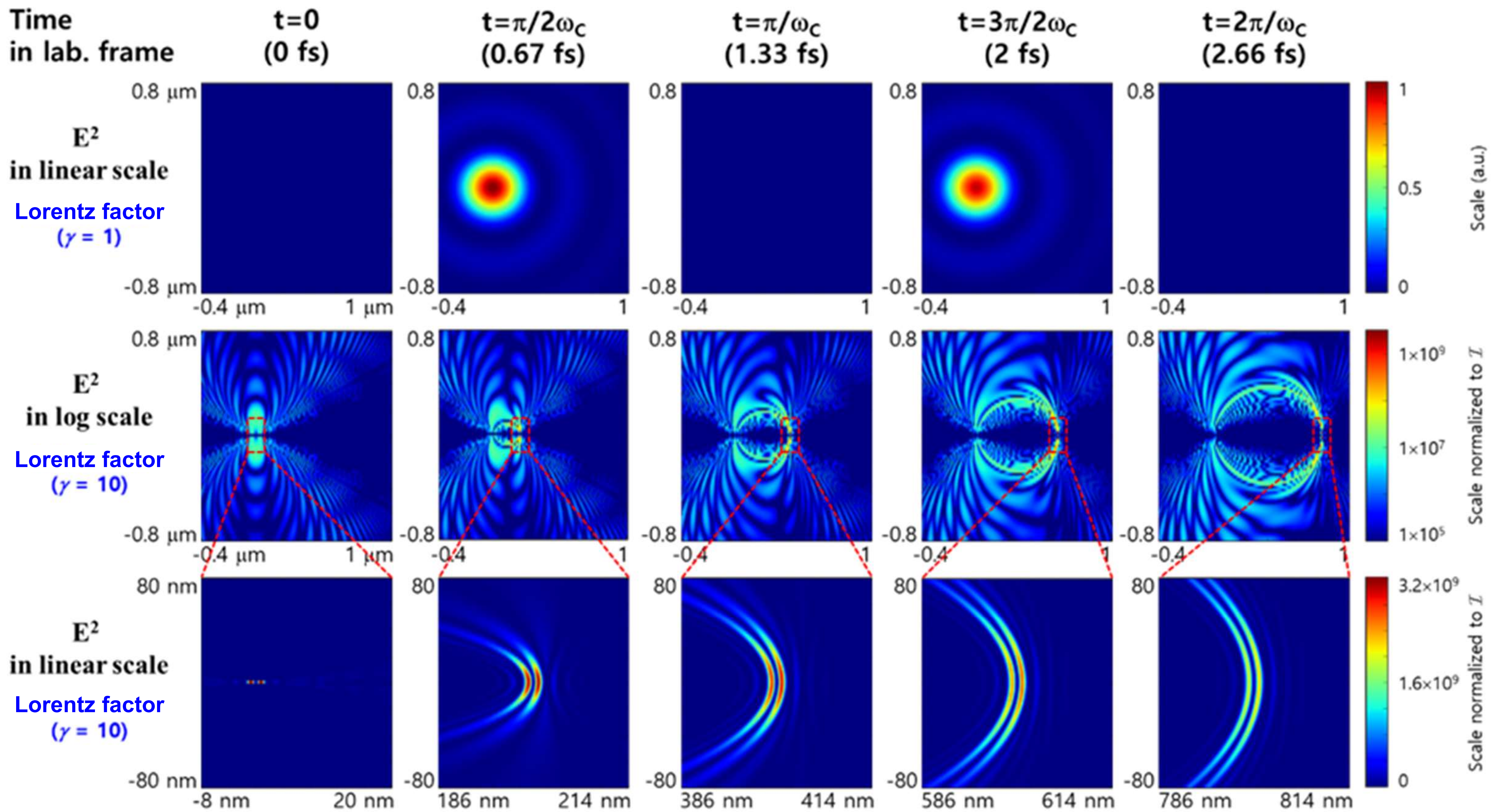
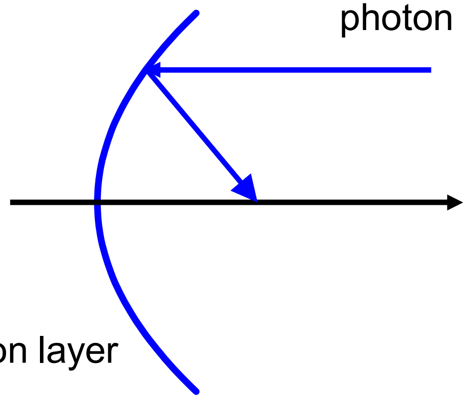
Squared Electric Field at different Lorentz γ



