A Reduced Model for Breit-Wheeler Pair Production by a Gaussian or Laguerre-Gaussian Laser Beam of Arbitrary Polarization and a Gamma flash

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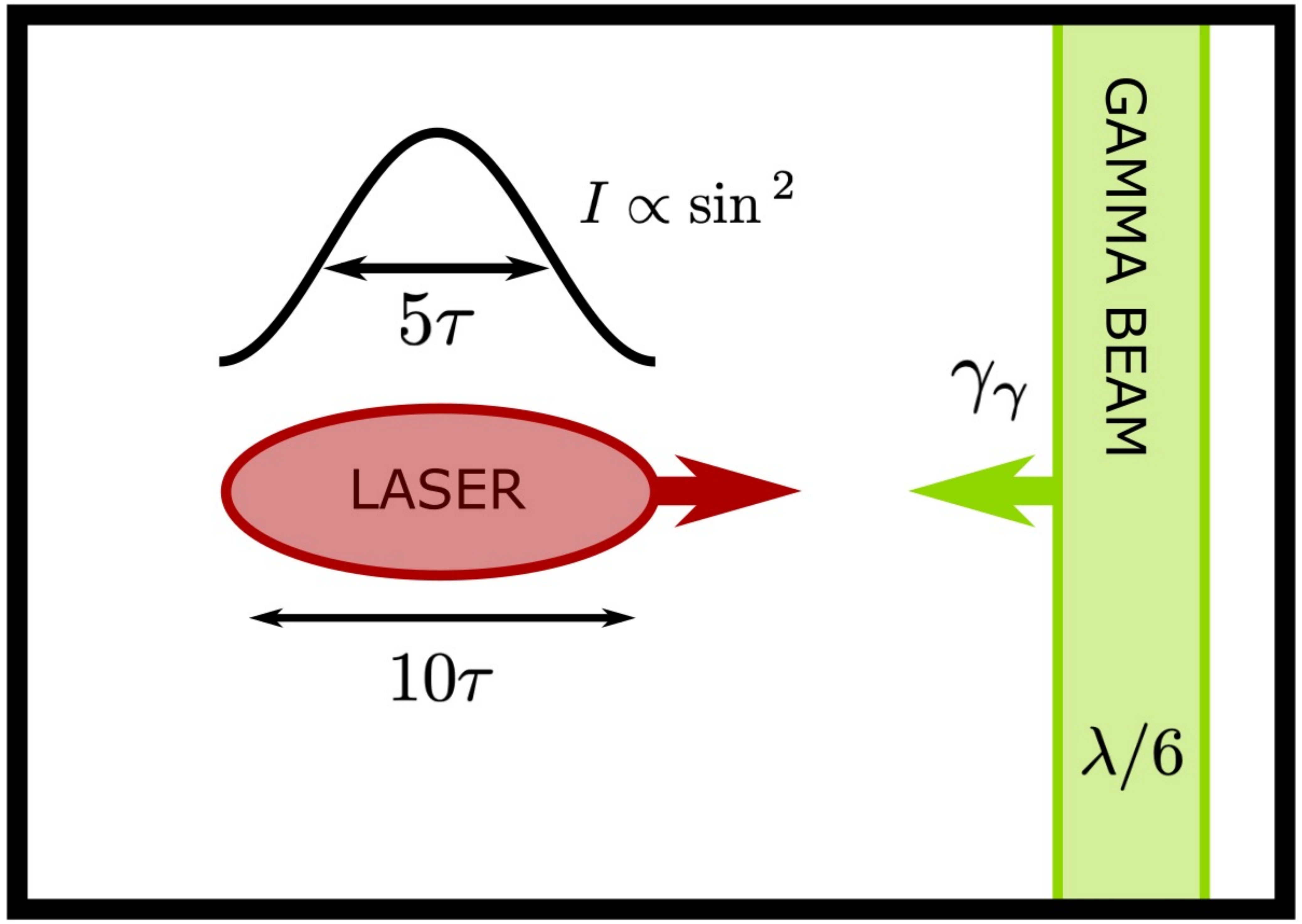
Reduced model for laser spatio-temporal shape

- A model for pair production in head-on collision of a laser pulse and gamma photons (electrons)

- Parametric study at fixed laser energy and focus on the soft shower regime of pair production (low secondary pairs)

  - Time dependence and 'building block'
  - Role of the laser pulse's spatial shape (Laguerre-Gauss beams)

Typical setup



Reminder : Nonlinear Breit-Wheeler

Gamma photon propagating in a laser field

Rate of pair production by photon decay (LCFA)

$$W_{\mathrm{BW}} = W_0 \frac{b_0(\chi_\gamma)}{\gamma_\gamma}$$

$$W_0 = \frac{2\alpha m c^2}{3\hbar} = 3.8\times 10^{18}\mathrm{s}^{-1}$$

$$\gamma_\gamma = \frac{\hbar\omega_\gamma}{m_e c^2}, \quad a_0 = \frac{eE_0}{mc\omega}, \quad \chi_\gamma \propto \gamma_\gamma \frac{E_0}{E_S} \propto \gamma_\gamma a_0$$

In the soft shower regime (no secondary gamma photons)

$$\frac{d}{dt} N_\gamma = -W_{\mathrm{BW}}(t) N_\gamma$$

The probability for a photon to decay in *Δt*

$$P(\Delta t) = 1 - \exp\left[-\int_{t_0}^{t_0+\Delta t} W_{\mathrm{BW}}(t')dt'\right]$$

Key element

Pair creation in a plane wave

Head-on collision with a plane wave

Field : $\mathbf{E}(z,t) = \frac{E_0}{\sqrt{1+\varepsilon^2}}\left[\sin(\omega t - kz)\hat{\mathbf{x}} + \varepsilon\cos(\omega t - kz)\hat{\mathbf{y}}\right]$

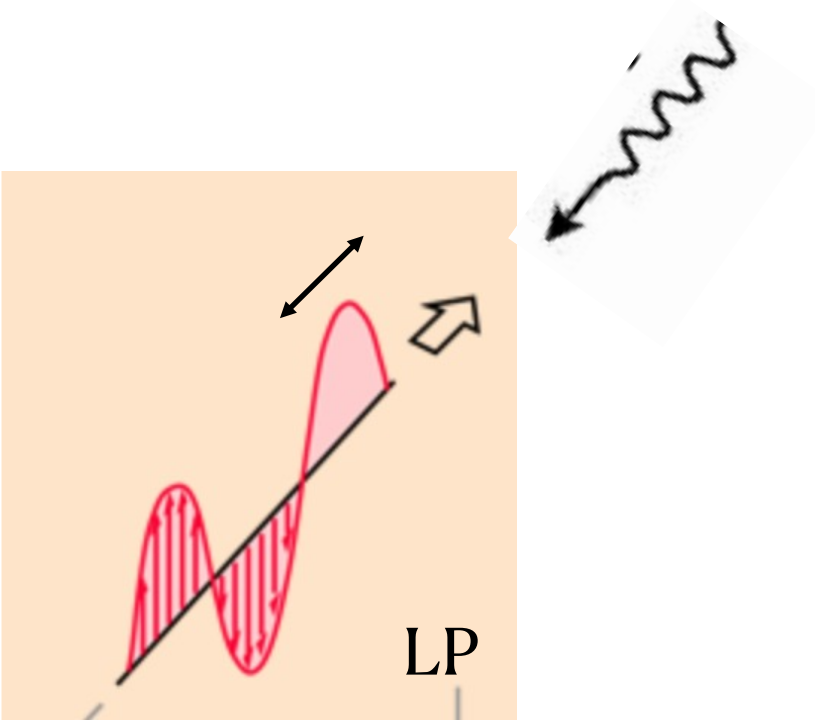
ε = polarisation
(normalisation constant energy)

