First-Order Strong-Field QED Processes Including the Damping of Particle States

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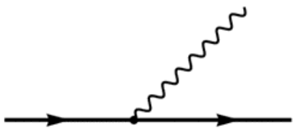
Contents

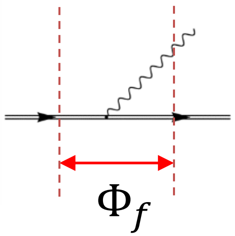


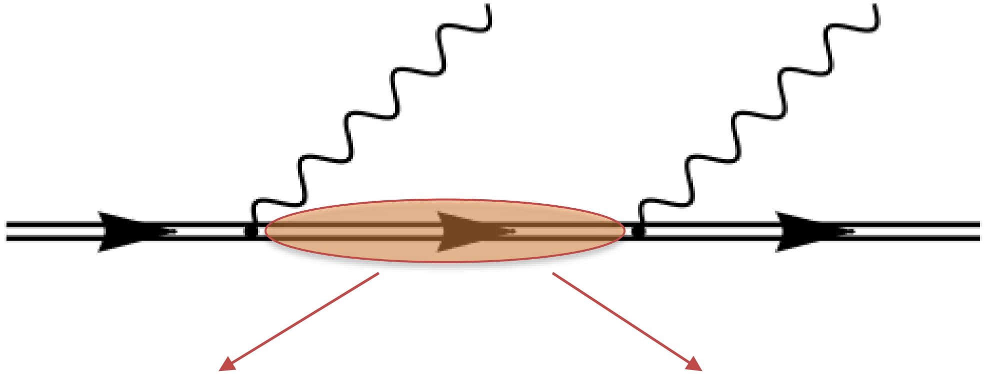
Detailed calculations in:

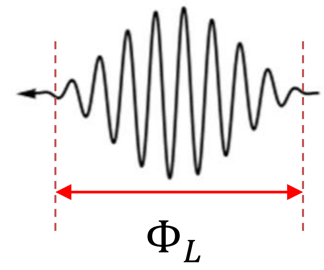
T. Podszus and A. Di Piazza, “First-order strong-field QED processes including the damping of particle states”, Phys. Rev. D **104**, 016014 (2021)

Motivation: Nonlinear Double Compton Scattering

<u>units:</u> $c = \hbar = \epsilon_0 = 1$ $\xi_0 \gg 1$ $\chi_0, \kappa_0 \sim 1$







„coherent“ or „one-step“

„incoherent“ or „two-step“

Intermediate electron off-shell

Intermediate electron on- and off-shell

$$P_{one-step} \sim \alpha^2 \Phi_L / \Phi_f$$

$$P_{two-step} \sim \alpha^2 \Phi_L^2 / \Phi_f^2$$

If $\Phi_L \gtrsim \Phi_f / \alpha = 137\ \Phi_f$ (long laser pulse length)

