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Electromagnetic Waves in a Vacuum

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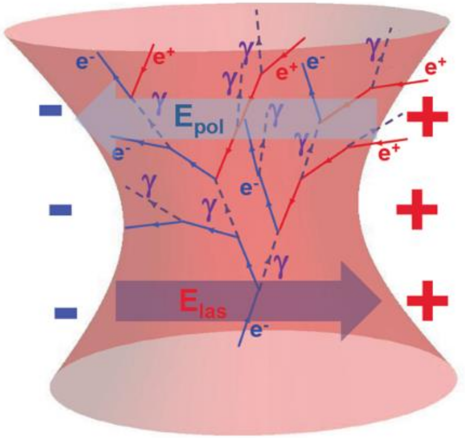
Extreme Field Limits: Nonlinear QED Vacuum

Towards studying of nonlinear QED effects with high power lasers.
Radiation dominated & QED regimes
in the high intensity electromagnetic wave interaction
with charged particles & vacuum.

$$eE_s\lambda\!\!\!^{-}_C = m_e c^2 \quad \Rightarrow \quad E_s = \frac{m_e^2 c^3}{e\hbar} \quad \Rightarrow \quad I_s = c\frac{E_s^2}{4\pi} \approx 10^{29}\frac{\mathrm{W}}{\mathrm{cm}^2}$$



Schwinger (Sauter, Bohr,…) field

QED parameters:

$$\chi_e = \frac{e\hbar\sqrt{-\left(F^{\mu\nu}p_\nu\right)^2}}{m_e^3c^4} \approx \frac{E}{E_s}\frac{p_e}{m_e c} \qquad [N\omega_0 + e \rightarrow e + \omega_\gamma]$$

$$\chi_\gamma = \frac{e\hbar^2\sqrt{-\left(F^{\mu\nu}k_\nu\right)^2}}{m_e^3c^4} \approx a\frac{\hbar^2\omega_0\omega_\gamma}{m_e^2c^4} \qquad [N\omega_0 + \omega_\gamma \rightarrow e^+e^-]$$

Dittrich W and Gies H, Probing the Quantum Vacuum. Perturbative Effective Action Approach in Quantum Electrodynamics and its Application, (Springer-Verlag, 2000)

G. A. Mourou, T. Tajima, and S. V. Bulanov, Rev. Mod. Phys. 78, 309 (2006)

M. Marklund and P. K. Shukla, Rev. Mod. Phys. 78, 591(2006)

F. Ehlotzky, K. Krajewska, and J. Z. Kaminski, Rep. Prog. Phys. 72, 046401 (2009)

A. Di Piazza, C. M. Muller, K. Z. Hatsagortsyan, and C. H. Keitel, Rev. Mod. Phys. 84, 1177 (2012)

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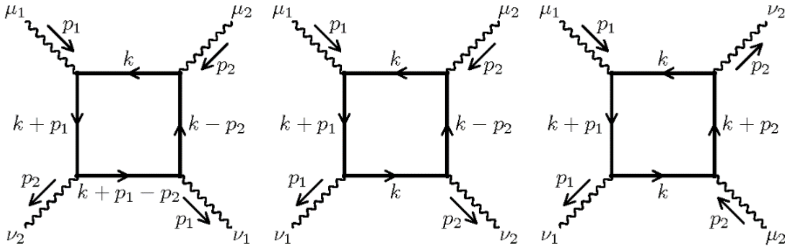
Outline

- Basic sources of modifications of high intensity e.m. waves behavior
- Main effects:
  - Phase shift (speed down)
  - Birefringence
  - High order harmonic generation
  - Nonlinear EM waves in the QED vacuum
  - Solitons
  - Nonlinear EM waves in a dispersive vacuum
  - Synergic Cherenkov radiation-Compton scattering
    - Reaching high laser intensity by a radiating electron
- Connections with experiments

Photon–Photon Scattering

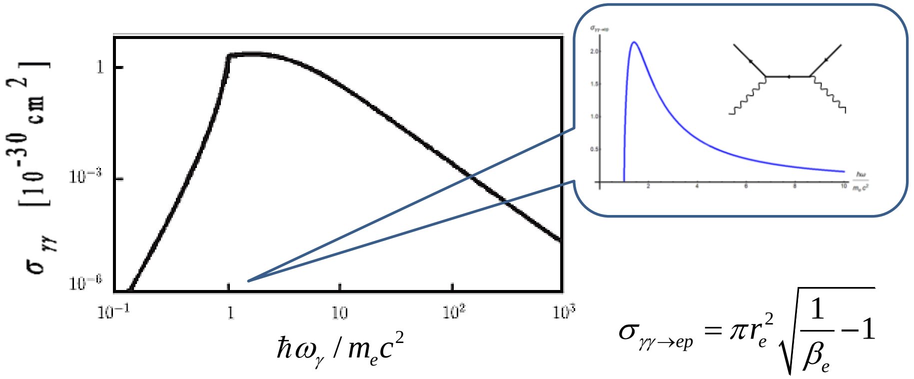
In the QED, photon-photon scattering occurs via interaction with the sea of virtual electron-positron pairs:



Dependence of the photon-photon scattering cross section on the photon frequency (assume that $\omega_1 = \omega_2 = \omega_\gamma$)

