TOMSK POLYTECHNIC UNIVERSITY

HI JENA
Helmholtz Institute Jena

DFG Deutsche Forschungsgemeinschaft
Research Unit FOR 2783

DAAD
German Academic Exchange Service

# Vacuum birefringence and diffraction at XFEL: from analytical estimates to optimal parameters

Elena Mosman

ExHILP 2021, September 16, Jena

# Outline

I. Introduction

II. Theory

III. Parameters

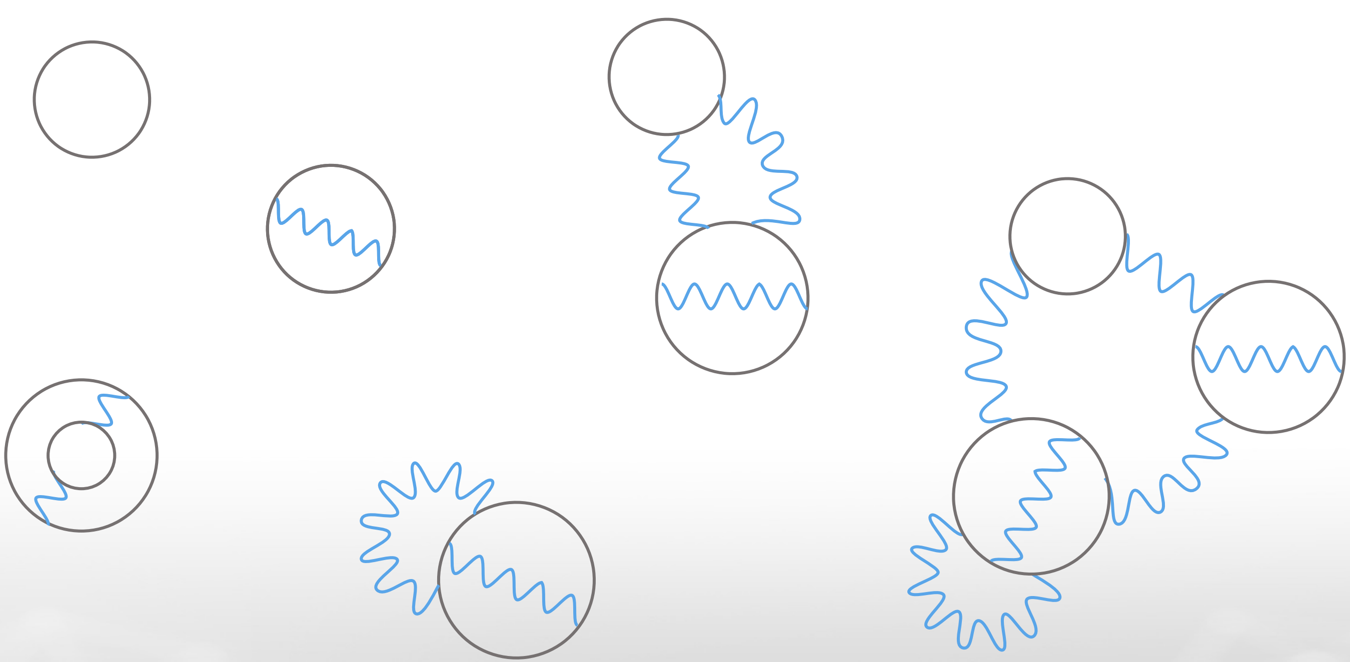
IV. Results

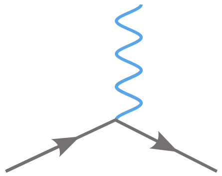
V. Conclusion & Outlook

# Introduction

The quantum vacuum

(in this talk: the QED vacuum)