Photon quantum parameter : $\chi_\gamma(t) = 2\gamma_\gamma \frac{E_0}{E_S}\sqrt{\frac{\sin^2(2\omega t)+\varepsilon^2\cos^2(2\omega t)}{1+\varepsilon^2}}$

χ_0 ; $\Psi_\varepsilon(2\omega t)$

Time dependent and $\tau/4$ periodic
(half period crossing, e.g. LP ->)

Examples :
$\chi_\gamma(t) = \chi_0|\sin(2\omega t)|$ for LP
$\chi_\gamma = \chi_0/\sqrt{2}$ for CP



Building block : the rate integral over $\tau/4$

$$\int_0^{\tau/4} W_{\mathrm{BW}}(t')dt' = \frac{W_0}{2\omega\gamma_\gamma}\underbrace{\int_0^{\pi} b_0(\chi_0\Psi_\varepsilon(\varphi))d\varphi}_{\mathcal{J}_\varepsilon(\chi_0)}$$

Probability in a LP plane wave

Example LP ($\varepsilon = 0$) : $\mathcal{J}_0(\chi_0) = \int_0^{\pi} b_0(\chi_0 \sin(\varphi))\mathrm{d}\varphi$ can be solved numerically or

analytical approximation :

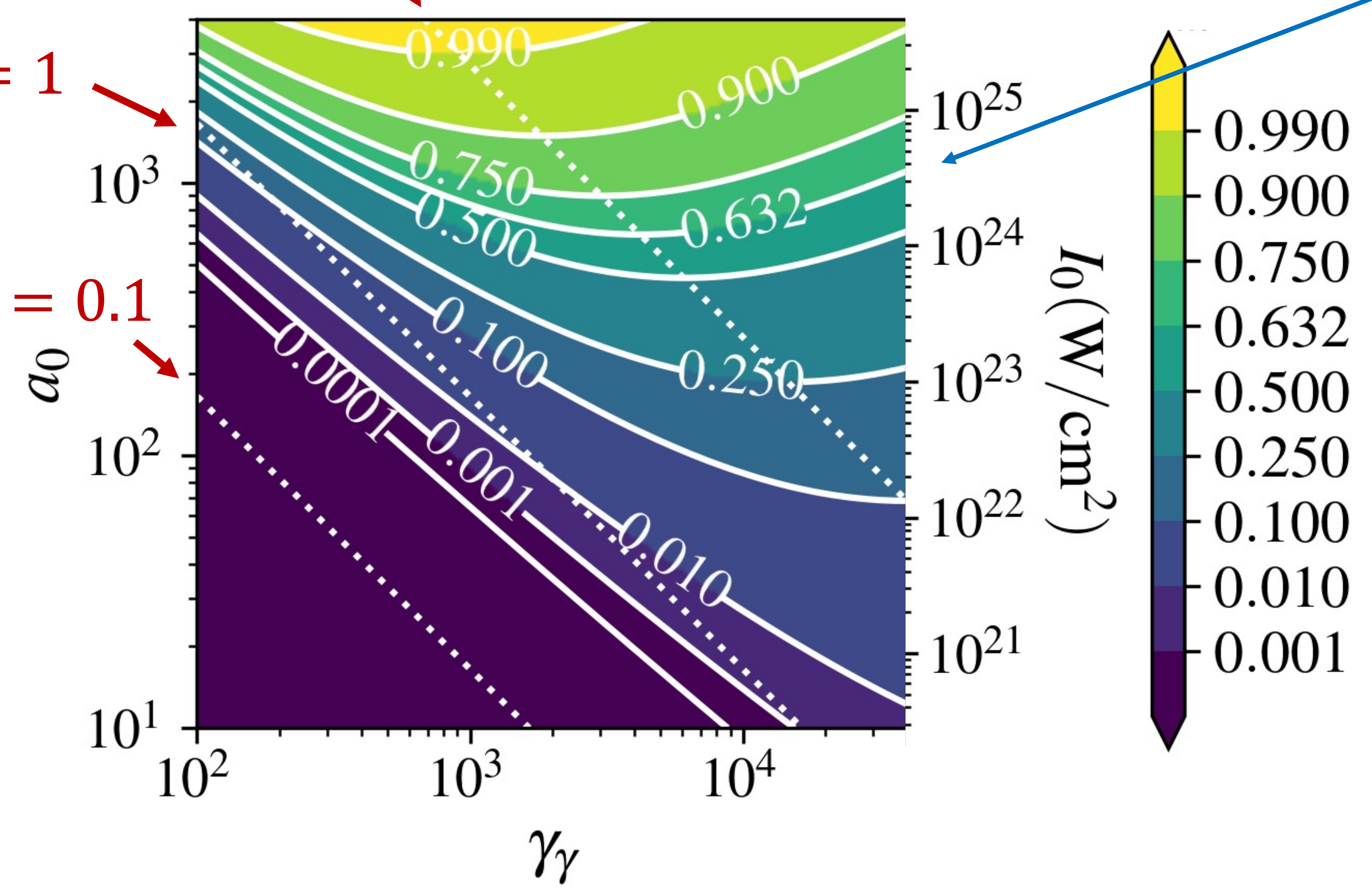
$$\mathcal{J}_0(\chi_0) \simeq \pi b_0(\chi_0) \min\left\{F\left(\sqrt{\frac{2b_0(\chi_0)}{3\chi_0 b_0'(\chi_0)}}\right), f\right\}$$

$$\left(\begin{array}{l} F(s) = \sqrt{2/\pi} s \, \mathrm{erf}\left(\pi\sqrt{2}/(4s)\right) \\ f = \frac{1}{\pi}\int_0^{\pi} \sin^{2/3}(\varphi)\mathrm{d}\varphi \end{array}\right.$$

The probability to create a pair after a half period crossing $P(\tau/4) = 1 - \exp\left[-\frac{W_0}{2\omega\gamma_\gamma}\mathcal{J}_0(\chi_0)\right]$

N.B. $P(\tau/4, a_0, \gamma_\gamma)$

Probability after half period crossing (LP)