R. Karplus and M. Neuman, Phys. Rev. 83, 776 (1951)

$$\sigma_{\gamma\gamma} = \begin{cases} \left(\dfrac{973}{10125\pi}\right)\alpha^2 r_e^2 \left(\dfrac{\hbar\omega_\gamma}{m_e c^2}\right)^6 & \text{for } \hbar\omega_\gamma << m_e c^2 \\ \left(\dfrac{3}{12\pi}\right)^2 \alpha^2 r_e^2 \left(\dfrac{m_e c^2}{\hbar\omega_\gamma}\right)^2 & \text{for } \hbar\omega_\gamma >> m_e c^2 \end{cases}$$



$$\sigma_{\gamma\gamma\to ep} = \pi r_e^2 \sqrt{\frac{1}{\beta_e} - 1}$$

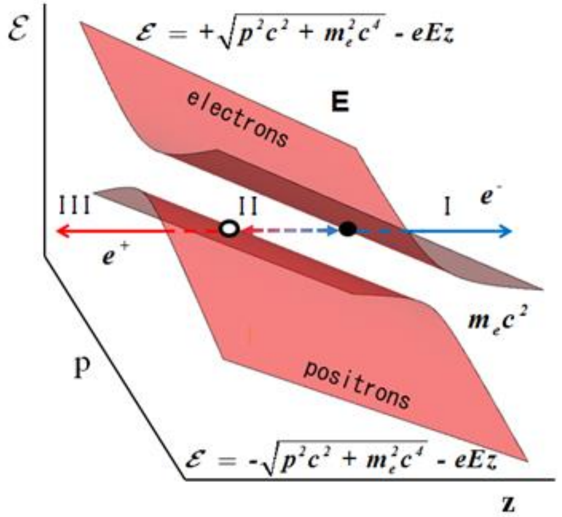
$$\beta_e = \sqrt{1 - \frac{2m_e^2 c^4}{\hbar^2 \omega_1 \omega_2 (1-\cos\varphi)}}$$

Experiment:

ATLAS Collaboration, Nature Physics 13, 852 (2017)

G. Breit and J. A. Wheeler, Physical Review 46, 1087 (1934)

Heisenberg-Euler Lagrangian



In the long wavelength and low frequency approximation ($|\partial_\mu A_\nu|/|A_\mu| << \lambda_C^{-1}$) the Lagrangian describing the electromagnetic field in vacuum is

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

where $\mathcal{L}_0 = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$ with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ gives the Maxwell equations.

The Heisenberg–Euler term

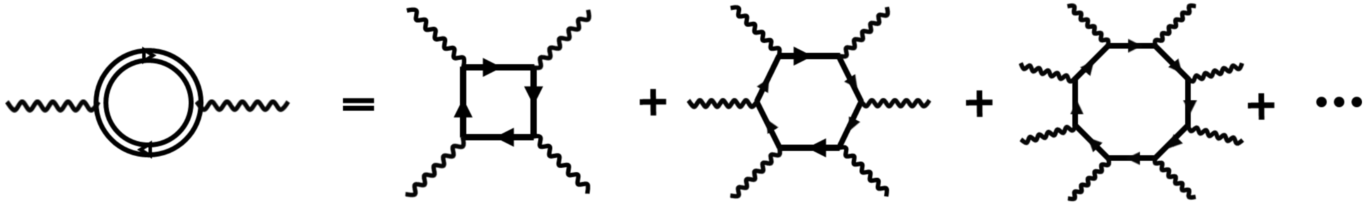
$$\mathcal{Re}\{\mathcal{L}'\} = -\frac{m_e^4}{8\pi^2}\,\text{p.v.}\int_0^\infty \frac{\exp(-\eta)}{\eta^3}\left\{1 - \frac{\eta^2}{3}(\mathfrak{a}^2 - \mathfrak{b}^2) - [\eta\mathfrak{a}\cot(\eta\mathfrak{a})][\eta\mathfrak{b}\coth(\eta\mathfrak{b})]\right\} d\eta$$

$$\mathcal{Im}\{\mathcal{L}\} = \frac{m_e^4}{4\pi^3}\mathfrak{a}^2 \exp\left(-\frac{\pi}{\mathfrak{a}}\right)$$

The invariant fields $\mathfrak{a}$ and $\mathfrak{b}$ can be expressed in terms the Poincare invariants $\mathfrak{F} = F^{\mu\nu}F_{\mu\nu}$ and $\mathfrak{G} = F^{\mu\nu}\tilde{F}_{\mu\nu}$ (dual tensor equals $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$) as

$$\mathfrak{a} = \sqrt{\sqrt{\mathfrak{F}^2 + \mathfrak{G}^2} + \mathfrak{F}} \text{ and } \mathfrak{b} = \sqrt{\sqrt{\mathfrak{F}^2 + \mathfrak{G}^2} - \mathfrak{F}}$$

Here we use the units $c = \hbar = 1$, and the e.m. field is normalized on the QED critical field E_S.



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Radiation Corrections

N. B. Narozhny, Sov. Phys. JETP 28, 371 (1969); V. I. Ritus, Sov. Phys. JETP, 30, 1181 (1970)

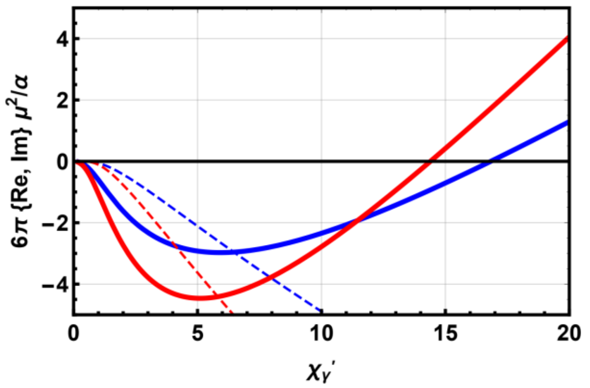
At the focus of 10 PW laser the field intensity can reach $10^{24}\mathrm{W/cm}^2$, i.e. $a_0 = 10^3$