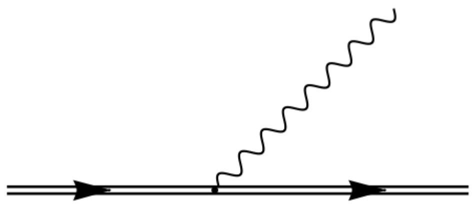
$$P_{one-step} \sim \alpha$$

$$P_{two-step} \sim 1$$

- $P_{two-step} \gg P_{one-step}$
- $P_{two-step}$ scales like leading order

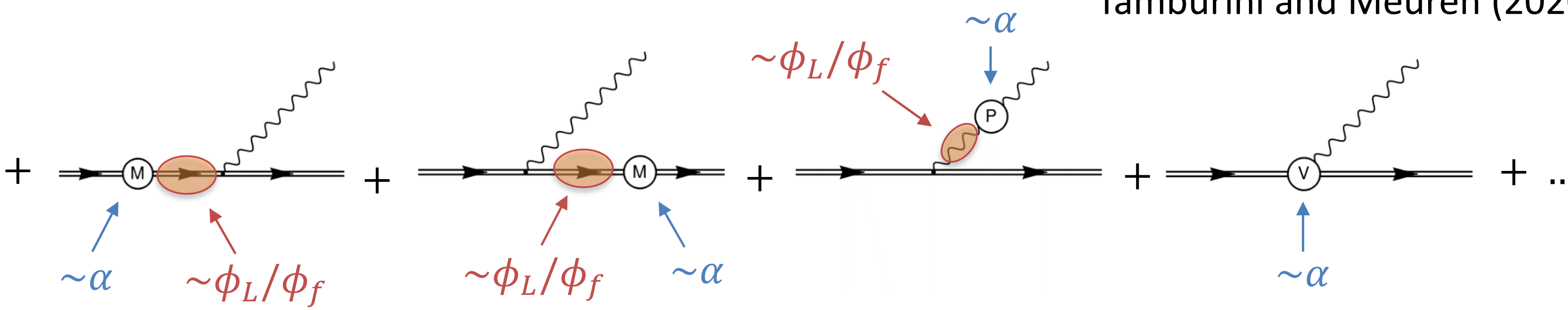
Lötstedt and Jentschura (2009)
Seipt and Kämpfer (2012)
Mackenroth and Di Piazza (2013)

Nonlinear Compton Scattering in Long Laser Pulses



For long phase duration ϕ_L of the background field one finds that $P_{NCS} > 1$

- ➢ in contradiction to unitarity of S-matrix
- ➢ perturbation theory not applicable
- ➢ Aim: compute exact description
- ➢ Previous investigations: Di Piazza et al. (2010)
  Tamburini and Meuren (2020)



- Intermediate electrons and photons can go on-shell
- ➢ receive contribution from plane wave
- ➢ becomes like leading order contribution if $\phi_L \gtrsim \phi_f/\alpha = 137\phi_f$
- Vertex correction can be neglected since $\sim\alpha$ and no contribution of phase
- For same reason one can use one-loop mass and polarization operator
- But have to take other higher one-particle reducible diagrams into account
- Resummation of one-particle reducible diagrams
- ➢ need exact electron and photon states → Schwinger-Dyson equation

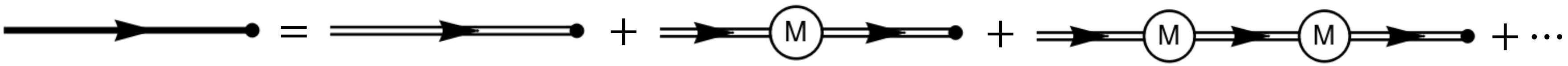
Exact Electron State

Solve Schwinger-Dyson equation, for electron in-state given by:

$$\left[\gamma^\mu\left(i\partial_\mu - A_\mu(x)\right) - m\right]\Psi_e^{(in)}(x) = \int d^4y\, M(x,y)\Psi_{\mathrm{e}}^{(in)}(y)$$

exact solution can be expanded with Volkov-states and –propagators:

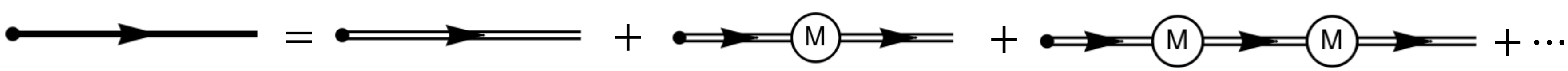
$$\Psi_{\mathrm{e}}^{(in)}(x) = \Psi_{e,V}^{(in)}(x) + \int d^4y\, d^4z\, G(x,y)\, M(y,z)\, \Psi_{e,V}^{(in)}(z)$$
$$+ \int d^4y\, d^4z\, d^4r\, d^4s\, G(x,y)\, M(y,z)\, G(z,r)\, M(r,s)\, \Psi_{e,V}^{(in)}(s) + \cdots$$



→ Solution contains resummation of all one-particle reducible diagrams

Analog for electron out-state:

$$\left[\gamma^\mu\left(i\partial_\mu - A_\mu(x)\right) - m\right]\Psi_e^{(out)}(x) = \int d^4y\, \overline{M}(y,x)\Psi_{\mathrm{e}}^{(out)}(y)$$