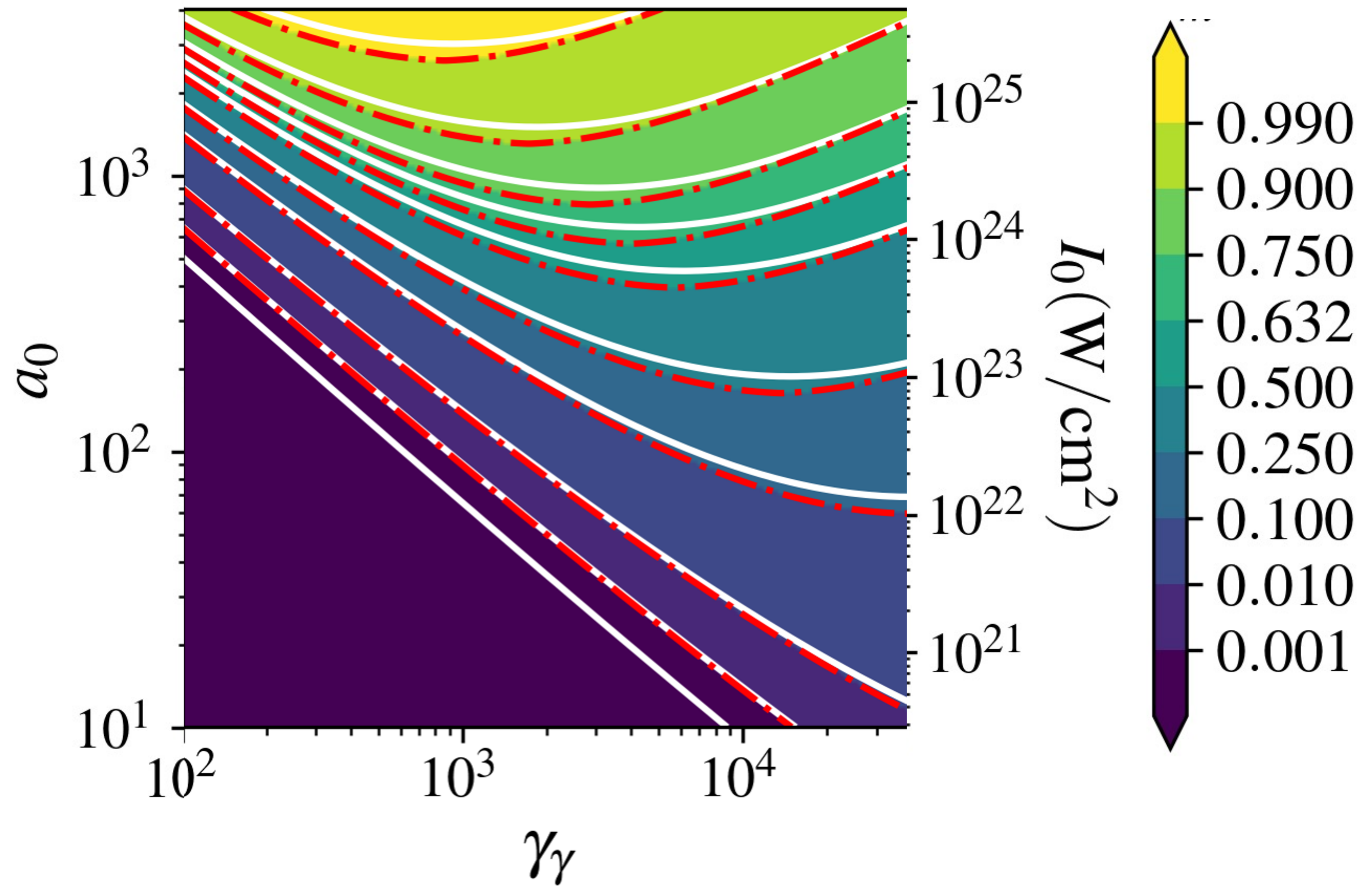
'High' probability
P > 0.63 (1-1/e)

Dotted lines :
constant χ_0 =(0.1; 1; 16.5 (

Numerical solution of the equicontours of constant probability :
straight lines at low a_0, γ_γ but cross values of constant χ_0
minimum a_o for a given probability, along the line χ_0 ~ 16.5

Analytical approximation

$P(\tau/4)$

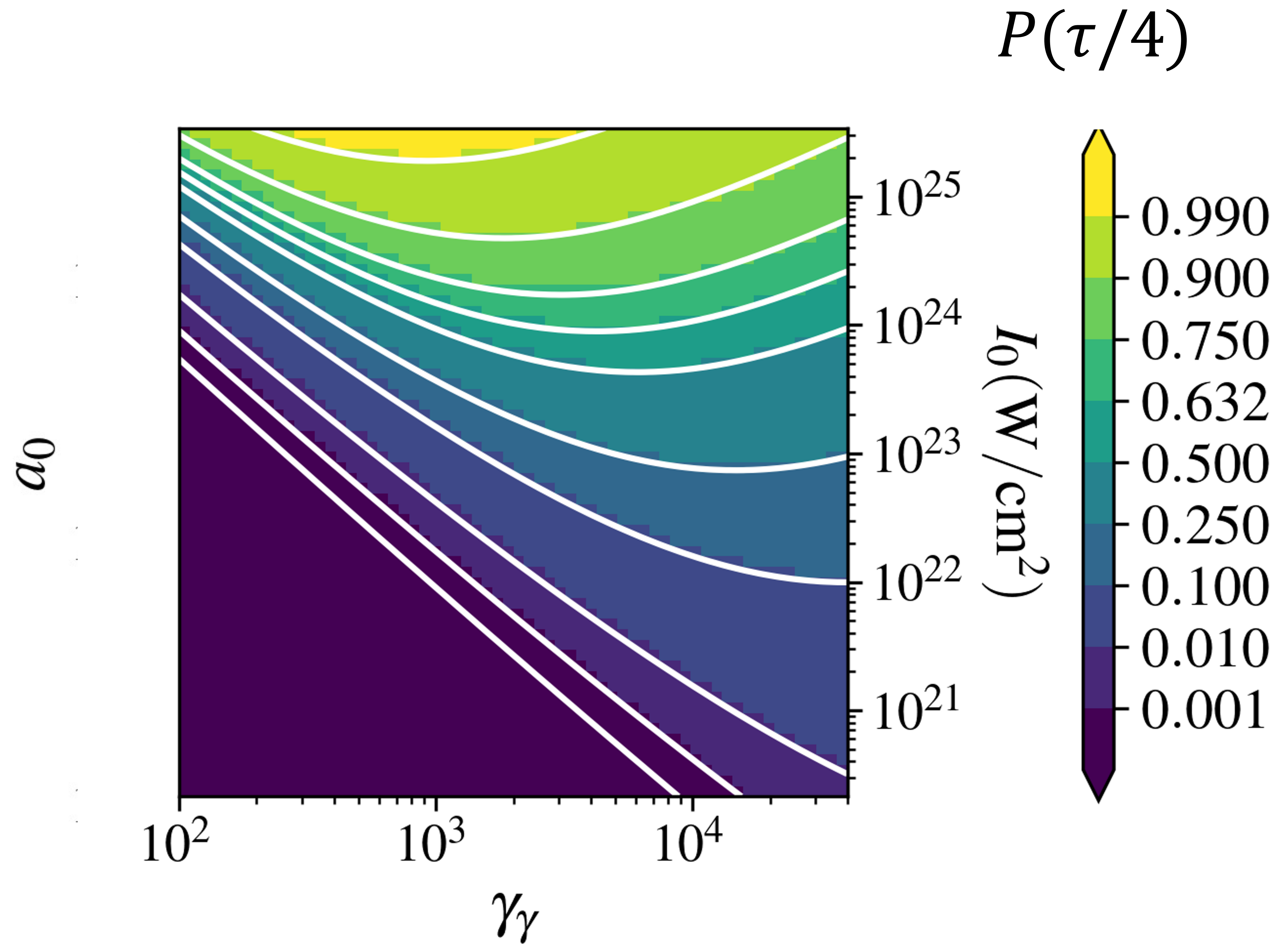


$$\mathcal{I}_0(\chi_0) \simeq \pi b_0(\chi_0) \min\left\{F\left(\sqrt{\frac{2b_0(\chi_0)}{3\chi_0 b_0'(\chi_0)}}\right), f\right\}$$

$$\left(\begin{array}{l} F(s) = \sqrt{2/\pi} s \,\mathrm{erf}\big(\pi\sqrt{2}/(4s)\big) \\ f = \frac{1}{\pi}\int_0^\pi \sin^{2/3}(\varphi)\mathrm{d}\varphi \end{array} \right.$$

