Challenges in characterizing QED processes in Ritus-Narozhny nonperturbative regime

Extremely High Intensity Laser Physics Conference (ExHILP'2021)

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Introduction

SOVIET PHYSICS JETP VOLUME 30, NUMBER 6 JUNE, 1970

RADIATIVE EFFECTS AND THEIR ENHANCEMENT IN AN INTENSE ELECTROMAGNETIC FIELD

V. I. RITUS

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Submitted June 27, 1969

Zh. Eksp. Teor. Fiz. **57**, 2176–2188 (December, 1969)

The radiative effects—the photon mass, the change in the electron mass, and the anomalous magnetic moment of the electron—are considered in the presence of an intense electromagnetic field where they are functions of the parameter $\chi = m^{-3}\sqrt{(eF_{\mu\nu}p_\nu)}$ which is proportional to the field and to the particle momentum. The effects become optimal at $\chi \sim 1$, that is, for example, at a field strength of 4×10^8 Oe and an energy of 25 GeV. At higher energies the square of the photon mass and the change of the electron mass increase like $\chi^{2/3}$, but the anomalous magnetic moment decreases like $\chi^{-2/3}$. For $\alpha\chi^{2/3} \sim 1$ an exact theory of the interaction with the radiation field is required, which takes all radiative corrections into account. This region is more accessible than the corresponding region of logarithmically large energies for the radiative effects in vacuum. The amplitudes found for the elastic scattering of a photon and an electron have an essential singularity at $\chi = 0$.

Introduction

ibid, page 6:

The most interesting property is the enhancement of the radiative corrections in a constant field at large energies and fields, which for $\alpha\kappa^{2/3} \sim 1$ leads to the necessity for an exact amount of the interaction with the radiation field. For fields of 10^8 Heaviside units, this corresponds to an energy of 25×10^3 GeV. In the classical theory one can treat the radiation as a perturbation as long as the radiative damping force is small in comparison with the Lorentz force in the particle's rest system, i.e., for $\alpha\chi \ll 1$. We see that the quantum theory changes this condition, and apparently $\alpha\chi^{2/3}$ is the universal parameter characterizing the applicability of perturbation theory to radiation in the presence of an external field for large energies and fields.

Introduction

ANNALS OF PHYSICS: **69**, 555–582 (1972)

Radiative Corrections in Quantum Electrodynamics with Intense Field and Their Analytical Properties

V. I. RITUS

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Received April 10, 1971

page 2:

The elastic scattering amplitude of an electron depends on the dynamic variable $\chi = \sqrt{(eF_{\mu\nu}p_\nu)^2}/m^3$ and its spin direction. Its imaginary part defines the probability of the processes caused by the electron in a field and its real part in particular determines the anomalous magnetic moment of the electron and its dependence on χ. The physical quantities attain their optimal values for $\chi \sim 1$ which for a field of 4.10^{8}Oe corresponds to an energy of electrons of 25 GeV. At higher energies ($\chi \gg 1$) the amplitude of second-order in the radiative field increases as $\alpha\chi^{2/3}$; in this connection it may be expected that at $\alpha\chi^{2/3} \sim 1$ the electrodynamic interaction becomes strong. The consideration of fourth-order radiative corrections shows that in this approximation the amplitude increases at high energies as $\alpha^2\chi \ln \chi$, the real part increasing more weakely. So the fourth-order and the second-order corrections are comparable at $\alpha\chi^{1/3} \ln \chi \sim 1$ and the second-order results are valid as long as $\alpha\chi^{1/3} \ln \chi \ll 1$.

Formal derivation

- Electron 1-loop self-energy as example:

$$\delta m_\pm^2(\chi_e) = \frac{\alpha m^2}{\pi}\int_0^\infty \frac{du}{(1+u)^3}\left[\frac{5+7u+5u^2}{3z}f'(z) \pm \chi_e z f(z)\right],$$

$$f(z) = \pi\left(\mathsf{Gi}\,(z) + i\,\mathsf{Ai}\,(z)\right), \quad u = \frac{\chi_\gamma}{\chi_e - \chi_\gamma}, \quad z = \left(\frac{u}{\chi_e}\right)^{2/3}$$

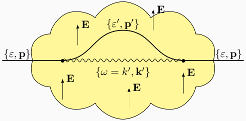
Ritus, JETP (1970); Ann. Phys. (1972)