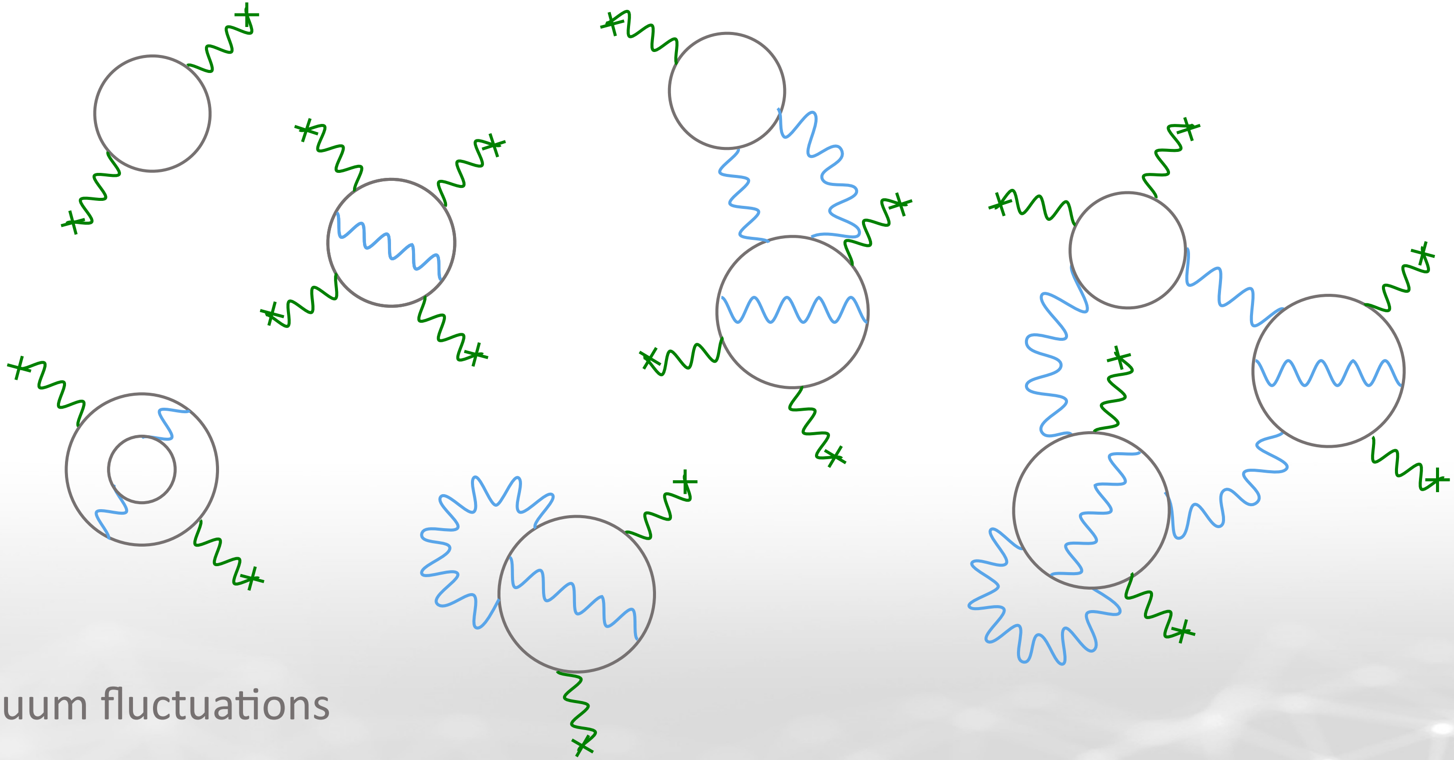


= vacuum fluctuations

TOMSK POLYTECHNIC UNIVERSITY

# Introduction

The quantum vacuum + prescribed electromagnetic fields



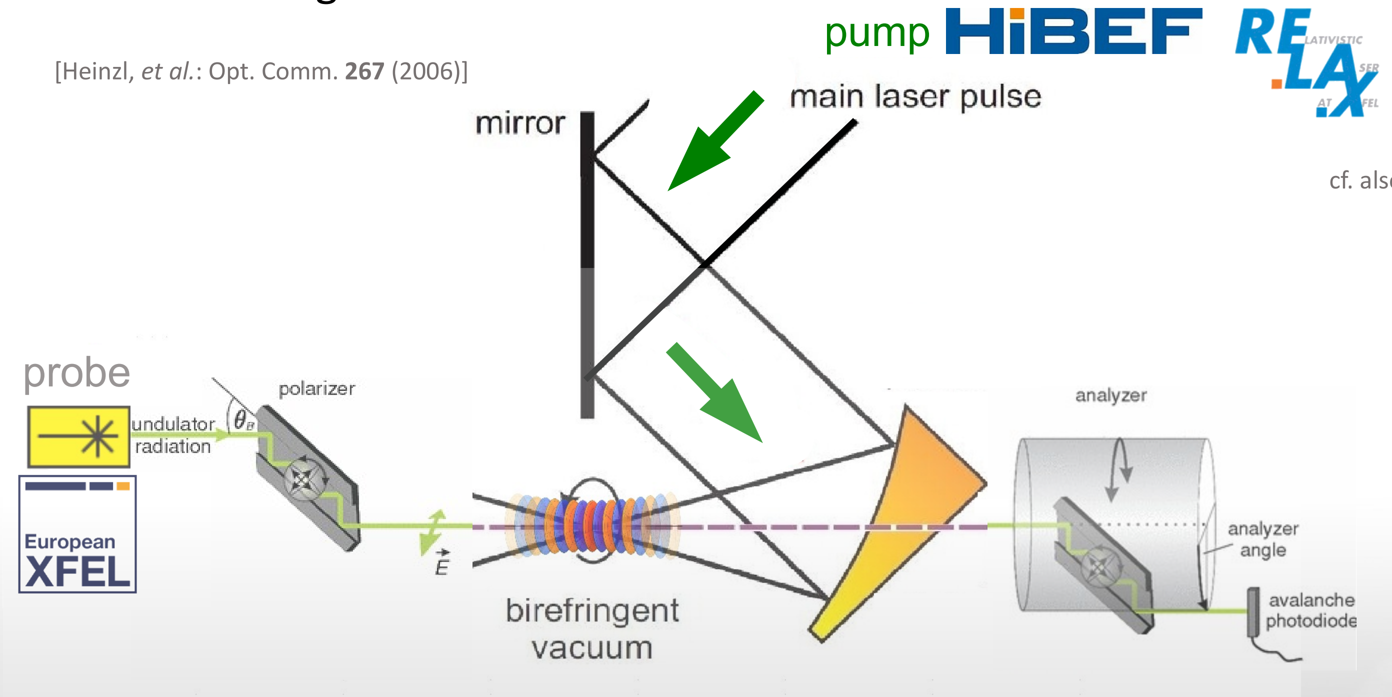
= vacuum fluctuations

# Introduction

## Vacuum birefringence in laser fields

[Becker, Mitter: J. Phys. A **8** (1975)],
[Aleksandrov, *et al.*: Sov. JETP **64** (1985)]

[Heinzl, *et al.*: Opt. Comm. **267** (2006)]




cf. also [Di Piazza, *et al.*: PRL **97** (2006)],
[Dinu, *et al.*: PRD **89** & **90** (2014)],
[Karbstein, *et al.*: PRD **92** (2015)],
[Schlenvoigt, *et al.*: Phys. Scr. **91** (2016)],
[King, Heinzl: HPL Sci. & Eng. **4** (2016)],
[Karbstein, Sundqvist: PRD **94** (2016)],
[Karbstein: PRD **98** (2018)],
[Ataman: PRA **97** (2018)],
[Briscese: PRA **97** (2018)],
[Shen, *et al.*: Plasma Phys. CF **60** (2018)],
[Huang, *et al.*: PRD **100** (2019)],
[Karbstein, EM: PRD **100** (2019)]

→ high-purity x-ray polarisation $\mathcal{P}\text{urity} = 1.4 \cdot 10^{-11} \,@\, \omega = 12914\,\text{eV}$

[Marx, *et al.*: PRL **110** (2013)]
[Marx, *et al.*: in preparation]

# Theory

For our consideration we use the leading nonlinear correction to classical Maxwell theory

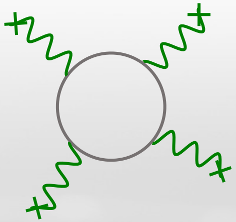
$(\hbar = c = 1)$

$$\mathcal{L}_{\rm HE} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overbrace{\frac{m_e^4}{360\pi^2}\left[\frac{(\vec{B}^2-\vec{E}^2)^2}{E_{\rm cr}^4} + 7\frac{(\vec{B}\cdot\vec{E})^2}{E_{\rm cr}^4}\right]} + \cdots$$

[Euler, Kockel: Naturwiss. **32** (1935)]
[Euler: Ann. Phys. **26** (1936)]

where

$$E_{cr} = \frac{m_e^2 c^3}{\hbar e} \simeq 1.3\cdot 10^{18}\frac{V}{m}$$



Higher terms are parametrically supressed by powers of:

Higher loops $\rightarrow \frac{\alpha}{\pi} \simeq 0.002 \ll 1$

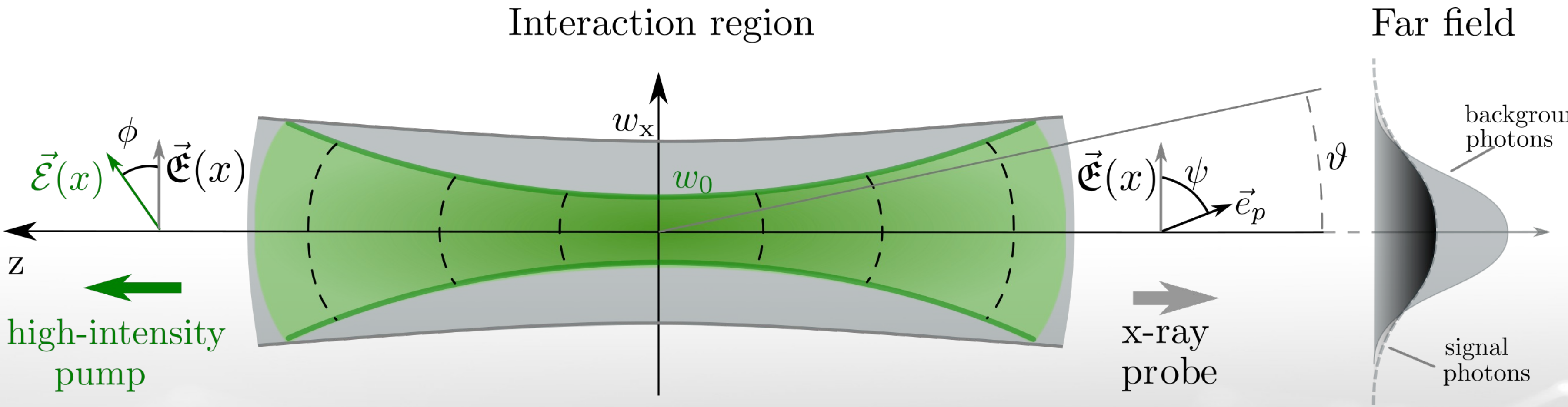
Higher derivatives $\rightarrow \left(\frac{\partial}{m_e}\right)^2 \sim \left(\frac{\omega}{m_e}\right)^2 \ll 1$

Higher terms in field $\rightarrow \left(\frac{eF}{m_e^2}\right)^2 \sim \frac{I}{I_{\rm cr}} \ll 1$

# Theory

We study QED vacuum birefringence and diffraction in the perfect head-on collision of a loosely focused x-ray laser beam and a tightly focused high-intensity laser beam:

[Karbstein: PRD **98** (2018)]
[Karbstein, Oude Weernink: PRD (2021)]
Poster of Ricardo Oude Weernink




The effective vacuum-fluctuation-mediated interaction of the two laser beams gives rise to signal photons mainly emitted in the forward direction of the driving beams.