Cross benchmark with PIC/MC

Excellent match with PIC/MC Smilei)* simulations



Generalization to more complex space and time dependences

*https://github.com/SmileiPIC/Smilei

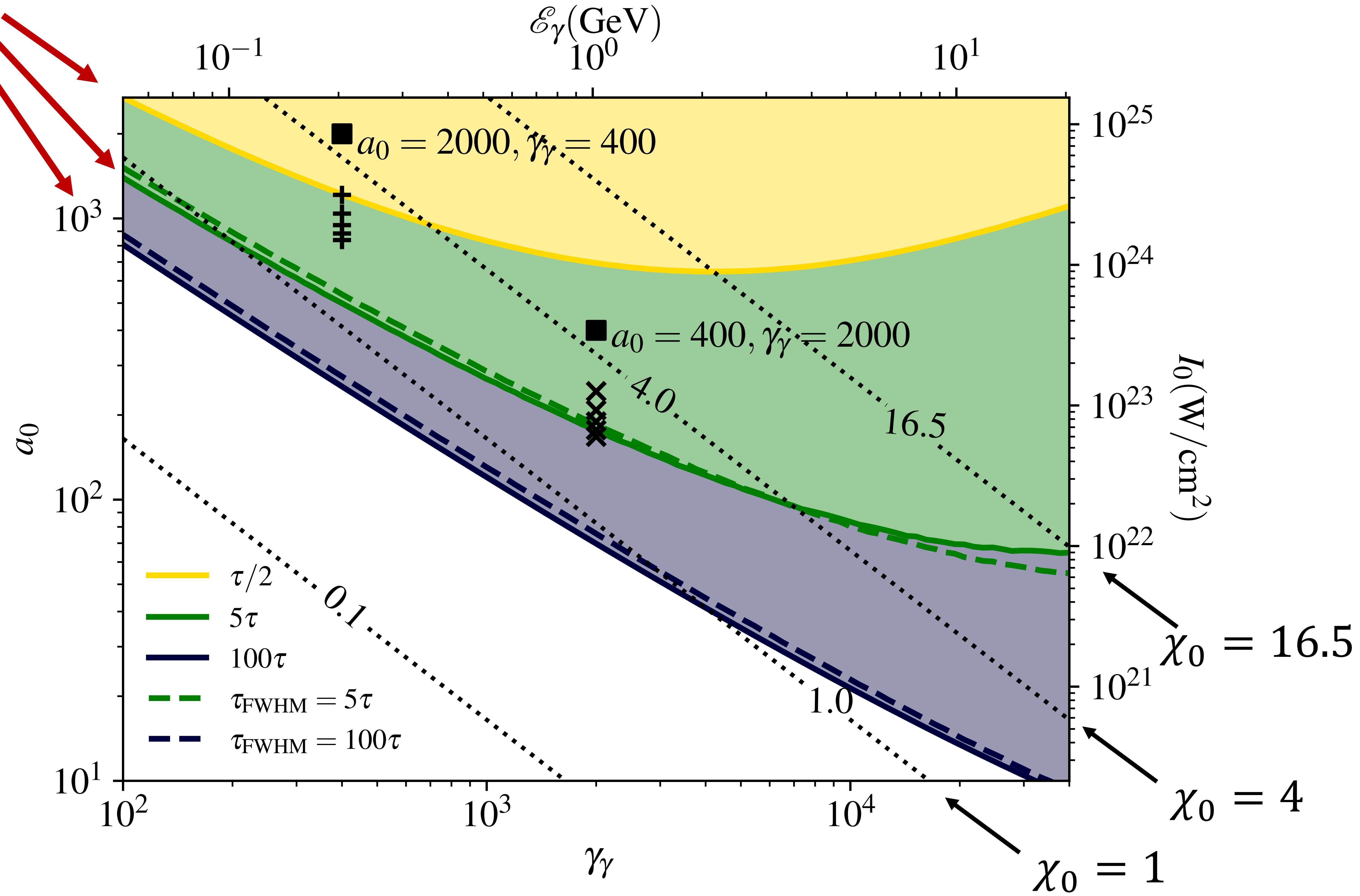
Finite pulse duration and parameter space

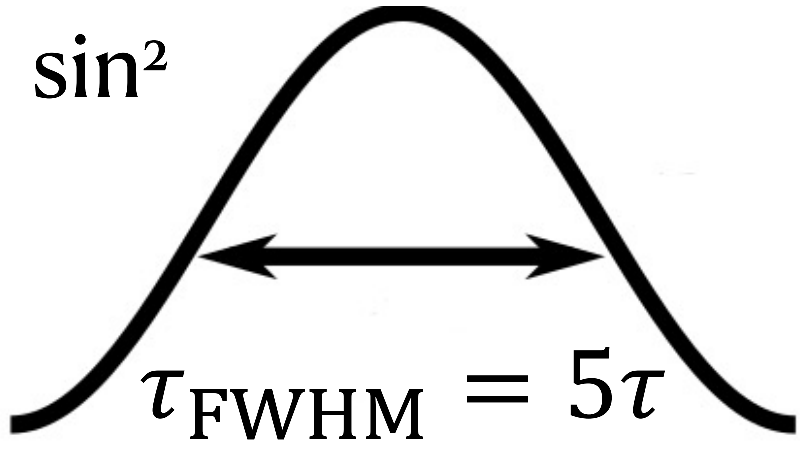
Probability $P_{tot}(t) \simeq 1 - e^{-Rt}$ with the (local) average rate $R = \frac{4}{n\tau} \sum_{m=1}^{n} \frac{W_0}{2\omega\gamma_\gamma} \mathcal{J}_0(\chi_m)$

\# of maxima crossed in t

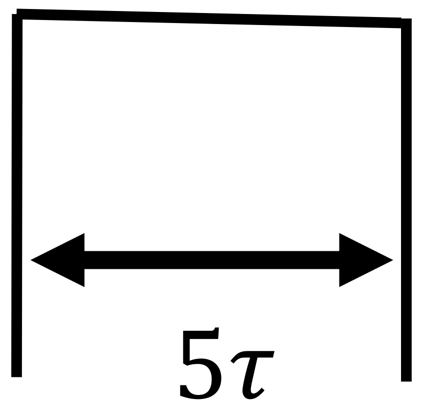
Building block, χ_m at local maximum

High probability isocontour (P>0.63) for different time durations





Plane wave of finite duration



Laguerre-Gauss beams and space dependence

Complex envelope $u_{p\ell}(\rho,\phi,z) = C_{p\ell}\,\frac{w_0}{w(z)}\left(\frac{\sqrt{2}\rho}{w(z)}\right)^{|\ell|} L_p^{|\ell|}\left(\frac{2\rho^2}{w^2(z)}\right)\exp\left[-\frac{\rho^2}{w^2(z)}\right]\exp\left[-i\psi_{pl}(z) + i\ell\phi + i\,\frac{z\rho^2}{w^2(z)}\right]$