- $\chi_e \to 0$: $u_{\text{eff}} = \mathcal{O}(\chi_e) \ll 1$, $z_{\text{eff}} \simeq 1 \implies \quad \delta m_\pm^2(\chi_e) = \mathcal{O}(\chi_e) \to 0$;
- $\chi_e \gg 1$: $u_{\text{eff}} \simeq 1$, $z_{\text{eff}} = \mathcal{O}\left({\chi_e}^{-2/3}\right) \ll 1$. Therefore:

$$f(z) \mapsto f(0); \quad f'(z) \mapsto f'(0); \quad \chi_e z = \mathcal{O}\left(\chi_e^{1/3}\right) \ll z^{-1};$$

$$\boxed{\begin{aligned}\delta m_\pm^2(\chi_e \gg 1) &\approx \frac{\alpha\chi_e^{2/3}m^2}{3\pi}f'(0)\int_0^\infty \frac{du}{(1+u)^3}\frac{5+7u+5u^2}{u^{2/3}} \\ &\approx 0.843\left(1 - i\sqrt{3}\right)\alpha\chi_e^{2/3}m^2 = \mathcal{O}\left(\alpha\chi_e^{2/3}\right)\end{aligned}}$$

Heuristic explanation



- Ultrarelativistic particle ($\chi \simeq eEp_\parallel/m^3 \gtrsim 1$):

$$\mathscr{E}(t) = \sqrt{p_\parallel^2 + e^2E^2t^2 + m^2} \approx p_\parallel + \frac{e^2E^2t^2}{2p_\parallel}$$

- Momentum conservation and energy mismatch:

$$\Delta\mathscr{E}(t) = \mathscr{E}'(t) + k' - \mathscr{E}(t) \simeq \frac{e^3E^3t^2}{m^3}\frac{\chi_\gamma}{\chi\chi'}, \quad p_\parallel = p'_\parallel + k', \quad \chi' = \chi - \chi_\gamma$$

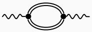
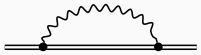
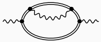
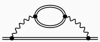
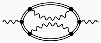
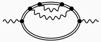
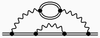
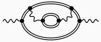
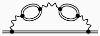
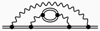
- Uncertainty principle $\Delta\mathscr{E} \cdot t \simeq 1$:

$$\Longrightarrow \quad \boxed{t_{\text{loop}} \simeq \frac{m}{eE}\left(\frac{\chi\chi'}{\chi_\gamma}\right)^{1/3} \equiv \frac{\gamma}{m}\left(\frac{\chi'}{\chi^2\chi_\gamma}\right)^{1/3} \underset{\chi_\gamma \sim \chi}{\simeq} \frac{\gamma}{m}\chi^{-2/3}}$$

- Mass shift estimation:

$$\frac{\delta m^2}{p_\parallel} \simeq \frac{e^2}{t_{\text{loop}}} \Longrightarrow \boxed{\delta m^2(\chi) \simeq \frac{e^2 p_\parallel}{t_{\text{loop}}} \simeq \alpha\chi^{2/3}m^2}$$

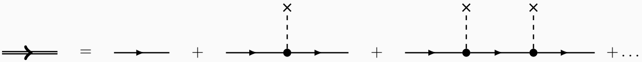
Higher-order corrections studied by Ritus group in 1969-1980

Polarization corrections			Mass corrections		
Diagram	**Asympotics ($\chi \gg 1$)**	**Reference**	**Diagram**	**Asympotics ($\chi \gg 1$)**	**Reference**
1 loop $\mathcal{O}(\alpha)$					
	$\boxed{\alpha\boldsymbol{\chi}^{\mathbf{2/3}}}$	Narozhny (1969)		$\boxed{\alpha\boldsymbol{\chi}^{\mathbf{2/3}}}$	Ritus (1970)
2 loops $\mathcal{O}(\alpha^2)$					
	$\boxed{\alpha^2\boldsymbol{\chi}^{\mathbf{2/3}}\log\chi}$	Morozov&Narozhny (1977)		$\boxed{\alpha^2\boldsymbol{\chi}\log\chi}$	Ritus (1972)
				$\alpha^2\chi^{2/3}\log\chi$	Morozov&Ritus (1975)
3 loops $\mathcal{O}(\alpha^3)$					
	$\alpha^3\chi^{2/3}\log\chi$	Narozhny (1979)		$\alpha^3\chi^{2/3}\log^2\chi$	Narozhny (1979)
	$\alpha^3\chi^{2/3}\log\chi$	Narozhny (1979)		$\alpha^3\chi^{4/3}$	Narozhny (1979)
	$\boxed{\alpha^3\boldsymbol{\chi}\log^2\chi}$	Narozhny (1980)		$\boxed{\alpha^3\boldsymbol{\chi}^{\mathbf{5/3}}}$	Narozhny (1980)
				$\alpha^3\chi\log^2\chi$	Narozhny (1980)