# Theory

For the driving laser fields we use solutions of the paraxial wave equation, namely linearly polarized fundamental Gaussian beam endowed with gaussian time envelope

- valid for small asymptotic beam divergences $\theta \ll 1$

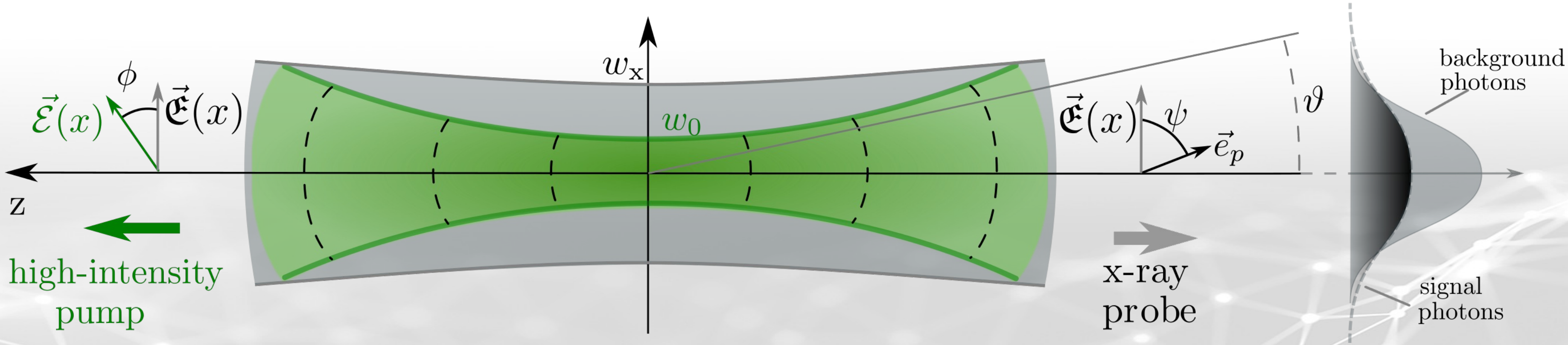
$$\rightarrow \theta^2 = \left(\frac{w_0}{z_R}\right)^2 \simeq 0.02 \ll 1$$

cf., e.g., [L.W. Davis: PRA **19** (1979)]
[J.P.Barton, D.R. Alexander: J.Appl. Phys. **66** (1989)]
[Salamin, *et al.*: Phys. Rev. ST Acc. B. **5** (2002)]

- valid for large pulse durations

$$\rightarrow \frac{1}{\tau\omega} \simeq 0.01 \ll 1$$

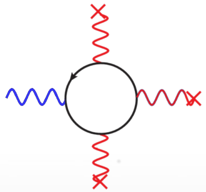
cf., e.g., [Karbstein, EM: PRD **96** (2017)]



# Theory

We aim at detecting signal photons far-outside the interaction region

- start configuration: vacuum plus driving laser fields $= |0\rangle$
- signatures of vacuum birefringence/diffraction are encoded in x-ray signal photons$= |\gamma_p(\vec{k})\rangle$
    - ○ direction $\vec{k}$
    - ○ energy $|\vec{k}|$
    - ○ polarization $p$
- transition amplitude of "vacuum emission"

$$\mathcal{S}_{(p)}(\vec{k}) = \langle\gamma_p(\vec{k})|$$  $$|0\rangle$$

[Galtsov, Skobelev: Phys. Lett. **36** (1961)]
[Karbstein, Shaisultanov: PRD **91** (2015)]

$$\mathrm{d}^3 N_{(p)}(\vec{k}) = \frac{\mathrm{d}^3 k}{(2\pi)^3} |\mathcal{S}_{(p)}(\vec{k})|^2$$

- the signal photon numbers are dominated by (quasi-)elastic contributions

[Karbstein, Gies, Reuter, Zepf: PRD **92** (2015)]

# Theory

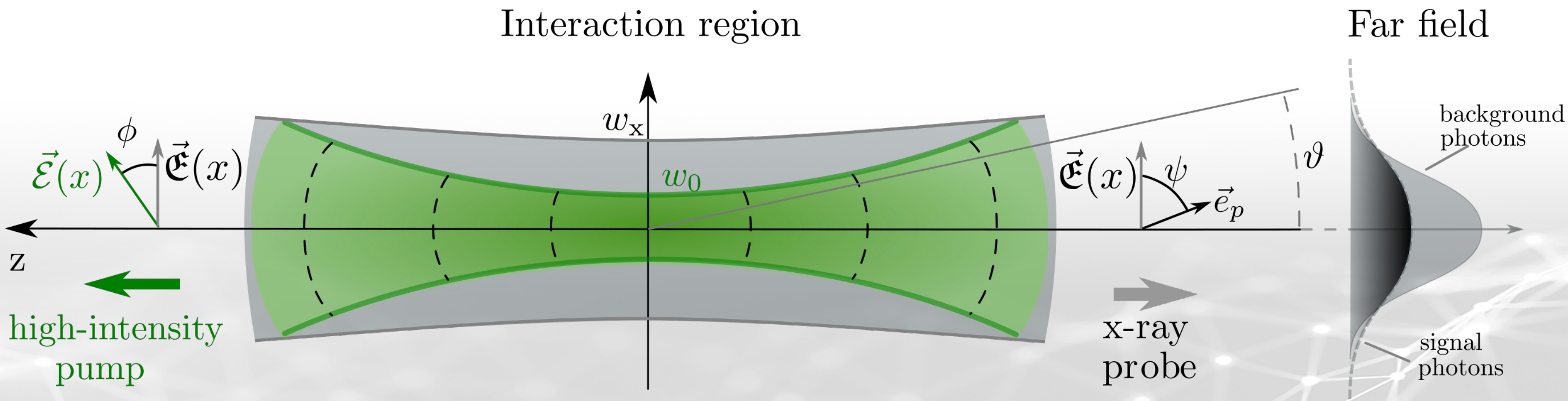
We are interested in two types of signals

- polarization-flipped photons $N_{(p)} = N_{\perp}$ becoming maximum for $\angle\phi = \frac{\pi}{4}$

  Talk of Annika Tamara Schmitt

- total photon number $N_{\text{tot}} = \sum_p N_{(p)}$ in polarization insensitive measurement becoming maximum for $\angle\phi = \frac{\pi}{2}$

  Talk of Yudai Seino



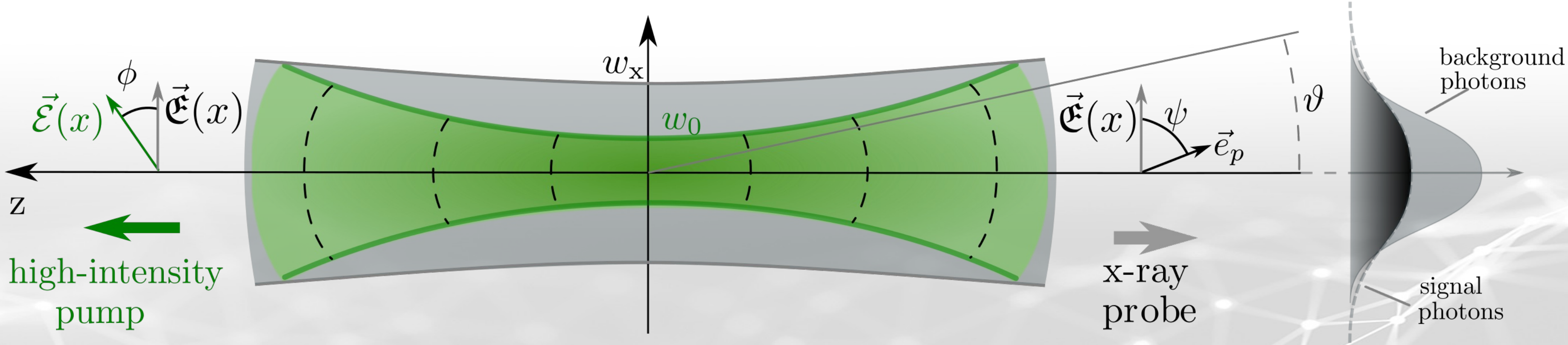
# Theory

The signal number is an integral expression depending on many experimental parameters

$$\begin{Bmatrix} \mathrm{d}N_{\mathrm{tot}} \\ \mathrm{d}N_{\perp} \end{Bmatrix} \simeq \begin{Bmatrix} 130 - 66\cos(2\phi) \\ 9\sin^2(2\phi) \end{Bmatrix} f(W, \Omega, \tau, w_0 | N, \omega, T, w_{\mathrm{x}} | \vartheta) \mathrm{d}\cos(\vartheta)$$

