Worldline fermion propagator dressed with N photons

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1. Introduction

The standard approach for calculating the S-matrix in quantum field theory was developed approximately seventy years ago. It is based on path integrals over field configurations, and can be formulated as a diagrammatic perturbative method. Despite its success, the computational effort required to describe several processes under this approach is non-negligible. Alternative to it, there is the Worldline formalism based on first-quantized relativistic particle path integrals, and which brings several advantages over the standard approach. However, a long-standing problem has been to extend this formalism to amplitudes involving open fermion lines. It was recently that we develop a suitable formalism for the case of quantum electrodynamics in vacuum based on second-order fermions and the symbol map [1, 2].

2. Dressed fermion propagator

The **fermion propagator** in the presence of an Abelian background field A, can be written as,

$$\begin{aligned} S^{x'x}[A] &= \langle x' | \left[m - i\not{D} \right]^{-1} | x \rangle \\ &= (i\not{D} + m)\, K^{x'x}[A], \end{aligned}$$

where the Kernel function $K^{x'x}[A]$ is defined as

$$K^{x'x}[A] = \langle x' | \left[-D^2 + m^2 + \frac{ie}{2}\gamma^\mu\gamma^\nu F_{\mu\nu} \right]^{-1} | x \rangle.$$

The worldline formalism uses the Schwinger proper time trick to exponentiate the inverse of the operator $-D^2 + m^2 + \frac{ie}{2}\gamma^\mu\gamma^\nu F_{\mu\nu}$. The resulting expression is a path integral representation of the Kernel

$$K^{x'x}[A] = \int_0^\infty dT e^{-m^2 T} \int_{x(0)=x}^{x(T)=x'} \mathcal{D}x \mathcal{P} e^{-\int_0^T d\tau \left(\frac{1}{4}\dot{x}^2 + ie\dot{x}\cdot A + \frac{i}{2} e\gamma^\mu\gamma^\nu F_{\mu\nu} \right)},$$

where $\mathcal{P}$ represents the path-ordering operator. It can be shown that

$$\mathcal{P} e^{-\int_0^T d\tau \frac{i}{2} e\gamma^\mu\gamma^\nu F_{\mu\nu}} = 2^{-\frac{D}{2}} \,\mathrm{symb}^{-1} \int_A D\psi\, e^{-\int_0^T \frac{1}{2}\psi_\mu \dot{\psi}^\mu d\tau} \times e^{\int_0^T ieF_{\mu\nu}(\psi+\eta)^\mu(\psi+\eta)^\nu d\tau}.$$

Here the symbol map, symb, is defined by

$$\mathrm{symb}(\gamma^{\alpha_1\alpha_2\ldots\alpha_n}) \equiv (-i\sqrt{2})^n \eta^{\alpha_1}\eta^{\alpha_2}\ldots\eta^{\alpha_n},$$

where $\gamma^{\alpha_1\alpha_2\ldots\alpha_n}$ denotes the totally antisymmetrised product of gamma matrices.

3. N-photon amplitudes

N-photon scattering amplitudes are generated by expanding $A(x)$ as a sum of planes waves with fixed polarization,

$$A^\mu(x) = \sum_{i=1}^N \epsilon_i^\mu e^{ik_i\cdot x}.$$

Substituting the plane wave expansion into the worldline version of the fermion propagator, taking only the $\mathcal{O}(N)$ multi-linear expression in polarisation vectors, and going to momentum space, we obtain the following version of the N-photon untruncated propagator,

$$S_N^{p'p} = (\not{p}' + m) K_N - e \sum_{i=1}^N \not{\epsilon}_i K_{N-1}^{p'+k_i,p},$$

where a global momentum conservation factor $(2\pi)^D \delta^D(p + p' + \sum_{i=1}^N k_i)$ has been omitted, and

$$K_N^{p'p} = (-ie)^N \mathrm{symb}^{-1} \int_0^\infty dT e^{-m^2 T} \int_0^T d\tau_1 \cdots \int \theta_N\, e^{\mathrm{Exp}} \Big|_{\epsilon_1\ldots\epsilon_N},$$

being

$$\begin{aligned} \mathrm{Exp} = & -p'^2 T - \sum_{i=1}^N \sqrt{2}\eta\cdot(\epsilon_i + i\theta_i k_i) + \frac{1}{2}\sum_{i,j=1}^N \theta_i\theta_j\sigma_{ij} k_i\cdot k_j \\ & + \sum_{i=1}^N (i\theta_i\epsilon_i - \tau_i k_i)\cdot(p' - p - \sum_{j=1}^N \sigma_{ij} k_j) - i\sum_{i,j=1}^N \sigma_{ij}\epsilon_i\cdot k_j\theta_j \\ & - \frac{1}{2}\sum_{i,j=1}^N (\sigma_{ij} + 2\theta_i\theta_j\delta_{ij})\epsilon_i\cdot\epsilon_j. \end{aligned}$$

($\sigma_{ij} = \mathrm{sign}(\tau_i - \tau_j)$, $\delta_{ij} = \delta(\tau_i - \tau_j)$).