Exact Electron State

$$\left[\gamma^\mu\left(i\partial_\mu - A_\mu(x)\right) - m\right]\Psi_e^{(in)}(x) = \int d^4y\, M(x,y)\Psi_e^{(in)}(y)$$

Not possible to solve Schwinger-Dyson equation exactly

- Use one-loop mass operator in constant crossed field
- Work in LCFA
- Neglect terms $\sim\alpha$ except those which scale like $\sim\alpha\Phi_L$

$M_s(p,\varphi) =$ 

$$\Psi_{e,s}^{(in)}(p,x) = \left[1 + \frac{\hat{n}\,\hat{A}(\phi)}{2p_-}\right] e^{-i(px) - i\int_{-\infty}^{\phi} d\varphi \left[\frac{(pA(\varphi))}{p_-} - \frac{A^2(\varphi)}{2p_-} + \frac{m}{p_-}M_s(p,\varphi)\right]} u_s(p)$$

Meuren and Di Piazza (2011)

Podszus and Di Piazza (2021)

Normalization: $$\frac{\bar{\Psi}_{e,s}^{(in)}(p,x)\,\Psi_{e,s}^{(in)}(p,x)}{\bar{u}_s(p)\,u_s(p)} = e^{\frac{2m}{p_-}\int_{-\infty}^{\phi} d\varphi\, \Im[M_s(p,\varphi)]}$$

Optical theorem: $\mathcal{P}_{NCS} = -\frac{2m}{p_-}\int_{-\infty}^{\phi} d\varphi\, \Im[M_s(p,\varphi)]$ → Decay of electron state

<u>Electron out-state</u>:

$$\Psi_{e,s}^{(out)}(p,x) = \left[1 + \frac{\hat{n}\,\hat{A}(\phi)}{2p_-}\right] e^{-i(px) + i\int_{\phi}^{\infty} d\varphi \left[\frac{(pA(\varphi))}{p_-} - \frac{A^2(\varphi)}{2p_-} + \frac{m}{p_-}M_s^*(p,\varphi)\right]} u_s(p)$$

Exact Photon State

Solve Schwinger-Dyson equation, for photon in-state given by:

$$-\partial_\mu \partial^\mu \mathcal{A}_\nu^{(in)}(x) = \int d^4y\, P_\nu{}^\lambda(x,y) \mathcal{A}_\lambda^{(in)}(y)$$

→ Solution contains resummation of all one-particle reducible diagrams

Not possible to solve Schwinger-Dyson equation exactly

- Use transverse part of one-loop polarization operator in constant crossed field
- Work in LCFA
- Neglect terms $\sim\alpha$ except those which scale like $\sim\alpha\Phi_L$

$P_j(q,\phi) =$ 

$(j = 1,2)$

$$\mathcal{A}_{j,\mu}^{(in)}(q,x) = e^{-i(qx) - i\frac{m}{q_-}\int_{-\infty}^{\phi} d\varphi\, P_j(q,\varphi)} \Lambda_{j,\mu}(q)$$

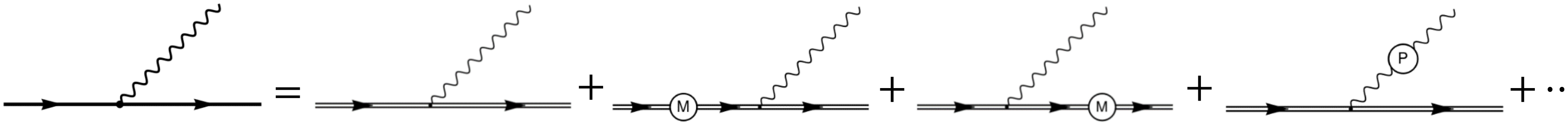
Meuren et al. (2015)