Evidence for bubble-chains dominance! For $\alpha\chi^{2/3} \gtrsim 1$ resummation needed!

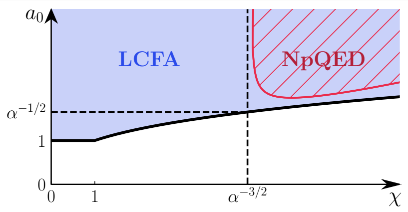
The Ritus-Narozhny conjecture

- For $a_0 \gtrsim \chi^{1/3} \gtrsim 1$
  - QED is *nonperturbative* w.r.t. **external field**
  - LCFA: arbitrary field $\mapsto$ CCF (exceptions: emission of soft γ's, motion at small angle with $\vec{S}$,...)
- SFQED = QED resummed over interactions with **external field**



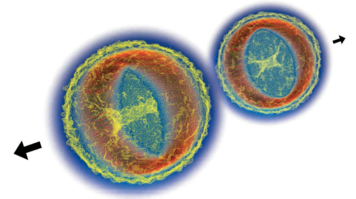
Ritus-Narozhny conjecture:

For $\alpha\chi^{2/3} \gtrsim 1$ SFQED becomes *nonperturbative* w.r.t. **radiative corrections** & further resummation needed



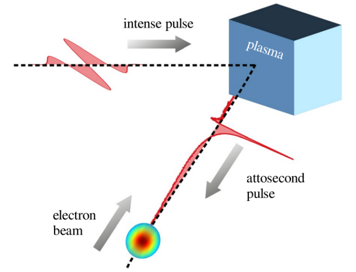
Recent proposals for attaining $\alpha\chi^{2/3} \sim 1$

- Beam-beam collisions at a 200GeV lepton collider



Yakimenko et al, PRL (2019)

- Collision with secondary attosecond pulses generated from $\sim 10^{23}$W/cm^2 short laser pulses



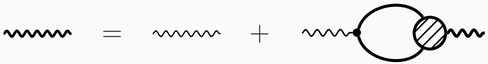
Baumann et al, Sci. Rep. (2019)

See also:

Blackburn et al., NJP (2019); Baumann & Pukhov, PPCF (2019); Di Piazza et al., PRL (2020)

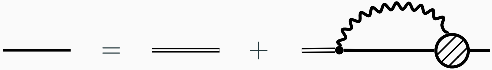
Path to resummation: Schwinger-Dyson equations

- Exact (fully dressed) **photon** propagator:



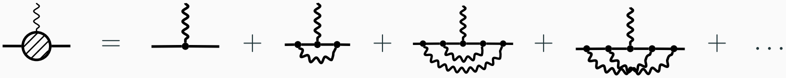
$(\star)$

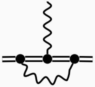
- Exact (fully dressed) **electron** propagator:



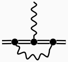
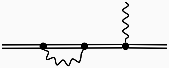
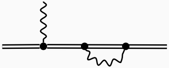
$(\star\star)$

- Non-closed without an input on a dressed vertex!
- Exact (fully dressed) dressed **vertex**:



- Dressed vertex at 1-loop:  $= \mathcal{O}\left(\alpha\chi^{2/3}\right)$

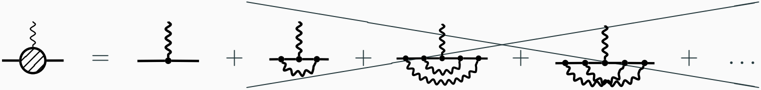
  Morozov, et al., JETP (1981); Di Piazza & Lopez-Lopez, PRD 102, 076018 (2020)

- But  $+$  $+$  $= \mathcal{O}\left(\alpha\chi^{1/3}\right)$

  Di Piazza & Lopez-Lopez, PRD (2020); cf. Gusynin, et al., PRL (1999)

  Implies that *in a certain gauge* vertices remain subdominant?