Vacuum polarization changes the refraction index: radiation correction to the "photon mass" results in the dispersion equation for the e.m. wave frequency and wave vector

$$\omega^2 - k^2c^2 - \mu_{||,\perp}^2 \frac{c^4}{\hbar^2} = 0$$

where

$$\mu_{||,\perp}^2 = -\alpha m_e^2 \begin{cases} \left[\dfrac{11 \pm 3}{90\pi}\chi_\gamma^2 + i\sqrt{\dfrac{3}{2}}\dfrac{3 \pm 1}{16}\chi_\gamma \exp\left(-\dfrac{8}{3\chi_\gamma}\right)\right] & \text{for } \chi_\gamma \ll 1 \\ \left[\dfrac{5 \pm 1}{28\pi^2}\sqrt{3}\Gamma^4\left(\dfrac{2}{3}\right)(1 - i\sqrt{3})(3\,\chi_\gamma)^{2/3}\right] & \text{for } \chi_\gamma \gg 1 \end{cases}$$



with $\alpha = e^2/\hbar c \approx 1/137$. When $\alpha\chi_\gamma^{2/3} \to 1$ the "photon mass" tends to m_e.
In the limit χ_γ<<1 the difference between the vacuum refraction index and unity is

$$\Delta n_{\parallel,\perp} = \alpha \frac{11 \pm 3}{45\pi}\left(\frac{E}{E_S}\right)^2$$

i. e. the normalized phase velocity of the e.m. wave equals $\beta_{||,\perp} = 1 - \varepsilon_{||,\perp}(E/E_S)^2$ with $\varepsilon_{||,\perp} = \alpha(11 \pm 3)/45\pi \approx 10^{-4}$.

Heuristic View of Perturbative Lagrangian

$$c = \hbar = m_e = 1 \qquad\qquad F^{\mu\nu} \to \frac{F^{\mu\nu}}{E_s}$$

Symbolic form of the expansion ($|F^{\mu\nu}| \ll 1; \quad \chi_\gamma \ll 1; \quad \alpha \ll 1$):

$$\alpha\mathcal{L} = F^2 \qquad\qquad + \alpha F^4 + \alpha F^6 + \cdots$$ (HE Lagrangian)

$$+ \alpha k^2 F^2 + \alpha k^2 F^4 + \alpha k^2 F^6 + \cdots$$ (finite χ corrections)

$$+ \alpha^2 F^4 + \alpha^2 F^6 + \cdots$$ (two-loop corrections)

$$+ \cdots$$

H. Gies and F. Karbstein, J. High Energ. Phys. **2017**, 108 (2017).
V. I. Ritus, Sov. Phys.-JETP **42**, 774 (1975)

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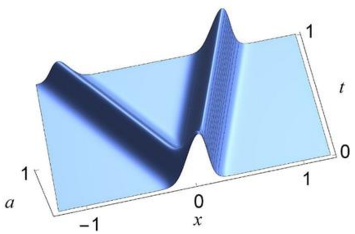
Probing the QED Vacuum

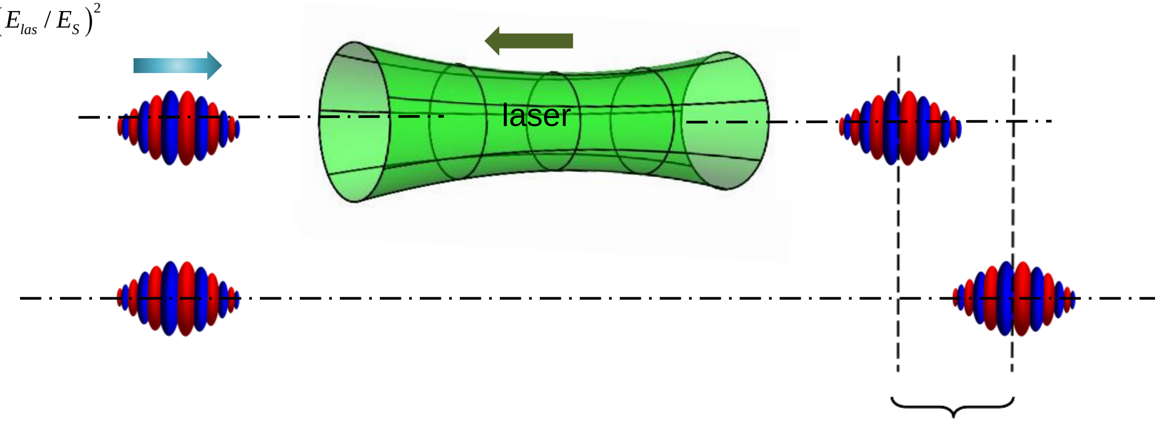
Vacuum polarization

$$v = c(1-2\delta n), \qquad \delta n = 2\varepsilon_2 W^2 \approx \alpha\left(E_{las}/E_S\right)^2$$

$$\alpha = e^2/\hbar c = 1/137, \quad E_S = m_e^2 c^3/e\hbar$$

$$(\omega + kc)(\omega - kc + 2kc\varepsilon_2 W^2) = 0$$





$$\Delta x = \alpha\left(E_{las}/E_S\right)^2 l_f$$

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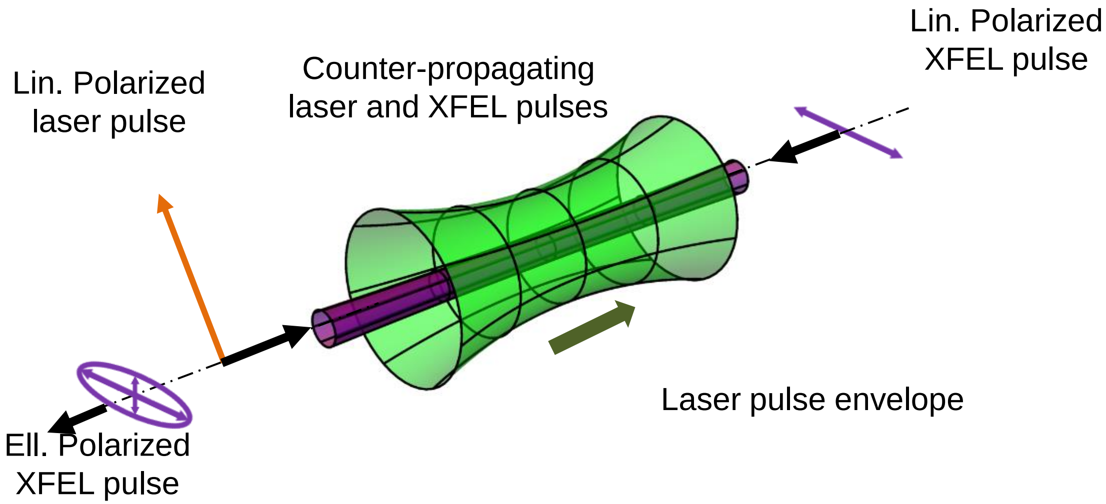
Vacuum birefringence