Optical theorem: $\mathcal{P}_{\mathrm{BW}} \sim -\frac{2m}{q_-}\int_{-\infty}^{\phi} d\varphi\, \Im\left[P_j(q,\varphi)\right]$ → decay of photon state into $e^-\, e^+$

photon out-state: $$-\partial_\mu \partial^\mu \mathcal{A}_\nu^{(out)}(x) = \int d^4y\, P^{*\lambda}{}_\nu(y,x) \mathcal{A}_\lambda^{(out)}(y)$$

$$\mathcal{A}_{j,\mu}^{(out)}(q,x) = e^{-i(qx) + i\frac{m}{q_-}\int_{\phi}^{\infty} d\varphi\, P_j^*(q,\varphi)} \Lambda_{j,\mu}(q)$$

Nonlinear Compton Scattering Including Particle States Decay



$$P^{(e^-\to e^-\gamma)} = \int \frac{d^3q}{16\pi^2}\,\frac{\alpha}{p_-p'_-\omega}\int d\phi_+\, e^{2\Im\left\{\frac{m}{p_-}\int_{-\infty}^{\phi_+} d\varphi M_s(p,\varphi)+\int_{\phi_+}^{\infty} d\varphi\left[\frac{m}{p'_-}M_{s'}(p',\varphi)+\frac{m}{q_-}P_j(q,\varphi)\right]\right\}}$$

$$\times\int d\phi_-\, e^{i\frac{m^2q_-}{2p_-p'_-}\left\{\left[1+\boldsymbol{\pi}^2_{\perp,e}(\phi_+)\right]\phi_-+\frac{\varepsilon^2(\phi_+)}{m^2}\frac{\phi_-^3}{12}\right\}}\,\frac{1}{4}\,\mathrm{tr}\left\{\left[1-\frac{\hat{n}\left[\hat{A}(\phi_+)+\hat{A}'(\phi_+)\,\phi_-/2\right]}{2p'_-}\right]\hat{\Lambda}_j(q)\right.$$

$$\left[1+\frac{\hat{n}\left[\hat{A}(\phi_+)+\hat{A}'(\phi_+)\,\phi_-/2\right]}{2p_-}\right](\hat{p}+m)\left(1+s\gamma^5\hat{\zeta}\right)\left[1-\frac{\hat{n}\left[\hat{A}(\phi_+)-\hat{A}'(\phi_+)\,\phi_-/2\right]}{2p_-}\right]$$

$$\left.\hat{\Lambda}_j(q)\left[1+\frac{\hat{n}\left[\hat{A}(\phi_+)-\hat{A}'(\phi_+)\,\phi_-/2\right]}{2p'_-}\right](\hat{p}'+m)\left(1+s'\gamma^5\hat{\zeta}'\right)\right\}$$

Podszus and Di Piazza (2021)

- New: damping term due to particles states decay
- Damping depends on:
  - the phase Tamburini and Meuren (2020)
  - momentum and spin/polarization of particles
- Decay becomes important if $\alpha\xi_0\Phi_L \gtrsim 1$ for $\chi_0, \kappa_0 \sim 1$

Nonlinear Compton Scattering Including Particle States Decay

$$P^{(e^-\to e^-\gamma)} = -\frac{\alpha m^2}{4p_-^2}\int_0^{p_-} dq_- \int d\phi_+ \, e^{2\Im\left\{\frac{m}{p_-}\int_{-\infty}^{\phi_+} d\varphi M_s(p,\varphi)+\int_{\phi_+}^{\infty} d\varphi\left[\frac{m}{p'_-}M_{s'}(p',\varphi)+\frac{m}{q_-}P_j(q,\varphi)\right]\right\}} T_{j,s,s'}$$

$$T_{1,s,s'} = \left[1+ss'\left(1-\frac{q_-^2}{2p_-(p_- - q_-)}\right)\right]\mathrm{Ai}_1(z) + (s+s')\left(2\frac{q_-}{p_-}+\frac{q_-^2}{p_-(p_- - q_-)}\right)\frac{\mathrm{Ai}(z)}{\sqrt{z}}\,\mathrm{sgn}(\psi'(\phi_+)) + \left[3+\frac{q_-^2}{p_-(p_- - q_-)}+ss'\left(3+\frac{q_-^2}{2p_-(p_- - q_-)}\right)\right]\frac{\mathrm{Ai}'(z)}{z}$$