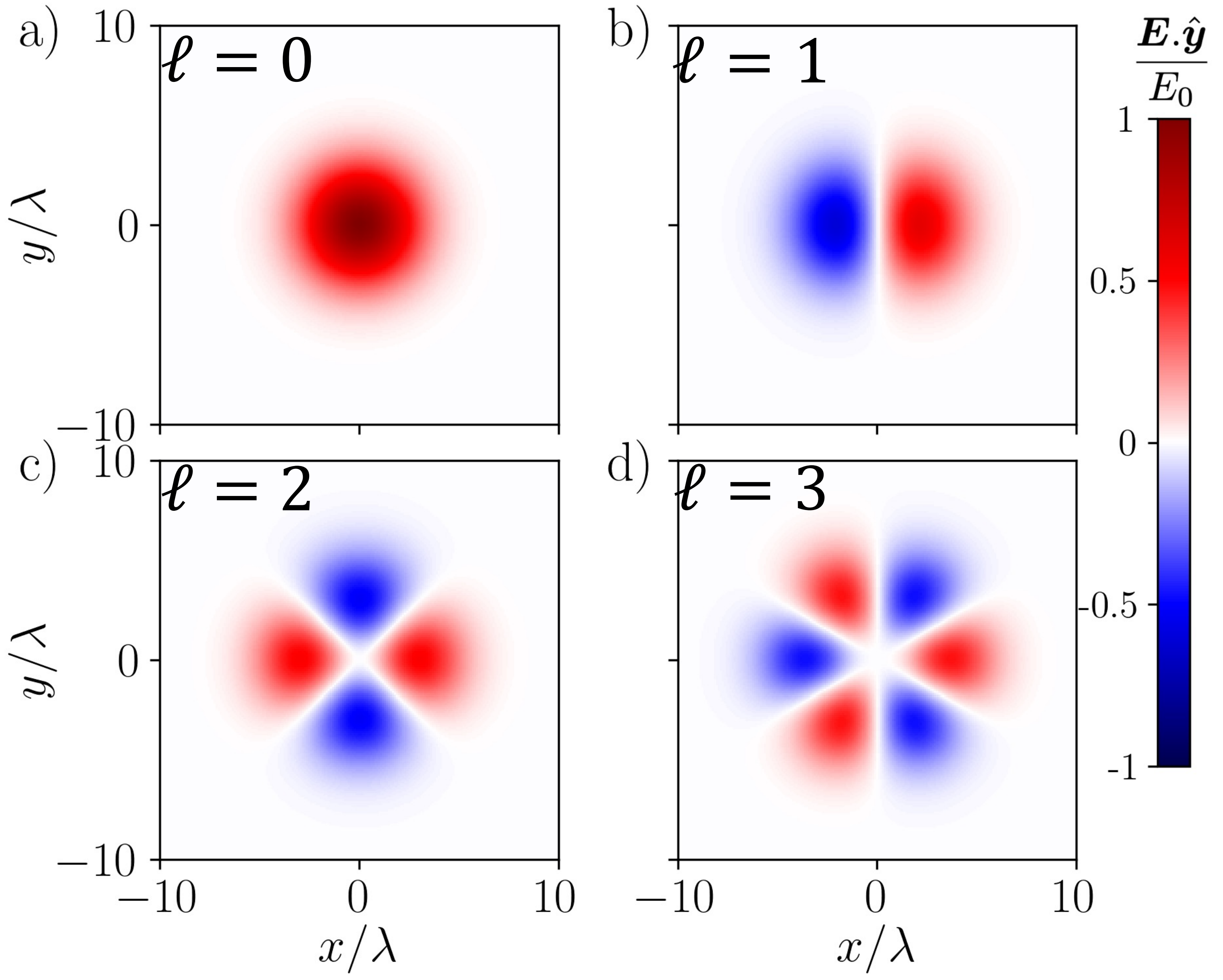
Polynomial factor

Generalised phase

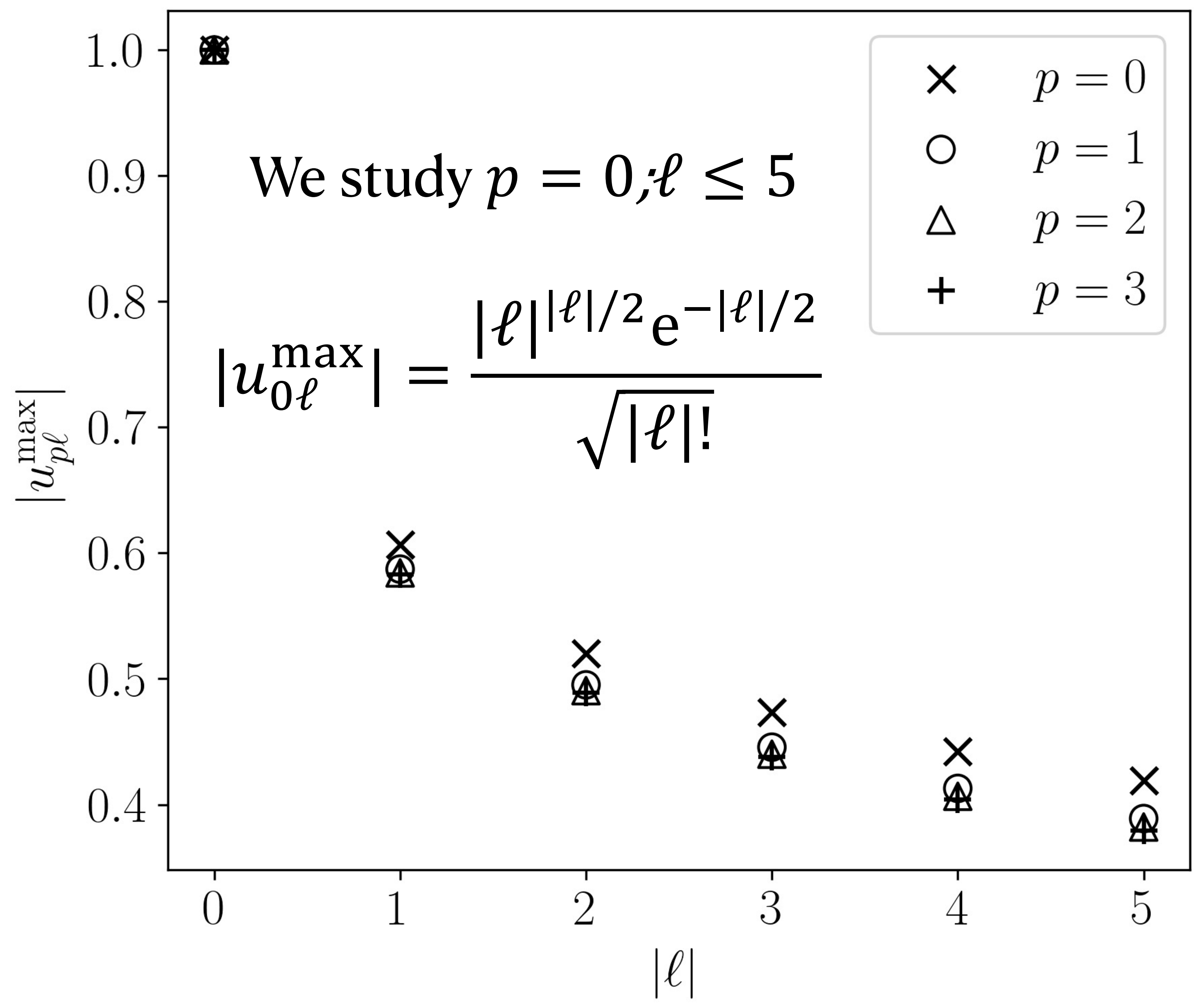
Solutions of the paraxial equation with constant total energy