Energy of the pump → $W$; Pump frequency → $\Omega$; Pulse duration → $\tau$; Pump waist → $w_0$

[Karbstein, EM: PRD **100** (2019)]



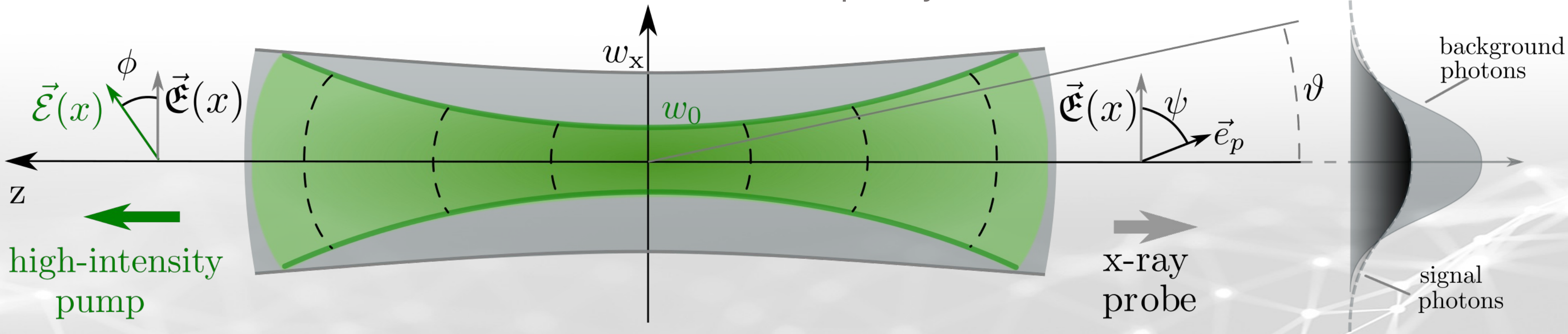
# Theory

The signal number is an integral expression depending on many experimental parameters

$$\begin{Bmatrix} \mathrm{d}N_{\mathrm{tot}} \\ \mathrm{d}N_{\perp} \end{Bmatrix} \simeq \begin{Bmatrix} 130 - 66\cos(2\phi) \\ 9\sin^2(2\phi) \end{Bmatrix} f(W,\Omega,\tau,w_0|N,\omega,T,w_{\mathrm{x}}|\vartheta)\mathrm{d}\cos(\vartheta)$$

Number of probe photons → $N$
Probe frequency → $\omega$
Pulse duration → $T$
Probe waist → $w_{\mathrm{x}}$

[Karbstein, EM: PRD **100** (2019)]

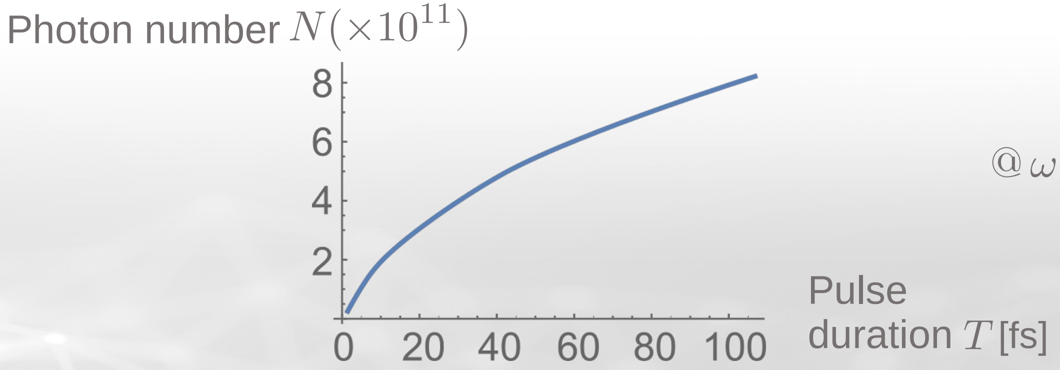


# Theory

The signal number is an integral expression depending on many experimental parameters

$$\left\{\begin{matrix} \mathrm{d}N_{\mathrm{tot}} \\ \mathrm{d}N_{\perp} \end{matrix}\right\} \simeq \left\{\begin{matrix} 130 - 66\cos(2\phi) \\ 9\sin^2(2\phi) \end{matrix}\right\} f(W,\Omega,\tau,w_0|N,\omega,T,w_{\mathrm{x}}|\vartheta)\mathrm{d}\cos(\vartheta)$$

In the experiment some of the given parameters depend on each other, e.g. at XFEL the photon number $N$ per pulse is a function of both the pulse duration $T$ and the photon energy $\omega$.




[Technical Design Report, European XFEL(2011)]

$@\,\omega = 12914\,\mathrm{eV}$

# Theory

The identification of the optimal experimental parameters is a complicated problem. Whithout additional simplifications this requires the use of numerical methods.

Building on several well-justified approximations and introducing the notion of an effective waist for the optical beam, we have constructed compact analytical approximations.

New approximation reproduces an exact numerical results for the signal driven by fundamental Gaussian beams on a one percent level accuracy in a wide range of parameters.

[EM,Karbstein: PRD **104** (2021)]

Due to both its analytical nature and high accuracy, our model can even be straightforwardly differentiated and allows for the transparent tracing of parameter dependencies.

# Parameters

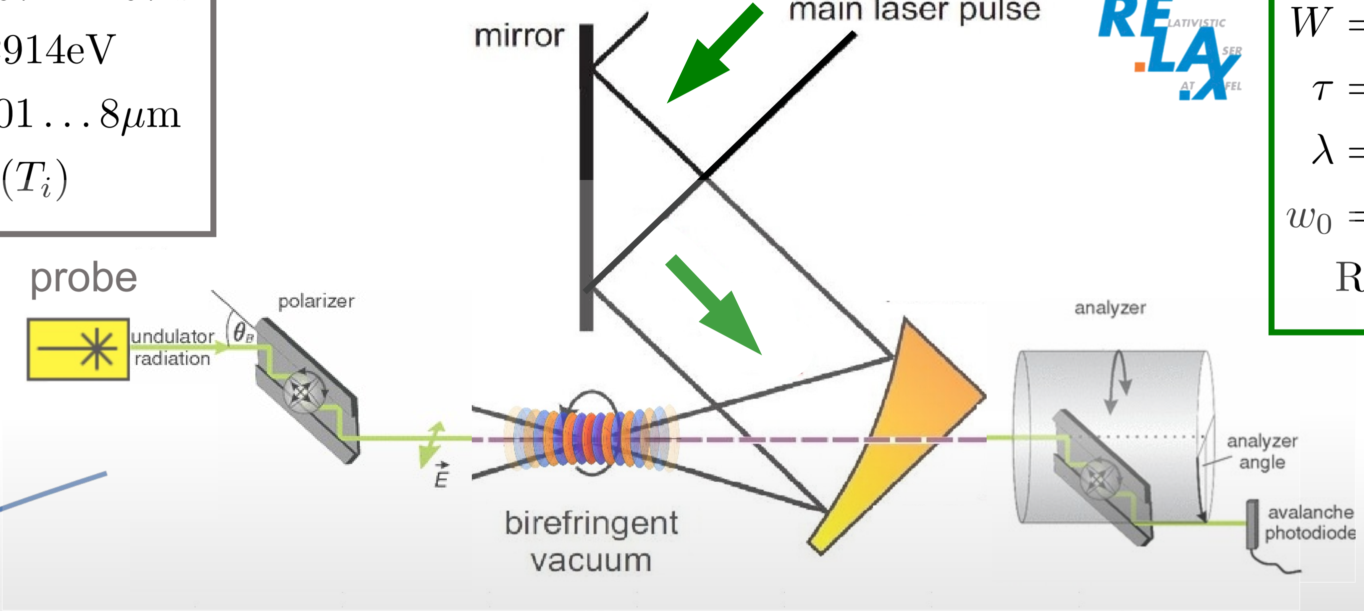
European XFEL

$$T_i \simeq 1.67\ldots 107\text{fs}$$
$$\omega = 12914\text{eV}$$
$$w_x \simeq 0.01\ldots 8\mu\text{m}$$
$$N = N(T_i)$$