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B. Shen, et al., Plasma Phys. Contr. Fus. 60, 044002 (2018)

T. Tajima and R. Li, SPIE (2018)

High Order Harmonics and Mixing in QED Vacuum

H-E Lagrangian in the Dirac Light Cone coordinates

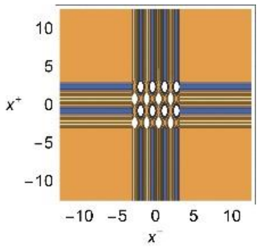
We use Dirac's light-cone coordinates

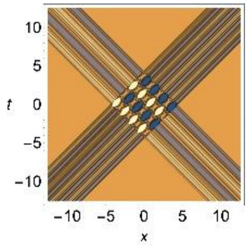
$$x^+ = \frac{x+t}{\sqrt{2}}, \quad x^- = \frac{x-t}{\sqrt{2}}, \quad u = -\frac{E_z - B_y}{\sqrt{2}}, \quad w = -\frac{E_z + B_y}{\sqrt{2}}, \quad \partial_\pm = \frac{\partial}{\partial x^\pm}$$

Lagrangian variation yields system of nonlinear equations for EM wave

$$\partial_+ \left(\frac{\partial \mathcal{L}}{\partial u}\right) + \partial_- \left(\frac{\partial \mathcal{L}}{\partial w}\right) = 0 \quad \text{and} \quad \partial_+ w - \partial_- u = 0$$

$$\mathcal{L} = uw + \alpha Q(uw), \quad \text{where} \quad Q(\zeta) = \sum_{m>1} b_m \zeta^m$$

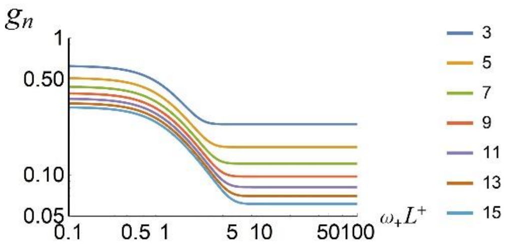


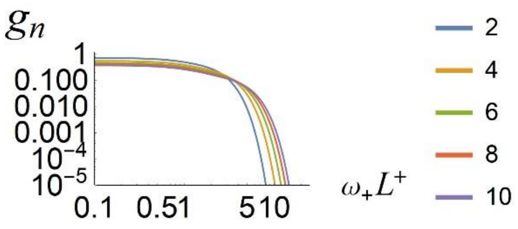


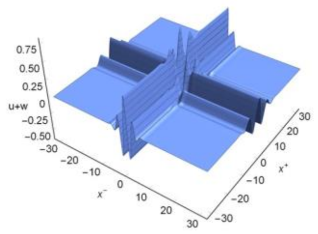
High Order Harmonic Generation

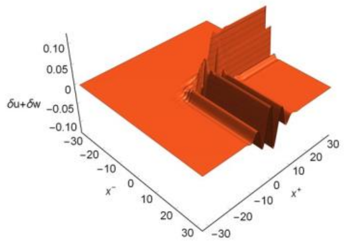
Perturbation theory yields

$$u^{out} = u^{in} - \frac{\alpha}{2}\left[\int_{-\infty}^{+\infty} (w^{in})^2\, Q''(u^{in} w^{in})\, dx^+\right] \partial_- u^{in} \qquad w^{out} = w^{in} + \frac{\alpha}{2}\left[\int_{-\infty}^{+\infty} (u^{in})^2\, Q''(u^{in} w^{in})\, dx^-\right] \partial_+ w^{in}$$









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Bohl P, King B and Ruhl H 2015 Vacuum high-harmonic generation in the shock regime Phys. Rev. A 92 032115

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Nonlinear EM waves in the QED vacuum

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F. Pegoraro and S. V. Bulanov, Hodograph solutions of the wave equation of nonlinear electrodynamics in the quantum vacuum, Phys. Rev. D 100, 036004 (2019)
F. Pegoraro and S. V. Bulanov, Nonlinear, nondispersive wave equations: Lagrangian and Hamiltonian functions in the Hodograph transformation, Phys. Lett.A 384,126064 (2020)
TM Jeong, SV Bulanov, PV Sasorov, G Korn, J Koga, SS Bulanov, Photon scattering by a 4pi-spherically-focused ultrastrong electromagnetic wave, Phys.Rev.A102,023504 (2020)
F. Pegoraro and S. V. Bulanov, Nonlinear electrodynamics at cylindrical “cumulation” fronts, Rendiconti Lincei. Scienze Fisiche e Naturali 31, 303 (2020)
F. Pegoraro and S. V. Bulanov, Nonlinear waves in a dispersive vacuum described with a high order derivative electromagnetic Lagrangian , Phys. Rev. D 103, 096012 (2021)

Lagrangian variation $\partial_+ \left(\partial L / \partial u\right) + \partial_- \left(\partial L / \partial w\right) = 0$ yields system of nonlinear equations for EM wave

$$\partial_+ w - \partial_- u = 0$$

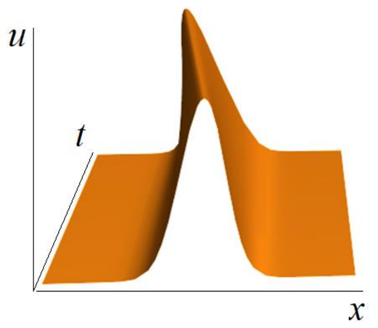
$$\left[1 - 4\varepsilon_2 \left(W_0 + w\right) u - 9\varepsilon_3 \left(W_0 + w\right)^2 u^2\right] \partial_+ u - \left[\varepsilon_2 \left(W_0 + w\right)^2 - 3\varepsilon_3 \left(W_0 + w\right)^3 u\right] \partial_- u - \left[\varepsilon_2 u^2 + 3\varepsilon_3 \left(W_0 + w\right) u^3\right] \partial_+ w = 0$$