The beam's transverse size increases with ℓ, the amplitude decreases

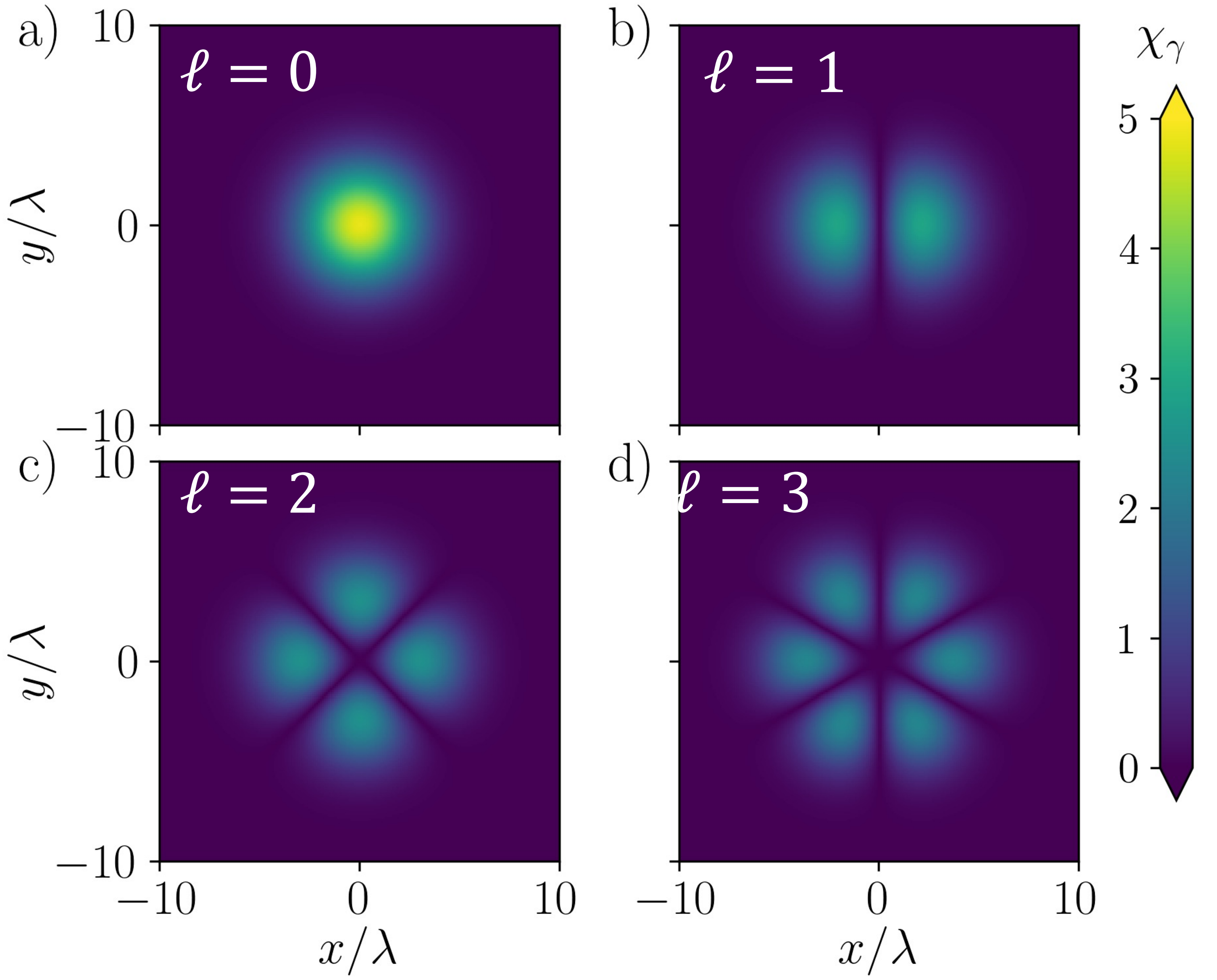
Normalised field



Maximum amplitude



Photon quantum parameter $a_0 = 2000,\ \gamma_\gamma = 400$



Total cross section of pair creation

Compute the time integrated probability for a finite time duration

with fields' spatial distribution given by the LG field $\frac{1}{\sqrt{\ell!}}\left(\frac{\sqrt{2}\rho}{w_0}\right)^{|\ell|} \exp\left[-\frac{\rho^2}{w_0^2}\right]$