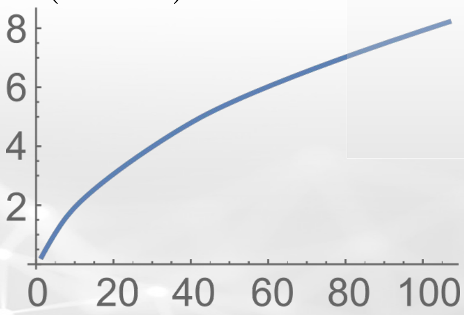
pump HiBEF

300TW

$$W = 10\,\text{J}$$
$$\tau = 25\,\text{fs (FWHM)}$$
$$\lambda = 800\,\text{nm}$$
$$w_0 = 1\,\mu\text{m (HWHM)}$$
$$\text{Rep.rate}\ 1\,\text{Hz}$$




[Technical Design Report, European XFEL(2011)]



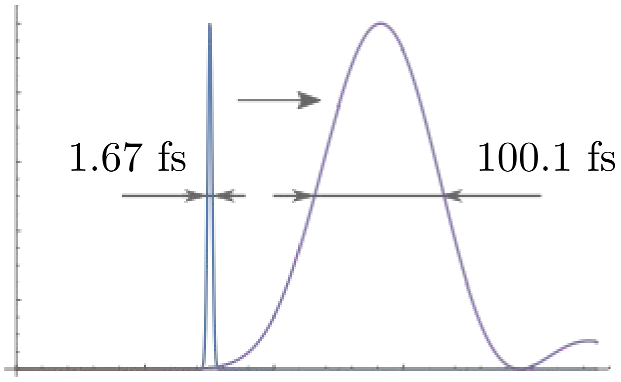

$$\mathcal{P} \simeq 1.4 \times 10^{-11}$$


[Marx, *et al.*: PRL **110** (2013)]
[Marx, *et al.*: in preparation]

# Parameters

The incident probe pulse duration is nonlinearly increased by the silicon polariser.

[Lindberg, Shvyd'ko, Phys. Rev. ST Accel.Beams **15**, (2012)]
[Shvyd'ko, Lindberg, Phys. Rev. ST Accel.Beams **15**, (2012)]
[Karbstein, Sundqvist, Schulze, Uschmann, Gies, Paulus: New J. Phys **23**(2021)]




$$T_i \to T_{\mathcal{P}}$$

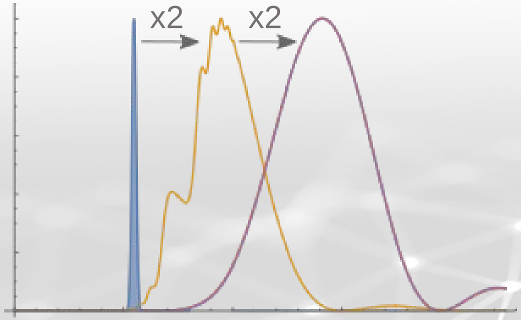
Incident XFEL pulse duration

Pulse duration in the interaction region

Typical FWHM pulse durations after the polariser are

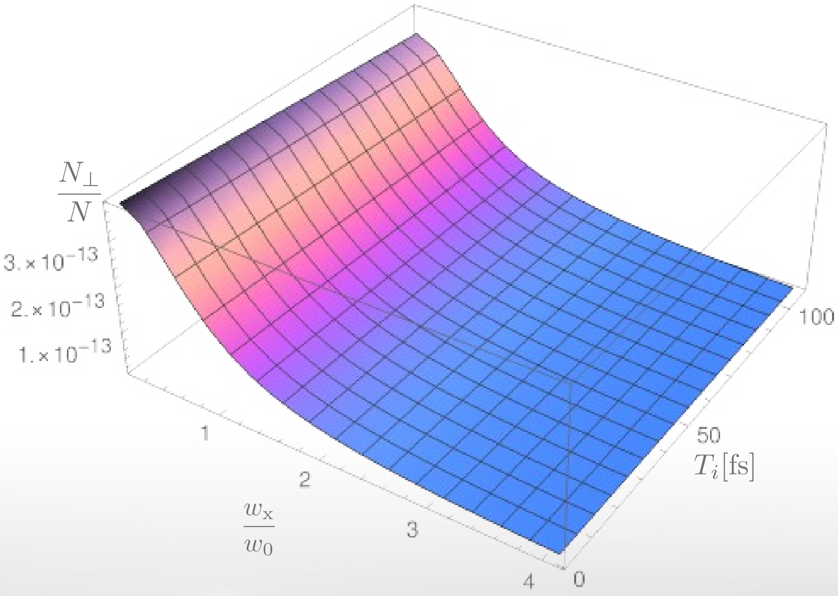
$$T_{\mathcal{P}} \simeq 100 - 130[\mathrm{fs}]$$

The temporal structure of realistic XFEL pulse is typically characterized by a large number of spikes; polarisers substantially smoothen the resulting pulse profile.



# Results: induced ellipticity

We identified the optimal parameters for maximizing the ellipticity of the outgoing x-ray beam.



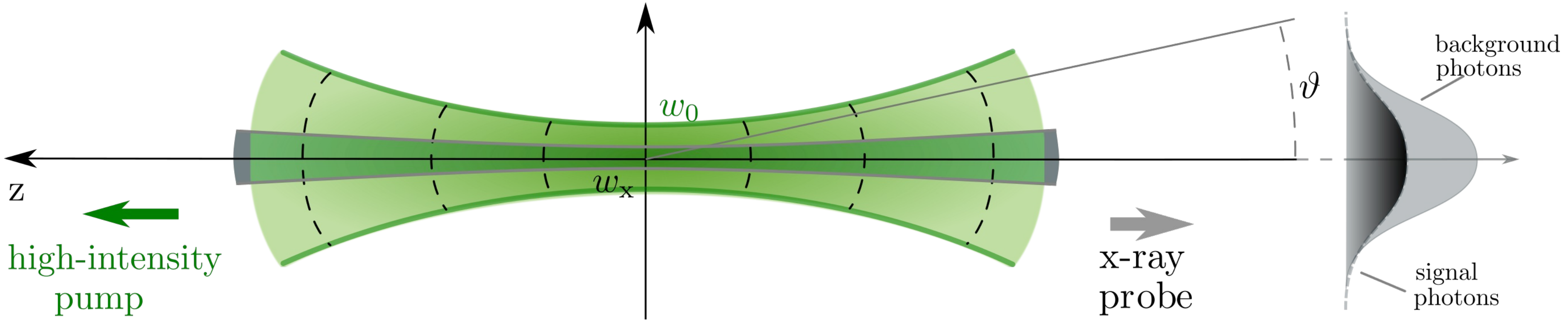

The ellipticity $\epsilon$ is directly related to the ratio of the photon numbers $\epsilon^2 \simeq N_\perp/N$

The ratio is maximum at minimum pulse duration $T_i = 1.7\,\mathrm{fs}$ and x-ray waist $w_\mathrm{x} \leq 0.1 w_0$

$$\frac{N_\perp}{N} \simeq \frac{4\alpha^4}{25(3\pi)^{\frac{3}{2}}} \left(\frac{W}{m_e}\frac{\omega}{m_e}\right)^2 \left(\frac{\lambda\!\!\!^-_C}{w_0}\right)^4 F\left(\frac{4z_R}{\sqrt{T^2+\frac{1}{2}\tau^2}}, \frac{T}{\tau}\right)$$