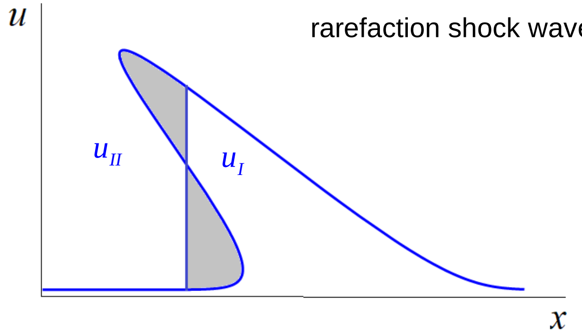
with $\varepsilon_2 = (2/45\pi)\alpha$ and $\varepsilon_3 = (32/315\pi)\alpha$ and $W_0 = E_0 / E_S$.

Analizing this system with the hodograph transform, self-similar solutions, etc. gives

$$\partial_t u + \left(v_W - 2\left(4\varepsilon_2^2 + 3\varepsilon_3\right) W_0^3 u\right) \partial_x u = 0$$

Riemann wave solution describes the nonlinear EM wave steepening and breaking leading to the shock wave formation: **the nonlinearity and dissipation make the shock wave**.





Shock wave front velocity equals $v_{SW} = c(1 - 2\delta n) - (4\varepsilon_2^2 + 3\varepsilon_3) W_0^3 (u_I + u_{II})$

Electromagnetic Solitons in QED Vacuum

S. V. Bulanov, P. V. Sasorov, H. Kadlecova, S. S. Bulanov, G. Korn, Electromagnetic solitons in quantum vacuum, Phys. Rev. D 101, 016016 (2020)

The nonlinearity and dispersion make the soliton

The photon invariant mass:

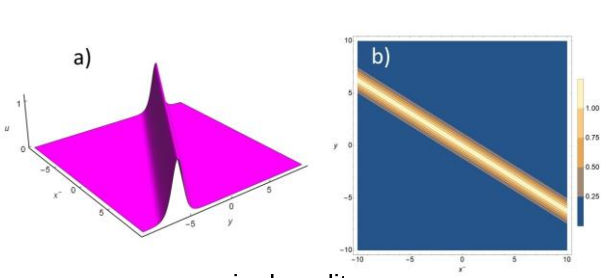
$$\mu^2 = -\alpha m_e^2 \left\{ \frac{7}{45\pi}\left[\chi_\gamma^2 + \frac{1}{3}\chi_\gamma^4\right] + i\sqrt{\frac{3}{32}}\,\chi_\gamma \exp\left(-\frac{8}{3\chi_\gamma}\right) \right\}$$

The term $\frac{1}{3}\chi_\gamma^4$ describes the dispersion effects.

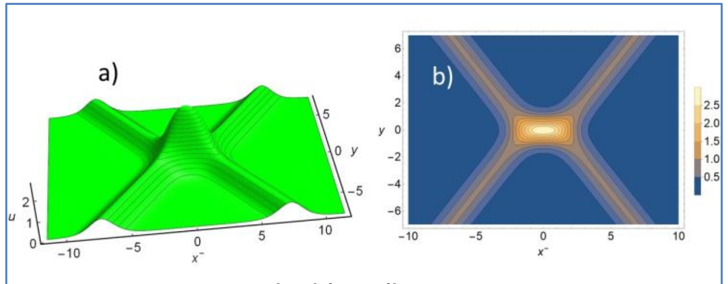
Implementation of nonlinearity and dispertion effects to the wave equation results in the Kadomtsev-Petviashvili equation (known as 2D or 3D Korteveg de Vries equation)

$$\partial_- \left[\partial_+ u + \left(\frac{4e^2}{45\pi} W_0^2 + \frac{32\sqrt{2}e^2}{105\pi} W_0^3 u \right) \partial_- u + \frac{8e^2}{135\pi m_e^2} W_0^4 \partial_{---} u \right] = -\frac{1}{2}\partial_{yy} u$$

Typical KP solitons:

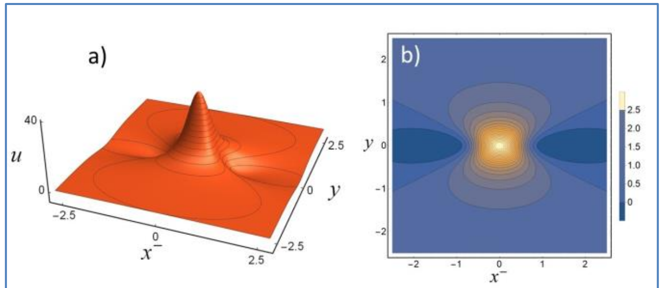


single soliton



double soliton

ROGUE WAVE



lump soliton

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G. Biondini and D. E. Pelinovsky, Kadomtsev-Petviashvili equation, Scholarpedia 3, 6539 (2018)

HIGH ORDER DERIVATIVE TERMS IN THE QED LAGRANGIANS

Lee-Wick Lagrangian [T. D. Lee and G. C. Wick, Phys. Rev. D 2, 1033 (1970)]

$$L = L_0 + L_{LW} \text{ with } L_{LW} = \frac{1}{M^2} F_{\mu\nu}(\partial^\alpha \partial_\alpha F^{\mu\nu})$$

MME-Lagrangian [S. G. Mamaev, V. M. Mostepanenko, and M. I. Eides, Sov. J. Nucl. Phys. 33, 569 (1981)]

$$L = L_{HE} + L_{MME} \text{ with } L_{MME} = \mu\left[-(\partial_\mu F_\nu^{\ \mu})(\partial_\mu F^{\mu\nu}) + F_{\mu\nu}\partial_\kappa \partial^\kappa F^{\mu\nu}\right]$$

with $\mu = \alpha / m_e^2$. The physical interpretation of higher order derivative Lagrangians presents some difficulties as these Lagrangians lead to "ghost" degrees of freedom and to instabilities.