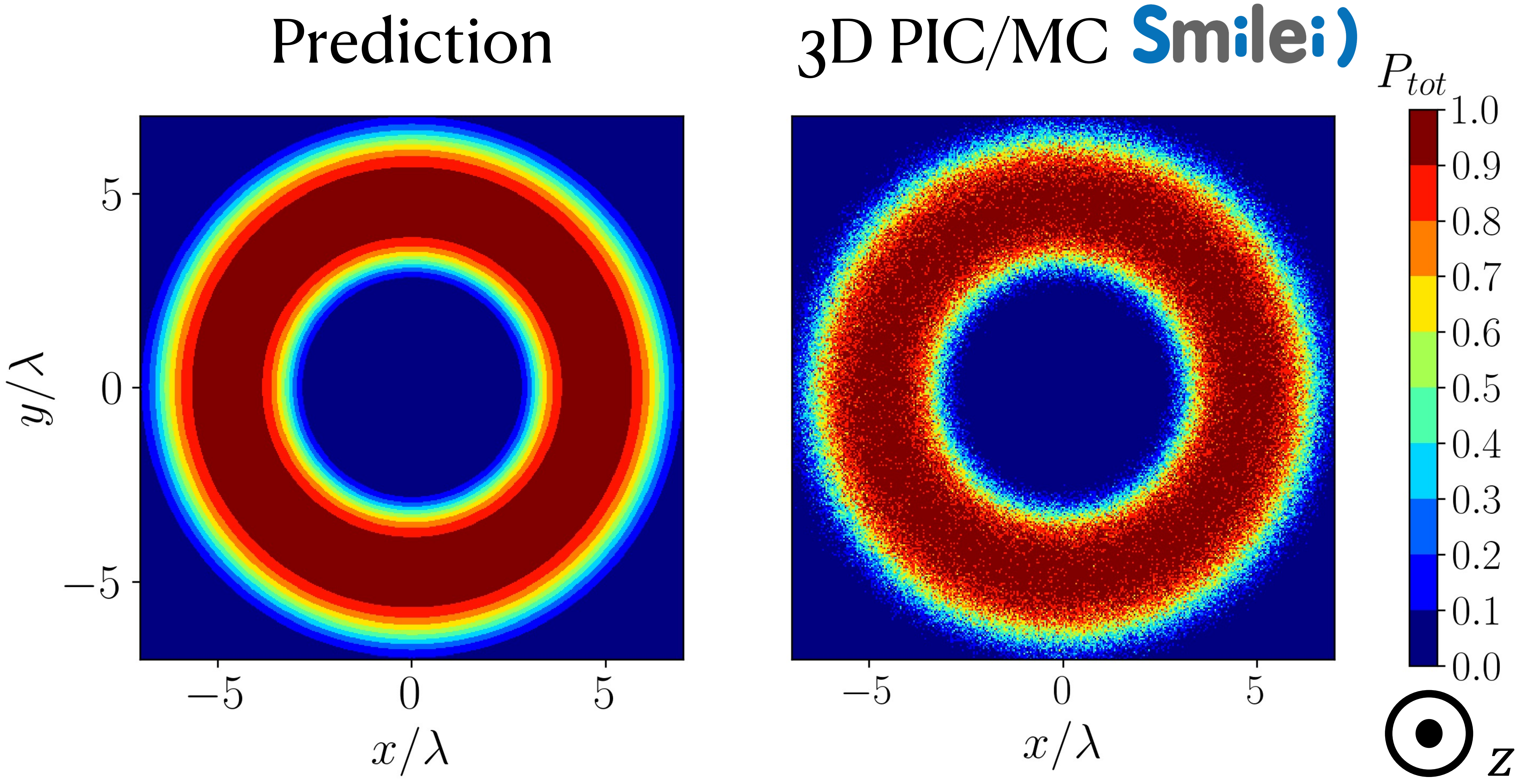
Example $\ell = 5$
$a_0 = 2000, \gamma_\gamma = 400$
$\tau_{\mathrm{FWHM}} = 5\tau$