[Karbstein: PRD **98** (2018)]

# Results: induced ellipticity




The divergence of the signal photons essenctially fit the one of the driving x-ray beam with approximately constant ratio $N_\perp / N$

For the current HiBEF parameters and a polarization purity of $\mathcal{P} = 1.4 \times 10^{-11}$ the maximum value for the ratio is

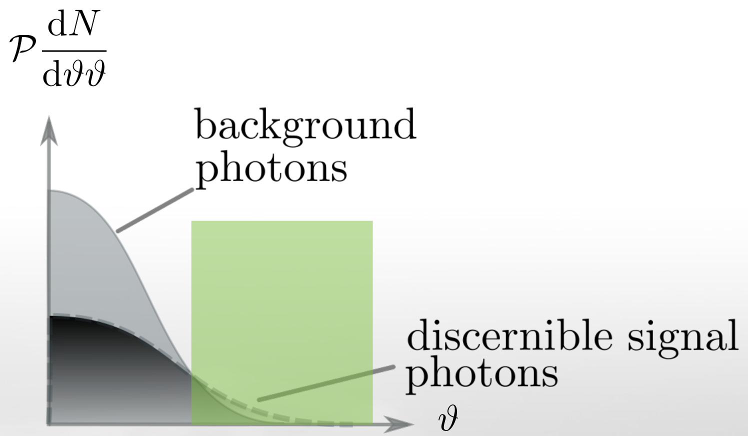
$$\frac{N_\perp}{N} = \frac{1}{40}\mathcal{P}$$

(above the background measurement requires $\frac{N_\perp}{N} > \mathcal{P}$ )

# Results: discernible number $N_{\perp>}$

The discernible number of polarization-flipped signal photons $N_{\perp>}$ is defined to fulfill the criterium

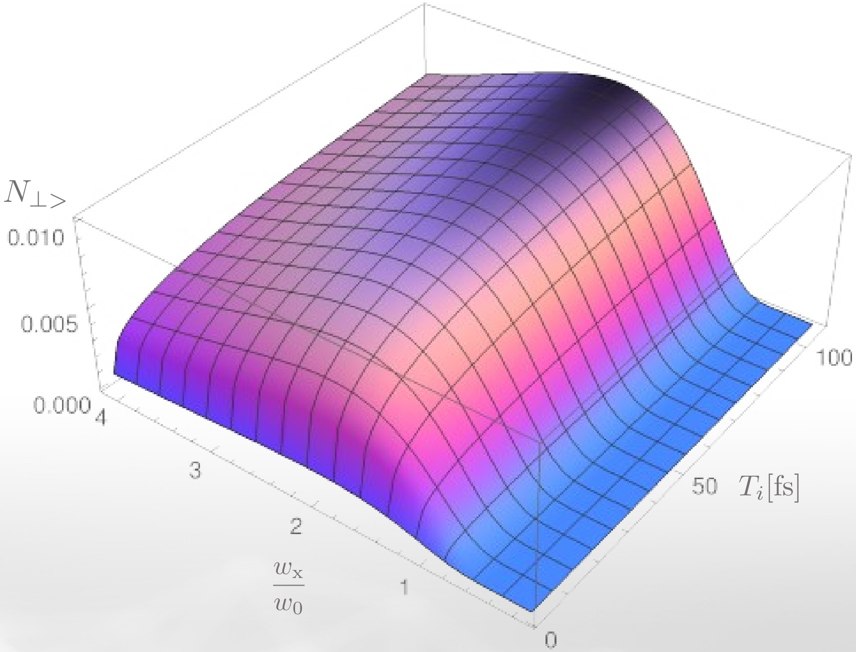
$$\frac{\mathrm{d}N_\perp}{\vartheta\,\mathrm{d}\vartheta} \geq \mathcal{P}\,\frac{\mathrm{d}N}{\vartheta\,\mathrm{d}\vartheta}$$




The discernible polarisation flipped photons appear due to the fact that vacuum birefringence is generically accompanied by a scattering phenomenon.

# Results: discernible number $N_{\perp>}$

The dependence of the discernible photon number $N_{\perp>}$ on the parameters $w_\mathrm{x}$ and $T_i$ is more complicated

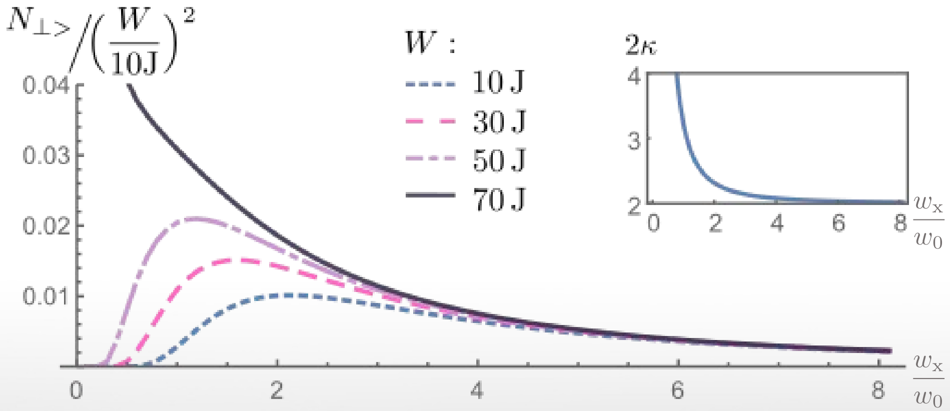



- maximum signal for maximum duration
- for current HiBEF parameters there is a negligible amount of discernible photons for waists fulfilling $w_\mathrm{x}/w_0 \leq 1$
- there exists an optimal x-ray waist maximizing the discernible signal $N_{\perp>}$
- for HiBEF parameters the maximum number of discernible photos is $N_{\perp>} \simeq 0.01/\mathrm{shot}$ at $w_\mathrm{x} = 2.1 w_0$

# Results: discernible number $N_{\perp>}$

Our analytic formulas allow to analyse the dependence of the signal on different input parameters.

The dependence of the signal on the pump energy is demonstrated below.




The figure shows the **normalized** discernible signal.

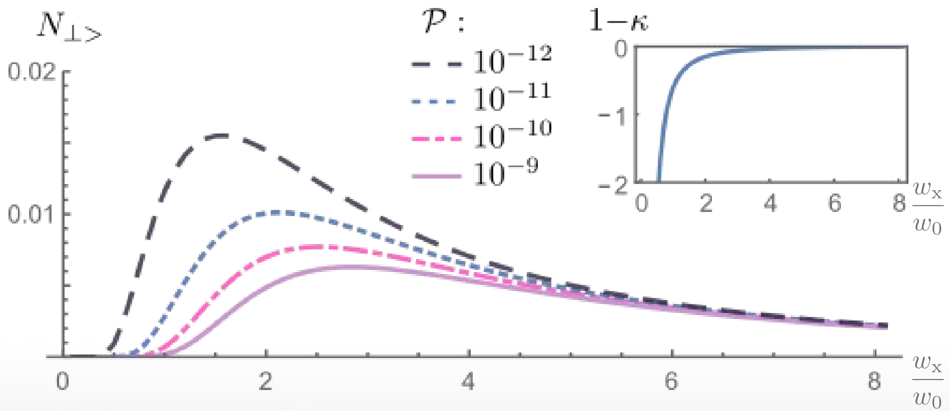
The optimal x-ray waist is changing with pump energy.

At waist $w_x = 2.1 w_0$ small variations of the energy will change the signal as

$$N_{\perp>} \sim W^{2\kappa} \simeq W^{2.3}$$

# Results: discernible number $N_{\perp>}$

The dependence of the discernible signal on the polarisation purity is less pronounced for current HiBEF parameters.