OSTROGRADSKY THEOREM

In 1850 Ostrogradsky [M. Ostrogradsky, Mem. Ac. St. Petersburg 6, 385 (1850)] proved in the context of classical mechanics that a Lagrangian of the form $L(q, \dot{q}, \ddot{q})$, which requires four initial conditions and thus involves four canonical variables, leads to a Hamiltonian that is not bounded from below with respect to a "ghost" degree of freedom.

CONSTANT CROSS FIELD: KdV SOLITON

F. Pegoraro and S. V. Bulanov, Nonlinear waves in a dispersive vacuum described with a high order derivative electromagnetic Lagrangian , Phys. Rev. D 103 (9), 096012 (2021)

SOLUTIONS IN CONSTANT CROSS FIELDS

From the Lagrangian $L = -uw + \varepsilon_2 (uw)^2 - \varepsilon_3 (uw)^3 - \mu\left(w^2\partial_- u + uw\partial_+ u + uw\partial_- w + u^2\partial_+ w\right)^2$

using the cross field approximation corresponding to $w = Su + W_0$ we obtain the nonlinear wave equation

$$2\mu W_0^4 \partial_{---} u = \partial_+ (u - \varepsilon_2 W_0 u^2) + \partial_- (Su - 2\varepsilon_2 W_0^2 u + 3\varepsilon_3 W_0^3 u^2)$$

Assuming dependence on the phase variable: $\psi = x^- + Sx^+ = \frac{1}{\sqrt{2}}\left(x(1+S) - t(1-S)\right)$,

i. e. the wave velocity is $V = \left(\frac{1-S}{1+S}\right)$, we find the soliton solution

$$u(\psi) = \frac{3(S - \varepsilon_2 W_0^2)}{2\varepsilon_2 W_0 - 3\varepsilon_3 W_0^3} \cosh^{-2}\left[\frac{\sqrt{S - \varepsilon_2 W_0^2}}{2\sqrt{\mu W_0^4}}\psi\right]$$

The soliton amplitude and width are $u_0 = \frac{3(S - \varepsilon_2 W_0^2)}{2\varepsilon_2 W_0 - 3\varepsilon_3 W_0^3}$ and $l_0 = \frac{2\sqrt{\mu W_0^4}}{\sqrt{S - \varepsilon_2 W_0^2}}$

The soliton propagation velocity depends on the soliton amplitude as $V \approx 1 - 2\varepsilon_2 W_0^2 - \varepsilon_2 W_0 u_0$

Synergic Cherenkov Radiation-Compton Scattering

I.M. Dremin, Cherenkov radiation and pair production by particles traversing laser beams, JETP Lett. 76, 151 (2002);
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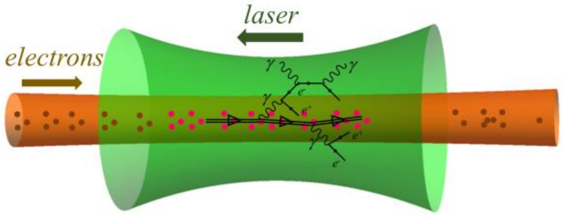
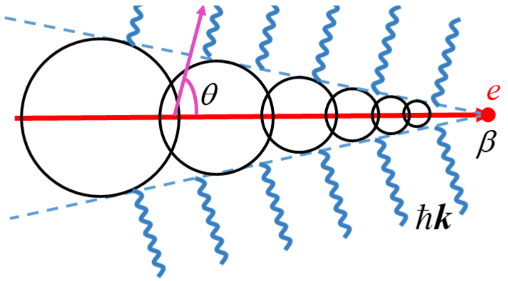
S. V. Bulanov, P. V. Sasorov, S. S. Bulanov, G. Korn, Synergic Cherenkov-Compton radiation, Phys. Rev. D 100, 016012 (2019)
I I Artemenko, E N Nerush, and I Yu Kostyukov, Quasiclassical approach to synergic synchrotron–Cherenkov radiation in polarized vacuum, New J. Phys. 22 093072 (2020)



$$\gamma_0 > \gamma_{Ch} = \frac{1}{\sqrt{2\Delta n_\pm}} = \sqrt{\frac{45\pi E_S^2}{\alpha(11\pm3)E_0^2}} \approx 30\sqrt{\frac{I_S}{I_0}}$$

The laser intensity $I_0 = cE_0^2/4\pi$ in the focus region of 10 PW laser is equal to 10^{24} W/cm^2;
$I_S = cE_S^2/4\pi \approx 10^{29} W/cm^2$, i. e. the Cherenkov radiation threshold is exceeded for the electron energy above 10 GeV.

The Cherenkov cone with the angle $\theta_{Ch} = 2\sqrt{\varepsilon_\pm I_0/I_S}$ in the focus of 10 PW laser is $\approx 2\times10^{-5}$.