4. Kernel decomposition in $D = 4$

For $D = 4$ the kernel $K_N^{p'p}$ can be written as

$$\begin{aligned} K_N^{p'p} &= (-ie)^N \frac{\mathfrak{K}_N^{p'p}}{(p'^2 + m^2)(p^2 + m^2)}, \\ \mathfrak{K}_N^{p'p} &= A_N + B_{N\mu\nu}\sigma^{\mu\nu} - iC_N\gamma_5, \qquad \sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu], \\ A_N &= A_N^{\text{scalar}} + A_N^{\text{spin}}. \end{aligned}$$

5. On-shell matrix element

The on-shell matrix element $\mathcal{M}_{Ns's}^{p'p}$ corresponding to the dressed electron propagator with fixed spins s, s', in the worldline formalism, reads as

$$\mathcal{M}_{Ns's}^{p'p} = \frac{(-ie)^N}{2m} \bar{u}_{s'}(-p')\mathfrak{K}_N^{p'p} u_s(p).$$

Also, in the on-shell case, we obtain the identities

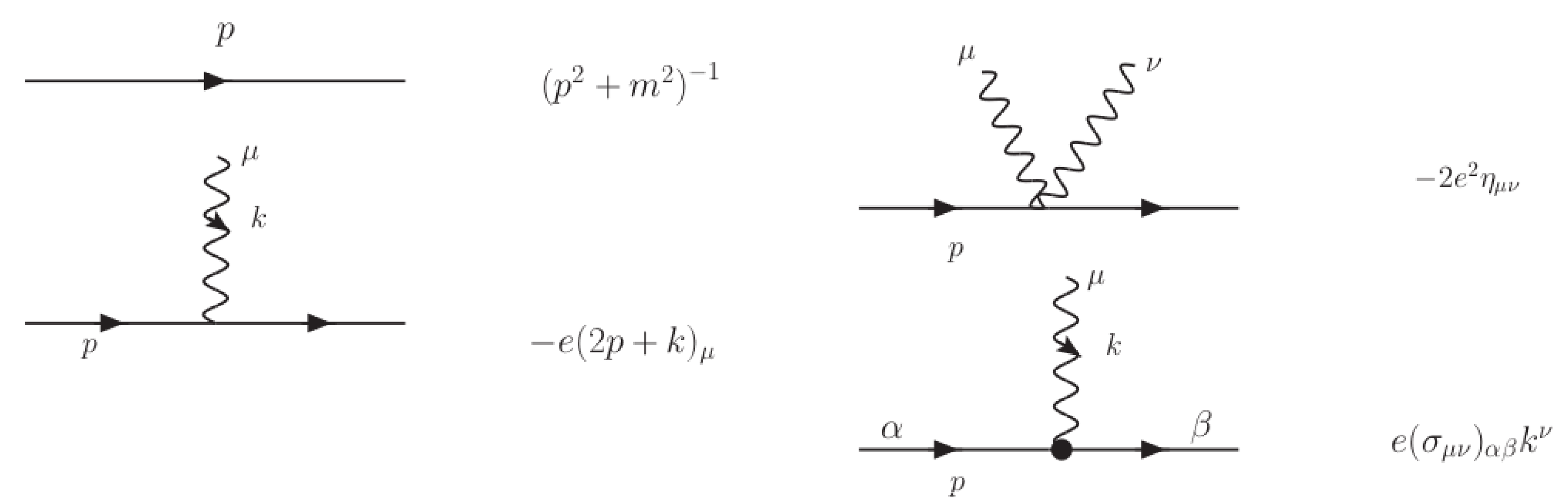
$$\begin{aligned} A_N(m^2 - p\cdot p') &= 2p^\mu B_{N\mu\nu} p'^\nu, \\ C_N(m^2 + p\cdot p') &= 2p^\mu \tilde{B}_{N\mu\nu} p'^\nu, \qquad \tilde{B}_{N\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} B_{N\alpha\beta}, \end{aligned}$$

from which the following spin-averaged cross sections is obtained,

$$\langle |\mathcal{M}_N^{p'p}|^2 \rangle = e^{2N}\left[|A_N|^2 + 2B_N^{\alpha\beta} B_{N\alpha\beta}^\star - |C_N|^2 \right].$$

6. Second order fermions

The Kernel $K_N^{p'p}$, and therefore the matrix element $\mathcal{M}_{Ns's}^{p'p}$, can equivalently be obtained with the aid of the less familiar second order rules shown below [3].



7. Conclusions

We have provided a new approach to the worldline path integral representation of the open Dirac-fermion line dressed with N photons. This approach presents several computational advantages compared to the standard first order formalism. One of these advantages is a compact representation for the spin-averaged cross section. We have also obtained general formulas for fully polarised amplitudes at tree level from the worldline version of the matrix element $\mathcal{M}_{Ns's}^{p'p}$ [2].

References.
1. N. Ahmadiniaz et al., JHEP 08 (2020) 049, arXiv:2004.01391 [hep-th]. **2.** N. Ahmadiniaz et al., arXiv:2107.00199 [hep-th]. **3.** A. Morgan, Phys. Lett. B 351 (1995) 249, arXiv:hep-ph/9502230 [hep-ph].