Recoil effect on the reflected frequency

Consider mirror reflectance, $\mathcal{R}$



Electron density in the mirror: n_e

Electron momentum: p_e (before), p_e" (after)

Electron energy: $\mathcal{E}_e$ (before), $\mathcal{E}_e$" (after)

Incident photon density: n_ω

Photon momentum: p_ω (before), p_ω" (after)

Photon energy: $\mathcal{E}_\omega$ (before), $\mathcal{E}_\omega$" (after)

Momentum conservation:

$$n_e p_e - n_\omega p_\omega = n_e p_e'' \cos\theta_e + \mathcal{R}(\theta) n_\omega p_\omega'' \cos\theta - \left[1 - \mathcal{R}(\theta)\right] n_\omega p_\omega$$

$$0 = n_e p_e'' \sin\theta_e - \mathcal{R}(\theta) n_\omega p_\omega'' \sin\theta$$

Energy conservation:

$$n_e \mathcal{E}_e + n_\omega \mathcal{E}_\omega = n_e \mathcal{E}_e'' + \mathcal{R}(\theta) n_\omega \mathcal{E}_\omega'' + \left[1 - \mathcal{R}(\theta)\right] n_\omega \mathcal{E}_\omega$$

Frequency shift of the reflected wave

$$\omega'' \approx \omega \frac{(1+\beta)}{(1-\beta\cos\theta)} \left[1 - \mathcal{R} \frac{\mathcal{I}_0}{\gamma n_e m_e c^3} (1+\cos\theta)^2 \exp\left(-\sin^2\theta / \sin^2\theta_0\right)\right]$$

$$\mathcal{R} \frac{\mathcal{I}_0 / c}{\gamma n_e m_e c^2} \ll 1$$

$$\frac{\text{Energy density of reflected EM wave}}{\text{Energy density of electron layer}}$$

e^+e^- Pair production rate

e^+e^- pair production rate for Schwinger mechanism

$$W_{ep} = \frac{e^2 E_{Sch}^2}{4\pi^3 \hbar^2 c} E_{imv} B_{inv} \coth\left(\pi \frac{B_{inv}}{E_{inv}}\right) e^{-\frac{\pi}{E_{inv}}}$$

Invariant fields and Poincare invariants

$$E_{imv} = \frac{\sqrt{\left(\mathcal{F}^2 + \mathcal{G}^2\right)^{1/2} - \mathcal{F}}}{E_{Sch}} \qquad B_{imv} = \frac{\sqrt{\left(\mathcal{F}^2 + \mathcal{G}^2\right)^{1/2} + \mathcal{F}}}{E_{Sch}} \qquad \mathcal{F} = \frac{c^2 B^2 - E^2}{2} \qquad \mathcal{G} = c\vec{B} \cdot \vec{E}$$

Peak strength of the focused laser field

$$E_f'' = \sqrt{\mathcal{R}} \cdot \gamma \frac{1+\beta}{1-\beta} \sqrt{\frac{3\pi}{c\varepsilon_0} \frac{\pi \omega w_0}{4c}} \sqrt{\mathcal{I}}$$

Required intensity for reaching Schwinger intensity

$$\mathcal{I} \approx \frac{\left(\lambda_0 / w_0\right)^2}{6\pi^5 \mathcal{R} \gamma^6} \mathcal{I}_{Sch} \sim 2.3\times10^{17}\ \mathrm{W/cm^2}$$

(γ = 12.2, $\mathcal{R} \sim 0.5\times\gamma^{-3}$, λ_0 = 0.2 μm, w_0 = 156 μm)

e^+e^- pair production rate with spatio-temporal distribution

$$W_{ep} \approx \mathcal{R} \cdot 12\pi^2 \alpha \gamma^4 \left(\frac{w_0}{\lambda_0}\right)^2 \left(\frac{\mathcal{I}_p}{\hbar c}\right) \left(-j_{\{0-1\}}^2\right) \exp\left[-\frac{1}{\gamma^2} \frac{\lambda_0}{w_0} \frac{E_{Sch}/E_p}{\sqrt{6\pi^3 \sqrt{-j_{\{0-1\}}^2}}}\right]$$