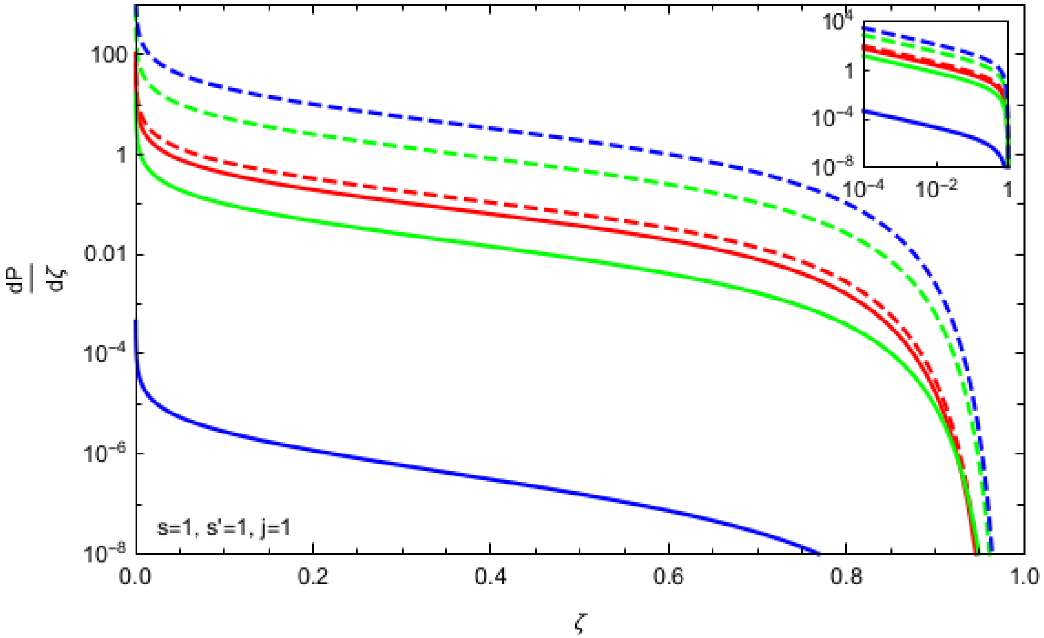
$$T_{2,s,s'} = \left[1+ss'\left(1+\frac{q_-^2}{2p_-(p_- - q_-)}\right)\right]\mathrm{Ai}_1(z) + (s'-s)\frac{q_-^2}{p_-(p_- - q_-)}\frac{\mathrm{Ai}(z)}{\sqrt{z}}\,\mathrm{sgn}(\psi'(\phi_+)) + \left[1+\frac{q_-^2}{p_-(p_- - q_-)}+ss'\left(1-\frac{q_-^2}{2p_-(p_- - q_-)}\right)\right]\frac{\mathrm{Ai}'(z)}{z}$$

$$z = \left[\left(\frac{p_-}{q_-}-1\right)\chi_p(\phi_+)\right]^{-\frac{2}{3}}$$

- Probability is Gauge invariant
- Without damping it reduces to result in Seipt and King (2020)
- Question: Is the result the same for different spin/polarization basis after summing over indices?

Note: With our choice Mass- and Polarization operator are diagonal

Nonlinear Compton Scattering Including Particle States Decay



$\xi_0 = 10, \ \chi_0 = 1, \zeta = \frac{kq}{kp}$

Gaussian pulse: $\sim \sin(kx) \exp\left[-\left(\frac{kx}{\phi_0}\right)^2\right]$,