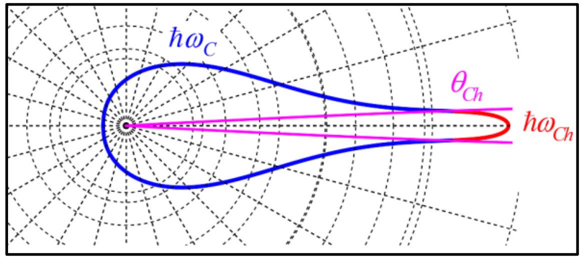


The photon invariant mass

$$\mu^2_{\|,\perp} = -\alpha m_e^2 \begin{cases} \left[\frac{11\pm3}{90\pi}\chi_\gamma^2 + \dots + i\sqrt{\frac{3}{2}}\frac{3\pm1}{16}\chi_\gamma \exp\left(-\frac{8}{3\chi_\gamma}\right)\right] & \text{for } \chi_\gamma<<1 \\ \left[\frac{5\pm1}{28\pi^2}\sqrt{3}\,\Gamma^4\left(\frac{2}{3}\right)(1-i\sqrt{3})(3\chi_\gamma)^{2/3}\right] & \text{for } \chi_\gamma>>1 \end{cases}$$

$$\mathbf{p}_0 + \hbar\mathbf{k}_0 = \mathbf{p} + \hbar\mathbf{k},$$
$$m_e c^2\gamma_0 + \hbar\omega_{\gamma,0} = m_e c^2\gamma + \hbar\omega_\gamma,$$
$$\mathbf{k} = \frac{\mathbf{k}}{|\mathbf{k}|c} n_\pm\,\omega$$

At the high photon energy end, when $\chi_\gamma = \frac{E_0}{E_S}\frac{\hbar(\omega + k_x c)}{m_e c^2} > 1$, the vacuum polarization effects and the Cherenkov radiation weaken.
As a result, the photons with the energy above $\hbar\omega_\gamma = m_e c^2 E_S/E_0$ are not present in the radiation. For 10 PW laser parameters this energy is 100 MeV.



Reaching high laser intensity by a radiating electron

J. Magnusson, A. Gonoskov, M. Marklund, T. Zh. Esirkepov, J. K. Koga, K. Kondo, M. Kando, S. V. Bulanov, G. Korn, and S. S. Bulanov, Laser-particle collider for multi-GeV photon production, Phys. Rev. Lett. 25, 254801 (2019)
S. V. Bulanov, Electron Dynamics in the Field of Strong Plasma and Electromagnetic Waves: A Review, Physics of Wave Phenomena, 29, 1 (2021)
M. Jirka, P. Sasorov, S. S. Bulanov, G. Korn, B. Rus, and S. V. Bulanov, Reaching high laser intensity by a radiating electron, Phys. Rev. A 103, 053114 (2021)

In the equations of electron motion the radiation friction force

$$\mathbf{g}_{\mathbf{rad}} = -\varepsilon_{\mathrm{rad}} a_0^2 G_e(\chi_e)\, \mathbf{p}\, \gamma_e = -\frac{2}{3}\alpha\, a_S \chi_e^2 G_e(\chi_e) \frac{\mathbf{p}}{\gamma_e}$$

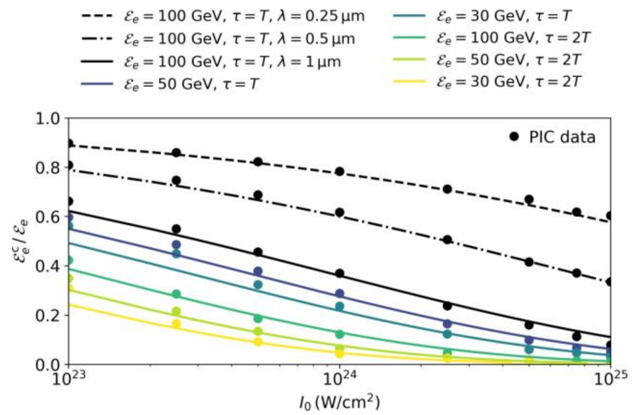
QED effects incorporated with the form-factor, $G_e(\chi_e)$, equal to the ratio of the full radiation intensity to the intensity emitted by a classical electron (the Gaunt factor)

$$G_e(\chi_e) = \underset{\chi_e >> 1}{\approx} \frac{0.6}{\chi_e^{4/3}}$$

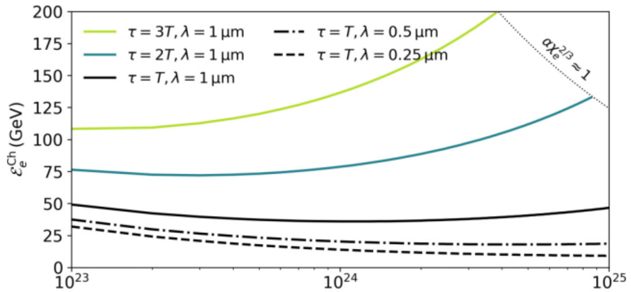
Electron equation of motion in the form $\dot{\chi}_e = -\frac{2}{3}\alpha a_0 \chi_e^2 G_e(\chi_e)$ yield

$\chi_e^{1/3} \approx \chi_e^{1/3}(0) - 0.1\alpha a_0$, i. e. for $a_0 = 10^3$, $0.1\alpha a_0 \approx 0.7$ and $\frac{p_0}{m_e c} = 2\times 10^5$

the parameter $\chi_e(0) \approx 2\frac{a_0}{a_S}\frac{p_0}{m_e c} \approx 800$ is substantially large to reach the high intensity region.



Electron energy in the center of the laser pulse as a function of laser intensity



The minimum required initial electron energy for the emission of the Cherenkov radiation in the center of the laser pulse

DISCUSSIONS

Birefringence and phase shifts: The phase difference between the e.m. pulse colliding with the counter-propagating wave and the pulse which does not interact with high intensity wave, equals

$$\delta\psi = \frac{8}{45}\alpha\,\frac{d}{\lambda}\left(\frac{E}{E_s}\right)^2$$

where λ is the pulse wavelength and is *d* the interaction length, plays a central role in discussion of experimental verification of …the QED vacuum birefringence.
10 PW laser the intensity can reach 10^{24} W/cm^2 for which the EM field is $(E/E_s)^2 = 10^{-5}$. For $d/\lambda = 10^4$ we have $\delta\psi = 10^{-4}$.

High-Order-Harmonics: Colliding two optical pulses of intensities 5×10^{26} W/cm^2 and of 30 fs durations inside the focus spot size of 1 μm, we obtain several quanta of the 3rd harmonics. Using of an x-ray pulse instead of one of the optical ones may diminish the required intensities by orders of magnitude, depending on details of the geometry.