- If so then the fully dressed vertex can be replaced with the bare one:

  making the system of $(\star)$ and $(\star\star)$ closed – so that it could be solved iteratively or even selfconsistently...

Bubble-chain photon propagator

- Resummed bubble-chain photon propagator in a CCF:

Narozhny, JETP (1969)

$$D^{(\mathsf{bc})}_{\mu\nu}(l) = \frac{-i}{l^2+i0}\left(g_{\mu\nu} - \sum_{\lambda=1}^{2}\frac{\Pi_\lambda}{l^2-\Pi_\lambda}\epsilon^{(\lambda)}_\mu(l)\epsilon^{(\lambda)}_\nu(l)\right)$$

$$\epsilon^{(1)}_\mu(l) \propto F_{\mu\nu}l^\nu, \quad \epsilon^{(2)}_\mu(l) \propto F^\star_{\mu\nu}l^\nu,$$
$$\Pi_\lambda\left(l^2 \ll m^2\chi_l^{2/3}, \chi_l \gtrsim 1\right) \simeq \alpha\chi_l^{2/3}m^2$$

(running coupling neglected).

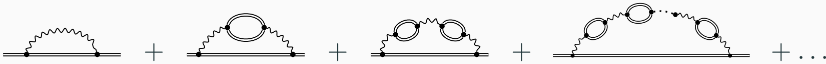
- Perturbative expansion:

$$\frac{\Pi_\lambda}{l^2-\Pi_\lambda} = \sum_{n=1}^{\infty} g^n(l^2,\chi_l), \quad g(l^2,\chi_l) \simeq \frac{\Pi(l^2,\chi_l)}{l^2+i0}$$

- Expansion parameter $g\left(l^2 \ll m^2\chi_l^{2/3}, \chi_l \gtrsim 1\right) \gg \alpha$

Bubble-chain contribution to electron MO

- Resummed bubble-type corrections to electron self-energy:



A.A. Mironov, S. Meuren & AMF, PRD 102, 053005 (2020)

- Main result:

$$\mathscr{M}^{(\mathrm{bc})}(\chi) - \underbrace{\mathscr{M}^{(0)}(\chi)}_{\text{no bubbles}} \simeq \underbrace{\mathscr{M}^{(\mathrm{II})}(\chi) + \mathscr{M}^{(\mathrm{III})}(\chi)}_{\text{resummed higher-order corrections}},$$

$$\mathscr{M}^{(0)}(\chi \gg 1) \simeq (0.843 - 1.46i)\alpha\chi^{2/3}m^2,$$

$$\mathscr{M}^{(\mathrm{II})}\left(\chi \gg \alpha^{-3/2}\right) \simeq (-0.995 + 1.72i)\alpha^{3/2}\chi^{2/3}m^2,$$

$$\mathscr{M}^{(\mathrm{III})}\left(\chi \gg \alpha^{-3/2}\right) \simeq (-0.103 - 1.18i)\alpha^{2}\chi m^2,$$

$$\frac{\mathscr{M}^{(\mathrm{II})}}{\mathscr{M}^{(0)}} = \mathcal{O}\left(\sqrt{\alpha}\right) \simeq 10\%, \quad \frac{\mathscr{M}^{(\mathrm{III})}}{\mathscr{M}^{(\mathrm{II})}} = \mathcal{O}\left(\sqrt{\alpha\chi^{2/3}}\right), \quad \frac{\mathscr{M}^{(\mathrm{III})}}{\mathscr{M}^{(0)}} = \mathcal{O}\left(\alpha\chi^{1/3}\right)$$

- MEANING & IMPLICATIONS?