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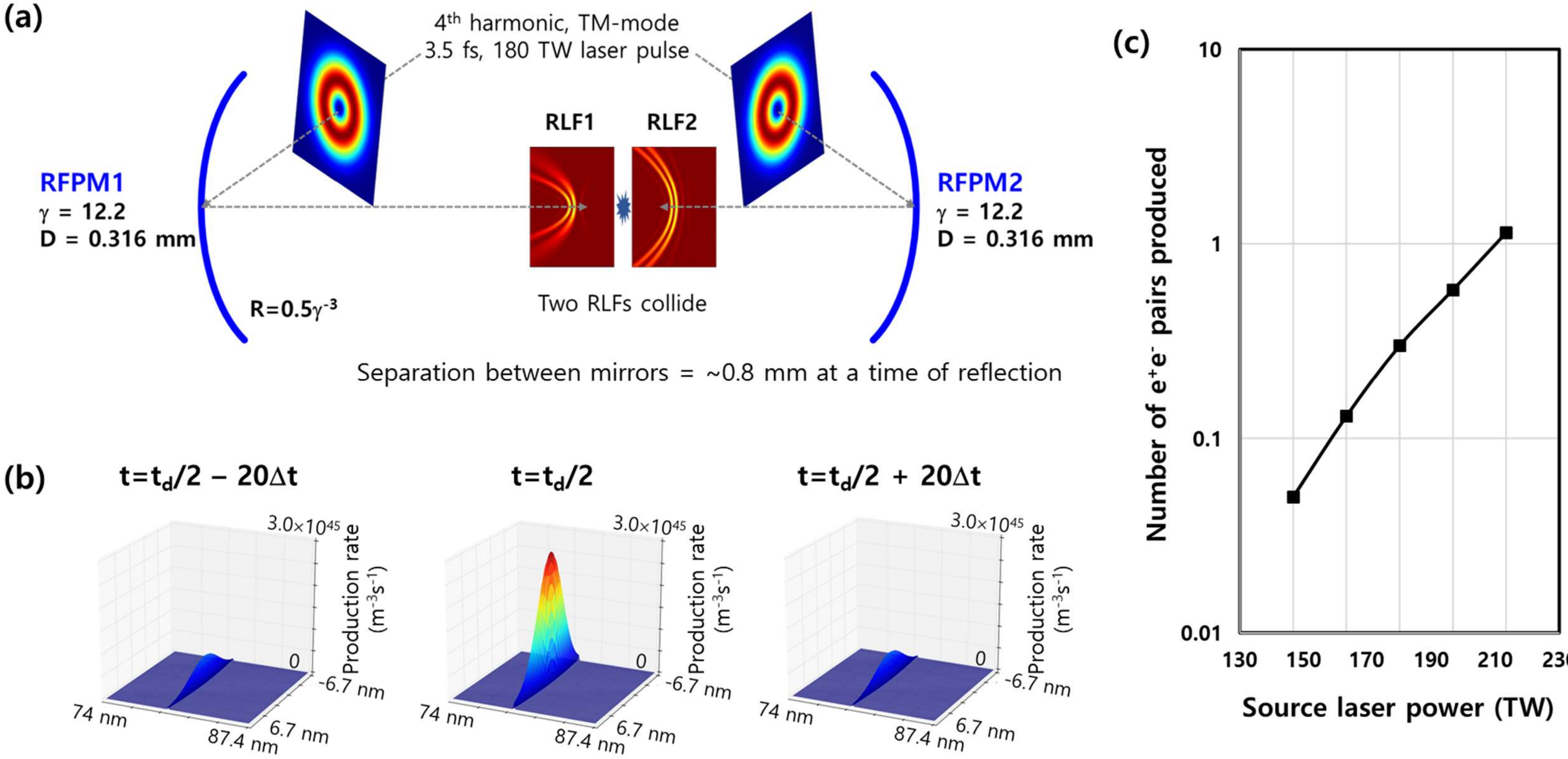
e^+e^- Pair production under relativistic flying laser focus



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Conclusion

- Mathematical expression for the laser focus formed by the relativistic-flying parabolic mirror (RFPM) was derived.
- Frequency shift, Field enhancement, and Shortening of pulse duration are examined.
- e^+e^- pair production is investigated using the field expression obtained by the relativistic flying parabolic mirror.

Thank you for your attention!!

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