dashed lines: without damping,

solid lines: with damping,

red: $\phi_0 = 5 \quad \rightarrow \alpha\xi_0\phi_{FWHM} \approx 0.6$,

green: $\phi_0 = 40 \quad \rightarrow \alpha\xi_0\phi_{FWHM} \approx 5$,

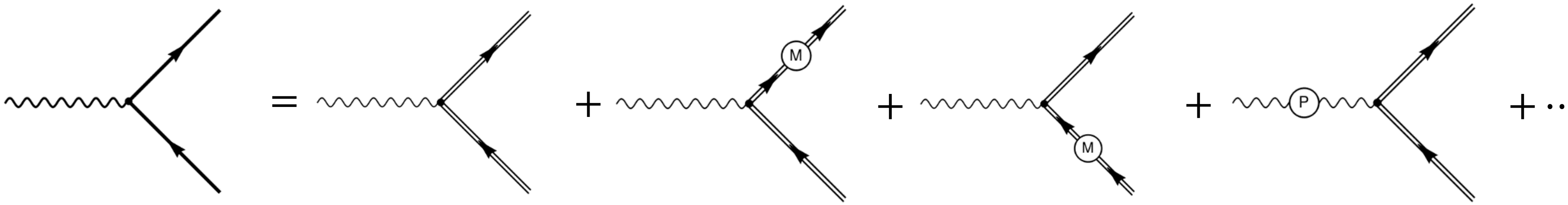
blue: $\phi_0 = 160 \rightarrow \alpha\xi_0\phi_{FWHM} \approx 19$

Total probability with damping < 1

Thanks to Victor Dinu for the numerical computations!

Cooperative paper in preparation.

Nonlinear Breit-Wheeler Pair Production Including Particle States Decay



$$P^{(\gamma\to e^- e^+)} = \int \frac{d^3p}{16\pi^2}\,\frac{\alpha}{q_- p'_- \varepsilon} \int d\phi_+\, e^{2\Im\left\{\frac{m}{q_-}\int_{-\infty}^{\phi_+} d\varphi\, P_j(q,\varphi) + \int_{\phi_+}^{\infty} d\varphi \left[\frac{m}{p'_-} M_{s'}(p',\varphi) + \frac{m}{p_-} M_s(-p,\varphi)\right]\right\}}$$

$$\times \int d\phi_-\, e^{i\frac{m^2 q_-}{2p_- p'_-}\left\{[1+\boldsymbol{\pi}^2_{\perp,p}(\phi_+)]\phi_- + \frac{\varepsilon^2(\phi_+)}{m^2}\frac{\phi_-^3}{12}\right\}}\,\frac{1}{4}\,\mathrm{tr}\left\{\left[1 - \frac{\hat{n}[\hat{A}(\phi_+)+\hat{A}'(\phi_+)\,\phi_-/2]}{2p'_-}\right]\hat{\Lambda}_j(q)\right.$$

$$\left[1 - \frac{\hat{n}[\hat{A}(\phi_+)+\hat{A}'(\phi_+)\,\phi_-/2]}{2p_-}\right](\hat{p}-m)\left(1+s\gamma^5\hat{\zeta}\right)\left[1 + \frac{\hat{n}[\hat{A}(\phi_+)-\hat{A}'(\phi_+)\,\phi_-/2]}{2p_-}\right]$$

$$\left.\hat{\Lambda}_j(q)\left[1 + \frac{\hat{n}[\hat{A}(\phi_+)-\hat{A}'(\phi_+)\,\phi_-/2]}{2p'_-}\right](\hat{p}'+m)\left(1+s'\gamma^5\hat{\zeta}'\right)\right\}$$

Podszus and Di Piazza (2021)

Conclusion

- Electrons/positrons and photons decay in plane wave field
- Damping is cumulative effect scaling with laser pulse duration
- It leads to exponential damping factor in probability of nonlinear Compton scattering and nonlinear Breit-Wheeler pair production
- Damping depends on momentum and spin/polarization
- Particle states decay becomes significant for long laser pulse length such that $\alpha\xi_0\Phi_L \gtrsim 1$ for $\chi_0, \kappa_0 \sim 1$

Detailed calculations in:

T. Podszus and A. Di Piazza, "First-order strong-field QED processes including the damping of particle states", Phys. Rev. D **104**, 016014 (2021)