At waist $w_x = 2.1 w_0$ small variations of the polarisation purity will change the signal as

$$N_{\perp>} \sim \mathcal{P}^{1-\kappa} \simeq \mathcal{P}^{-0.14}$$

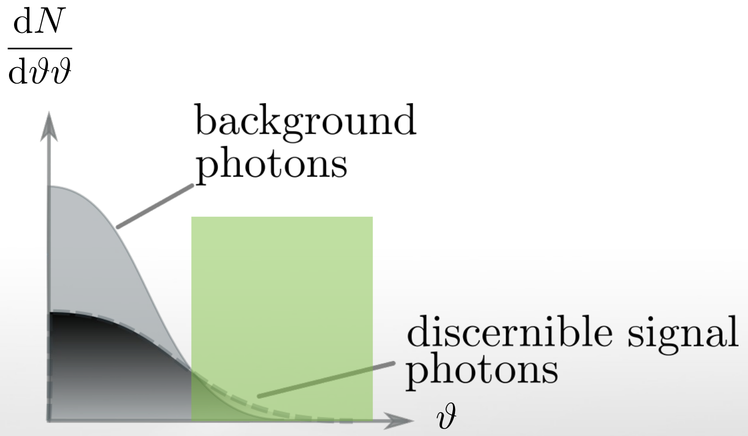
The number of signal photons per background photon scattered outside the discernibility angle is

$$\frac{N_{\perp>}}{N_{>}\mathcal{P}} \simeq 8 \qquad @\,\mathcal{P}\text{urity} = 1.4 \cdot 10^{-11}$$

# Results: discernible number $N_{\mathrm{tot}>}$

The discernible number of total signal photons $N_{\mathrm{tot}>}$ is defined to fulfill the criterium

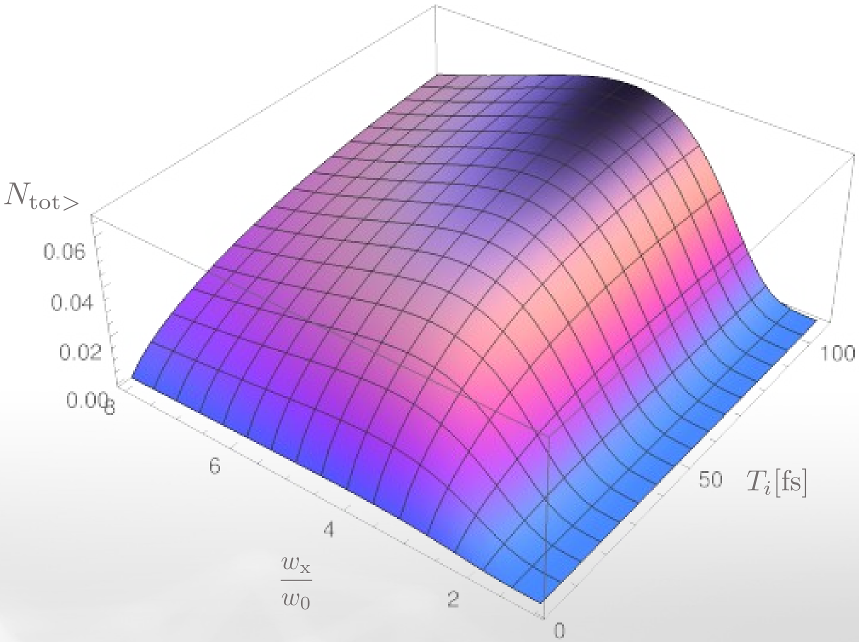
$$\frac{\mathrm{d}N_{\mathrm{tot}}}{\vartheta\,\mathrm{d}\vartheta} \geq \frac{\mathrm{d}N}{\vartheta\,\mathrm{d}\vartheta}$$




- No x-ray polariser is needed
- Probe pulse duration in the interaction region is the incident pulse duration of the XFEL $\mathrm{T}_i \simeq 1.67\ldots 107\mathrm{fs}$

# Results: discernible number $N_{\mathrm{tot}>}$

The dependence of the discernible photon number $N_{\mathrm{tot}>}$ on the parameters $w_{\mathrm{x}}$ and $T_i$ is more complicated

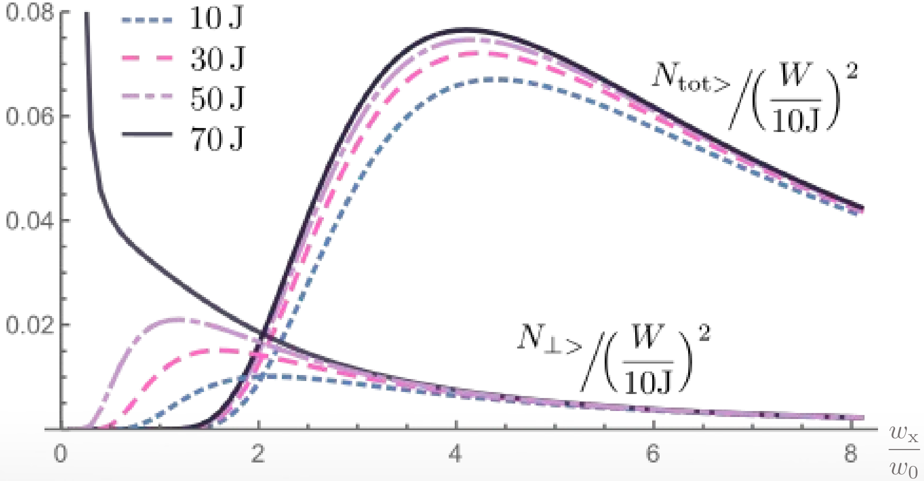



- The optimal pulse duration is the maximum one
- There is an optimal x-ray waist maximizing the total discernible signal $N_{\mathrm{tot}>}$
- For current HiBEF parameters the maximum number of discernible photons is $N_{\mathrm{tot}>} \simeq 0.07/\mathrm{shot}$ at $w_{\mathrm{x}} = 4.5 w_0$

TOMSK POLYTECHNIC UNIVERSITY

# Results: discernible number $N_{\mathrm{tot}>}$

The dependence of the signal on the pump energy




The figure shows the dependence of the **normalized** discernible signal on the pump energy.

At waist $w_x = 4.5 w_0$ small variations of the energy will change the signal as

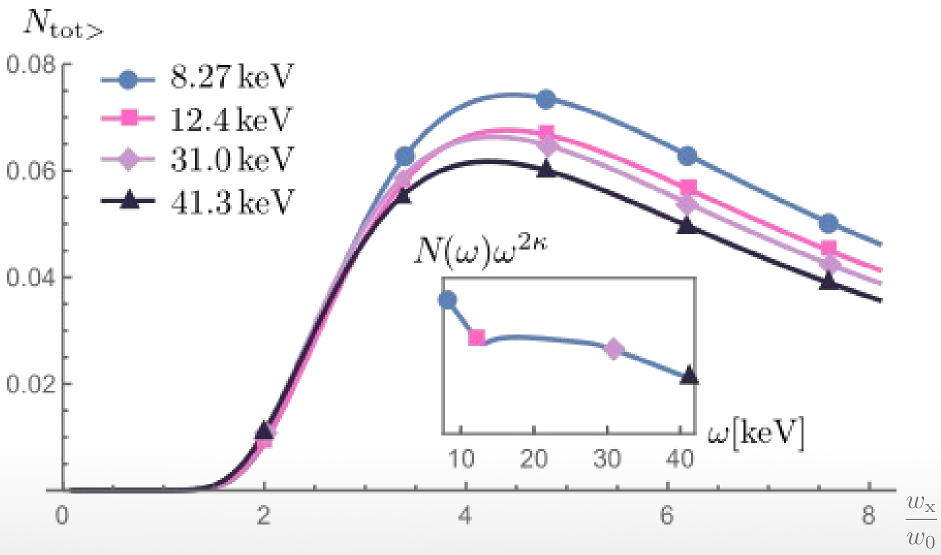
$$N_{\mathrm{tot}>} \sim W^{2\kappa} \simeq W^{2.06}$$