Emergent expansion parameter (perturbative treatment)

- Off-shell photon: $l_0 = \sqrt{l_\parallel^2 + l^2} \simeq l_\parallel + l^2/2l_\parallel$
- Energy mismatch in the loop: $\Delta\mathscr{E} = \mathscr{E}' + l_0 - \mathscr{E} \mapsto \Delta\mathscr{E} + l^2/2l_\parallel$
- Effective values of virtuality: $l^2/l_\parallel \simeq \Delta\mathscr{E} \simeq t_{\text{loop}}^{-1}$

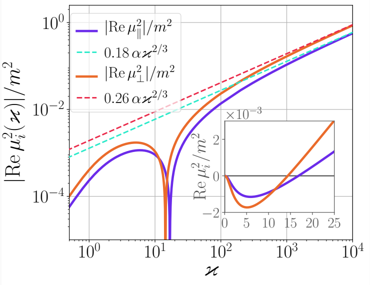
$$\implies \quad l^2 \simeq \frac{l_\parallel}{t_{\text{loop}}} \simeq \frac{l_\parallel}{\frac{m}{eE}\left(\frac{\chi\chi'}{\chi_\gamma}\right)^{1/3}} \simeq \frac{m^2\chi_\gamma^{4/3}}{(\chi\chi')^{1/3}}$$

- Emergent all-order expansion parameter:

$$g(\chi) \simeq \frac{\Pi(l^2,\chi_\gamma)}{l^2} \simeq \frac{\Pi(0,\chi_\gamma)}{m^2\chi_\gamma^{4/3}}\,(\chi\chi')^{1/3}$$

$$\Bigg]\ \max_{\chi_\gamma} \frac{\Pi(0,\chi_\gamma)}{m^2\chi_\gamma^{4/3}} \simeq \alpha\ \Bigg[\ \boxed{\simeq \alpha\chi^{2/3}}$$

for $\chi_\gamma \sim 1 \ll \chi$



- Resulting scales in bubble-chain diagrams: $\boxed{\chi_\gamma \simeq 1 \ll \chi}$,

$$\Pi(\chi_\gamma \simeq 1) \simeq \alpha m^2, \ \boxed{l^2 \simeq m^2\chi^{-2/3} \ll m^2} \implies \quad g \simeq \Pi/l^2 \simeq \alpha\chi^{2/3}$$

Nonperturbative resummation:

- Resumed higher-order corrections:

$$\delta\mathscr{M}^{(\geq 1\text{bubble})}(\chi) \simeq \int d\chi_\gamma d\sigma dl^2 e^{-i(\sigma^3/3+l^2\tau(\sigma,\chi_\gamma))} \cdots \frac{\Pi(l^2,\chi_\gamma)}{l^2-\Pi(l^2,\chi_\gamma)} \cdots,$$

  where $\tau(\sigma,\chi_\gamma) = \frac{\sigma}{m^2}(\chi\chi')^{1/3}/\chi_\gamma^{4/3}$ – dressed photon proper time.

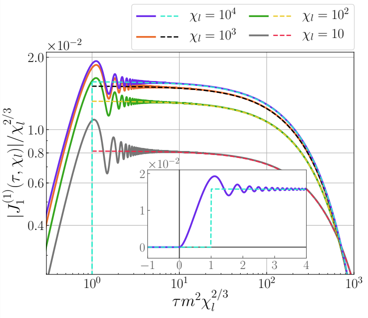
- Nonperturbatively, for $\tau \gg m^{-2}\chi^{-2/3}$ the integral over l^2 effectively sits onto the pole:

$$\int dl^2 \frac{\Pi(l^2,\chi_\gamma)e^{-il^2\tau}}{l^2-\Pi(l^2,\chi_\gamma)} \approx -2\pi i \Pi(0,\chi_\gamma)\, e^{-i\Pi(0,\chi_\gamma)\tau}$$

$$l^2 \simeq \frac{m^2\chi_\gamma^{4/3}}{(\chi\chi')^{1/3}},$$

$$\Pi(0,\chi_\gamma \gg 1) \simeq \alpha\chi_\gamma^{2/3}m^2,$$

$$l^2 \approx \Pi(0,\chi_\gamma) \implies \boxed{\chi_\gamma \simeq (\alpha\chi^{2/3})^{3/2}}$$



- Modified effective loop scale: $t_{\text{loop}} \simeq (\gamma/m)(\chi'/\chi^2\chi_\gamma)^{1/3} \simeq \gamma/m\chi^{2/3}\sqrt{\alpha}$. Hence $\delta\mathscr{M} \propto t_{\text{loop}}^{-1} \simeq \sqrt{\alpha}\mathscr{M}^{(0)}$ (cf $\mathscr{M}^{(\text{II})}$, similar for $\mathscr{M}^{(\text{III})}$).