Cherenkov-Compton: 10 -100 GeV LWFA electrons accelerated by 10 PW pulse laser collide with the 10^{24} W/cm^2 e.m. field. Threshold electron energy is of 10 GeV. Traversing the laser focus the electron emits 0.2 photons. The Cherenkov formation length is $l_f = 5\times10^{-5}$ cm, whereas the non-linear Compton formation length is much less. Hence, Compton and Cherenkov emissions interfere with each other, forming Synergic Compton-Cherenkov effect. The angular distribution of radiation differs by about 2-5% from the pure Compton process for scattered γ-quanta (with $\chi_\gamma \sim 1 \Rightarrow \varepsilon_\gamma \sim 100$ MeV). Interaction of the Compton scattered photons with the laser field will result in the Breit-Wheeler electron-positron pair plasma generation.

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1-10-100 PW LASERS

Danson C N, Haefner C, Bromage , et al.,
Petawatt and exawatt class lasers worldwide,
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Peng Y, Xu Y, Yu L, Wang X, Li Y, Lu X, Wang C,
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Overview and status of station of extreme light
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LWFA ELECTRONS

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Acceleration to 8 GeV in a Laser-Heated
Capillary Discharge Waveguide
Physical Review Letters 122, 084801 (2019)

DISCUSSIONS

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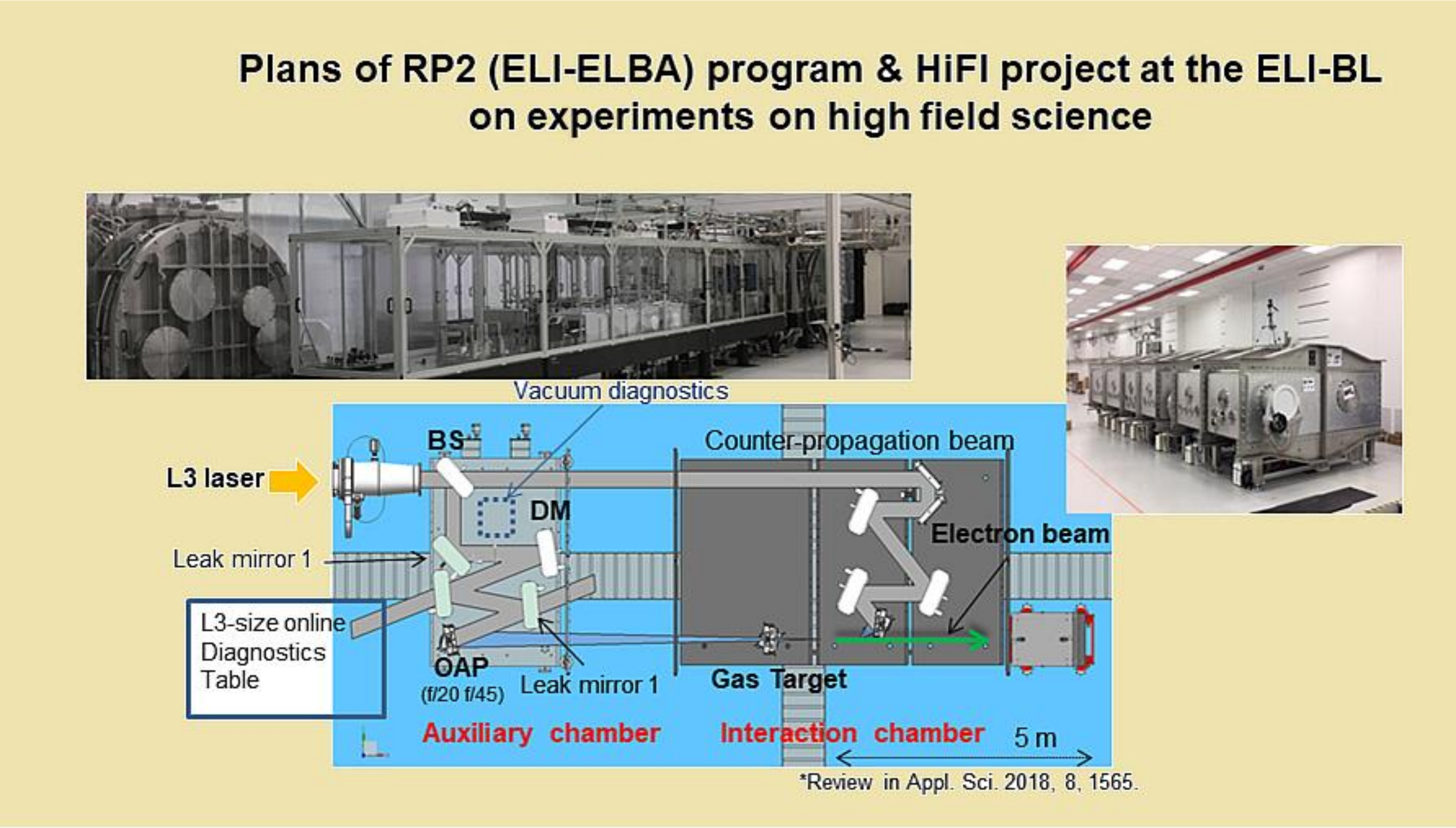

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Physical Review Letters 122, 084801 (2019)

CONCLUSION

- The QED vacuum in the long-wavelength limit shows properties of continuous medium with the optical properties determined by the refraction index dependent on the electromagnetic wave amplitude.

- In the high photon energy range the QED vacuum is a medium possessing dispersion and dissipation.

- The vacuum can be considered as a "plasma" of virtual electron-positron pairs. The electron-positron pairs may become real when the electric field approaches the Schwinger limit: $E \to E_S$ ($\chi_\gamma >> 1$).

- We extend the field of applications of the methods of Nonlinear Wave Theory to the QED vacuum.

Thank you for listening to me!