The number of signal photons per background photon scattered outside the discernibility angle is

$$\frac{N_{\mathrm{tot}>}}{N_{>}} \simeq 32.6$$

# Results: discernible number $N_{tot>}$

Dependence of the signal photon number on different probe photon energies




The probe photon number at XFEL raises with decreasing photon energy. A smaller energy may increase the effect

$$N_{tot>} \sim N(\omega) \times \omega^{2.06}$$

# Conclusions & Outlook

Consider generic non-linear corrections to classical Maxwell theory

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m_e^4}{360\pi^2}\left[a\,\frac{(\vec{B}^2-\vec{E}^2)^2}{4E_{\rm cr}^4} + b\,\frac{(\vec{B}\cdot\vec{E})^2}{E_{\rm cr}^4}\right] + \cdots$$

Heisenberg-Euler at one loop:

$$a = 4, \quad b = 7$$

The corresponding signal photon distributions read:

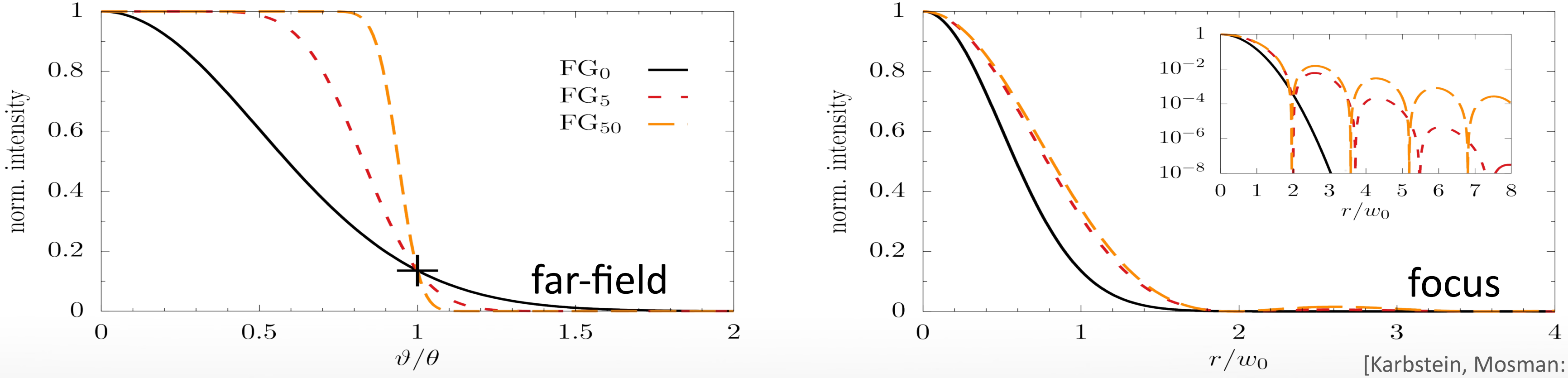
$$\begin{Bmatrix} {\rm d}N_{\rm tot} \\ {\rm d}N_\perp \end{Bmatrix} \simeq \begin{Bmatrix} 2(a^2+b^2) + 2(a^2-b^2)\cos(2\phi) \\ (a-b)^2\sin^2(2\phi) \end{Bmatrix} f(W,\Omega,\tau,w_0|N,\omega,T,w_{\rm x}|\vartheta)\,{\rm d}\cos(\vartheta)$$

Measurement of both signals (or total number at different polarization angles) allows to identify separately the coefficients *a* and *b*.

Talk of Luis Roso

# Conclusions & Outlook

- Analytic scalings are important to provide guidance for experiments.
- Fundamental Gaussian beams are certainly an idealisation
  More realistic beam models are to be considered (e.g., flattened-Gaussian beams)




[Karbstein, Mosman: PRD **101** (2020)]

- Quantitatively accurate predictions eventually require detailed numerical simulations of these quantum vacuum signatures in experimentally realistic laser fields

[Blinne *et al*: PRD **99** (2019)]

- This could for instance be done with the vacuum emission solver of the Jena group, or an Adapted Yee scheme (FDTD) to solve the nonlinear Maxwell equations.

[Grismayer, *et al*: NJP **23** (2021)]

# THANK YOU!

TOMSK
POLYTECHNIC
UNIVERSITY

# Appendix

Let us come back to the original expression for signal photons

$$\begin{Bmatrix} \mathrm{d}N_{\mathrm{tot}} \\ \mathrm{d}N_{\perp} \end{Bmatrix} \simeq \vartheta\,\mathrm{d}\vartheta \begin{Bmatrix} 2(a^2+b^2)+2(a^2-b^2)\cos(2\phi) \\ (a-b)^2\sin^2(2\phi) \end{Bmatrix} \frac{4\alpha^4}{225(3\pi)^{\frac{3}{2}}} N\Big(\frac{W}{m_e}\frac{\lambda\!\!\!^{-}_{\mathrm{C}}}{w_0}\Big)^2\Big(\frac{\omega}{m_e}\Big)^4$$

$$\times \frac{(\frac{w_{\mathrm{x}}}{w})^2}{[1+2(\frac{w_{\mathrm{x}}}{w})^2]^2}\,\mathrm{e}^{-\frac{1}{2}\frac{(\omega\vartheta w_{\mathrm{x}})^2}{1+2(\frac{w_{\mathrm{x}}}{w})^2}} F\Big(\frac{4\mathrm{z}_R}{\sqrt{T^2+\frac{1}{2}\tau^2}},\frac{T}{\tau}\Big)$$

$$F(\chi,\rho) := \sqrt{\frac{1+2\rho^2}{3}}\,\chi^2\,\mathrm{e}^{2\chi^2}\int_{-\infty}^{\infty}\mathrm{d}\kappa\,\mathrm{e}^{-\kappa^2}\Big|\sum_{\ell=\pm1}\mathrm{e}^{2\ell\rho\kappa\chi}\mathrm{erfc}\big(\ell\rho\kappa+\chi\big)\Big|^2$$

# Appendix

For the high-intensity laser pump we consider the following field profile

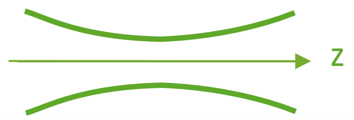
transverse profile

Gouy phase

$$\mathcal{E}(x) = \mathcal{E}_0 \, \mathrm{e}^{-\frac{(\mathrm{z}-t)^2}{(\tau/2)^2}} \frac{w_0}{w(\mathrm{z})} \, \mathrm{e}^{-\frac{r^2}{w^2(\mathrm{z})}} \cos\Big(\Omega(\mathrm{z}-t) + \frac{\Omega r^2}{2\,\mathrm{z}\big[1+(\frac{\mathrm{z}_R}{\mathrm{z}})^2\big]} - \arctan(\tfrac{\mathrm{z}}{\mathrm{z}_R})\Big)\,,$$

pulse envelope

For the x-ray probe field is justified to use in the infinite Rayleigh length approximation




$$w(\mathrm{z}) = w_0\sqrt{1+(\tfrac{\mathrm{z}}{\mathrm{z}_R})^2}$$

$$\mathfrak{E}(x) = \mathfrak{E}_0 \, \mathrm{e}^{-\frac{(\mathrm{z}+t)^2}{(T/2)^2}} \, \mathrm{e}^{-\frac{r^2}{w_{\mathrm{x}}^2}} \cos\big(\omega(\mathrm{z}+t)\big)$$

The peak field amplitudes can be expressed as

$$\mathcal{E}_0^2 = 8\sqrt{\frac{2}{\pi}}\frac{W}{\pi w_0^2 \tau}\,, \qquad \mathfrak{E}_0^2 = 8\sqrt{\frac{2}{\pi}}\frac{N\omega}{\pi w_{\mathrm{x}}^2 T}$$

[Karbstein,Mosman: PRD **96** (2017)]