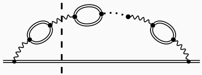
Relation to radiation processes

- Optical theorem:

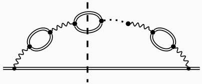
$$R_{e\to \text{all}} = -\sum_{\text{cuts}} \frac{1}{p_0} \text{Im}\,\mathscr{M}$$

- Initial (naive) guess:

$$\mathscr{M}^{(\text{II})} \simeq (-0.995 + 1.72i)\alpha^{3/2}\chi^{2/3}m^2 \quad \longleftrightarrow$$



$$\mathscr{M}^{(\text{III})} \simeq (-0.103 - 1.18i)\alpha^2\chi m^2 \quad \longleftrightarrow$$



- However, **only stable** states should be included as the final states!

M. Veltman, Physica (1963)

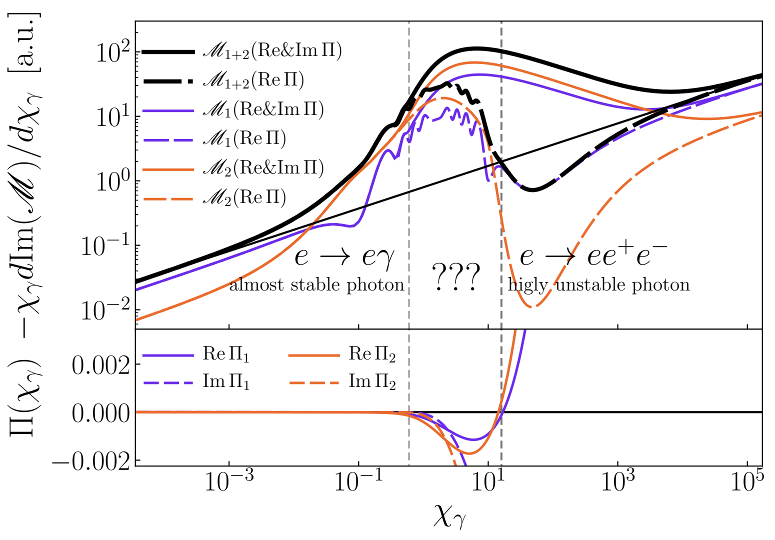
$$\text{Im}\,\frac{1}{p^2 - \Sigma' + i\Sigma''} \underset{\Sigma''\to 0}{\mapsto} -\pi\delta\left(p^2 - \Sigma'\right)$$

E. Sozinov, LPHYS'21

- Since a bubble-dressed photon with $\chi_\gamma \simeq (\alpha\chi^{2/3})^{3/2} \gtrsim 1$ is highly unstable, photon emission is excluded

Spectral signature (under development)

- Modification of the radiation spectrum ($\chi = 10^7$)

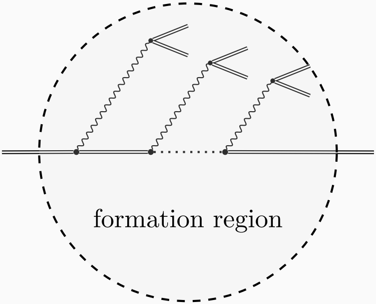


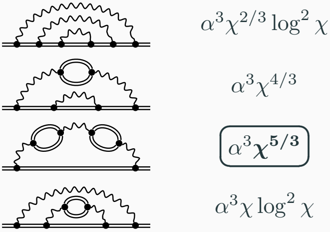
Radiation ($\tau_\gamma \sim \tau_e$) vs formation (τ_f) times

- Morever, e.g.

$$\frac{\tau_\gamma}{\tau_f} \simeq \left(\frac{R_{\gamma\to e^+e^-}}{\Delta\mathscr{E}}\right)^{-1} \simeq \left(\frac{|\mathrm{Im}\,\Pi|/2l_0}{l^2/2l_0}\right)^{-1} \simeq \frac{l^2}{\Pi} \simeq \frac{1}{g} \lesssim 1 \quad \text{for } g \gtrsim 1$$

probably implies that a relevant observable process should be coherent production of multiple ‘soft’ ($\chi_f \simeq 1 \ll \chi$) pairs:





- First step: general γ-matrix/tensor structure of a MO in a CCF

Mironov & AMF, arXiv:2109.00634

Questions?