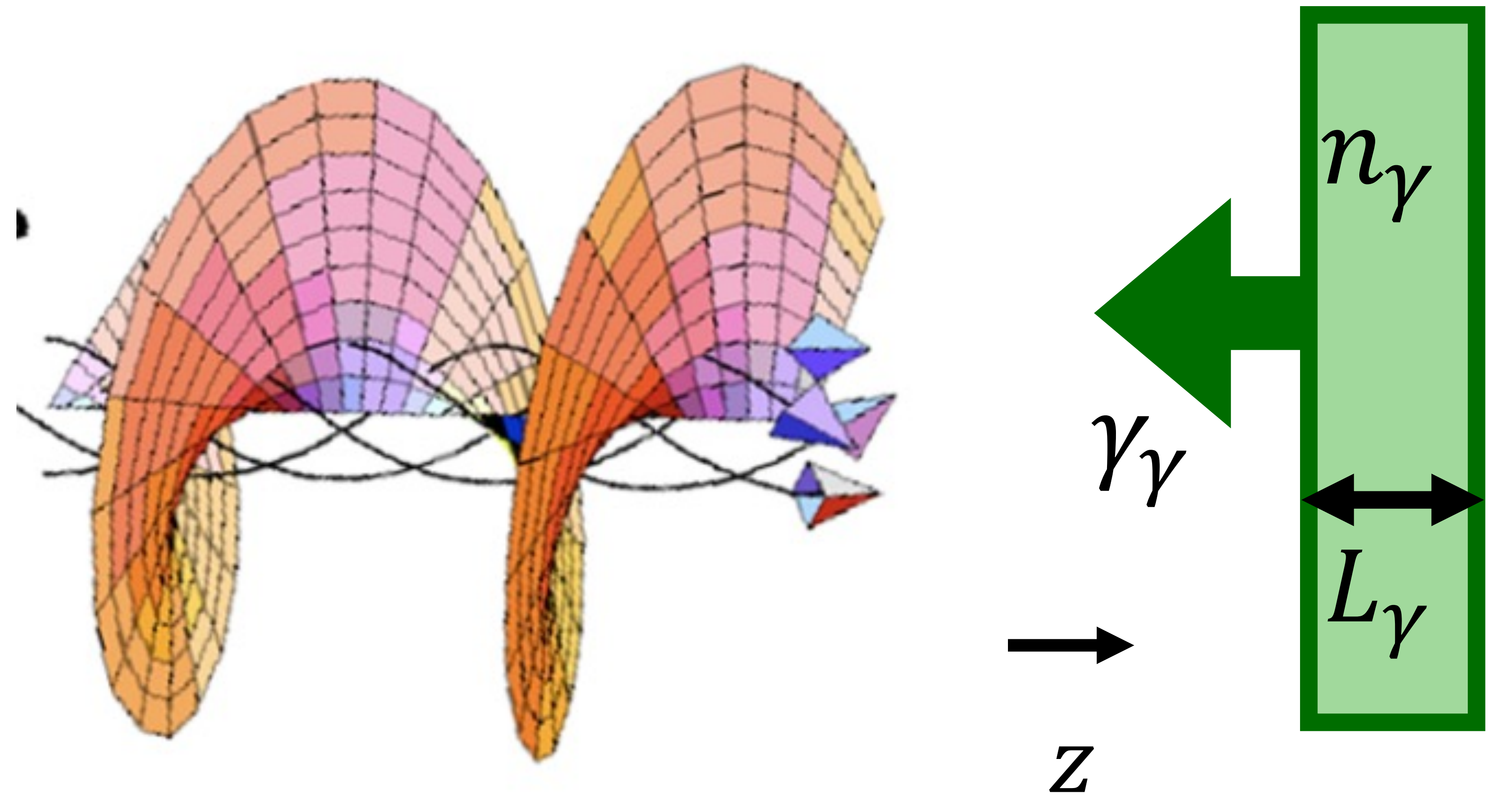
We obtain a probability map

Total cross section $\sigma_{tot} = \int\int P_{\mathrm{tot}}\mathrm{dxdy}$

N.B. Integral over interaction region

Number of pairs $N_{pair} = \sigma_{tot} n_\gamma L_\gamma$

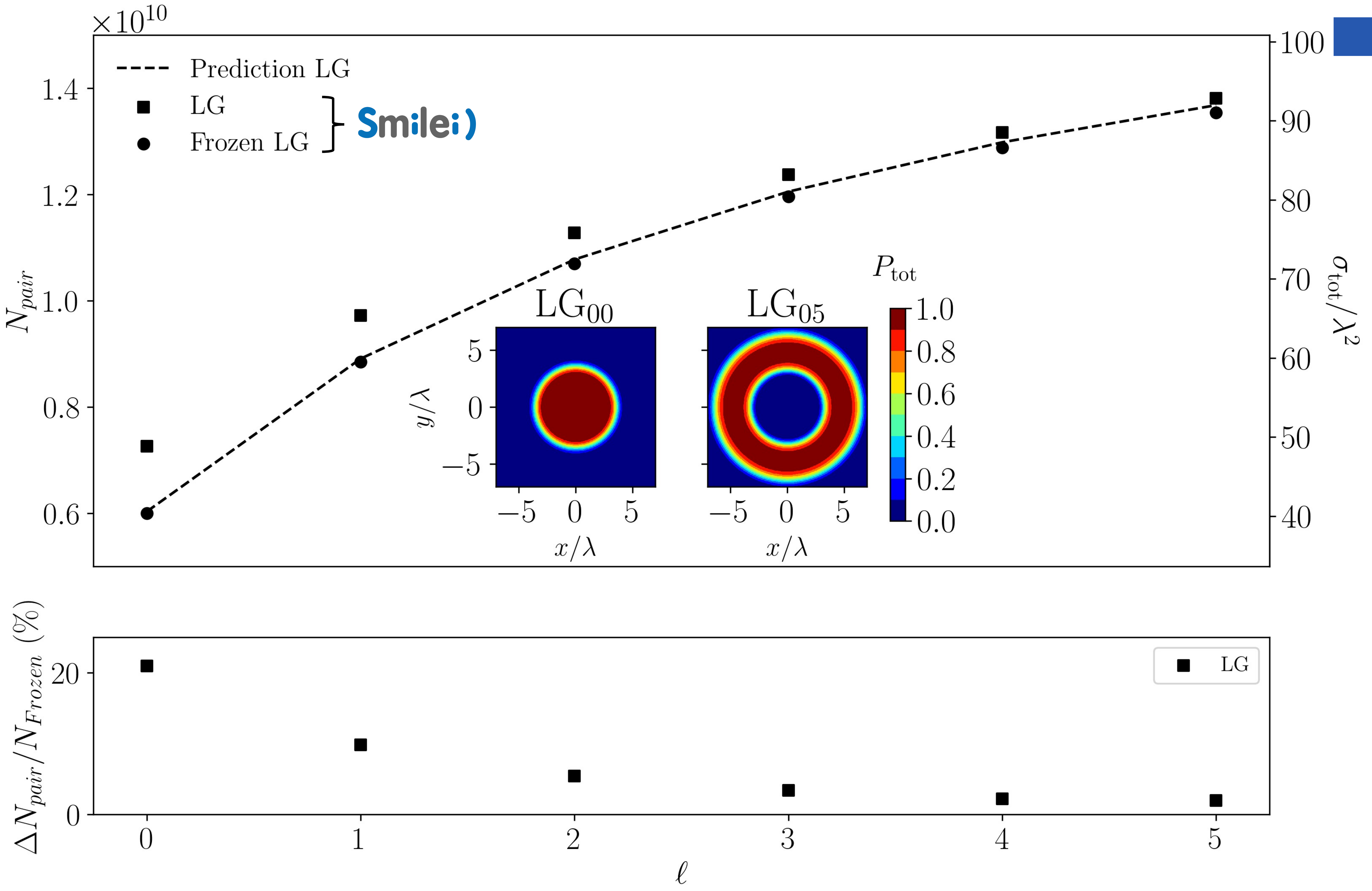




Pair creation in LG beams

First set of PIC/MC Smilei) simulations

$a_0 = 2000, \gamma_\gamma = 400, \tau_{\mathrm{FWHM}} = 5\tau, \chi_0 = 4.85$



Excellent agreement with model

The cross section is increasing with ℓ

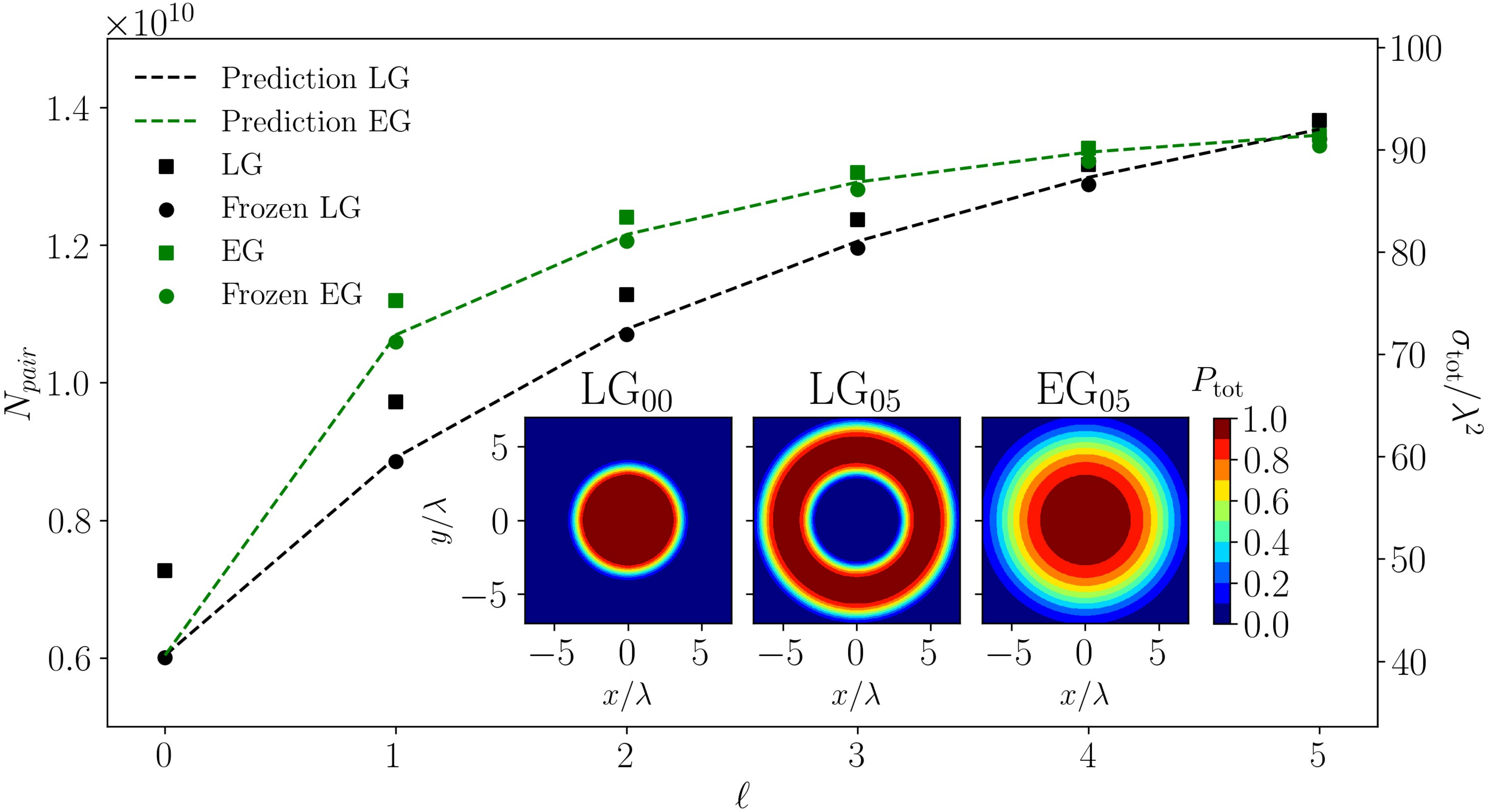
Geometrical effect suggest to consider defocused Gaussian beams with the same energy and same peak value as LG

Few secondary pairs (maximum 20 %)

Pair creation in less focused Gaussian beams

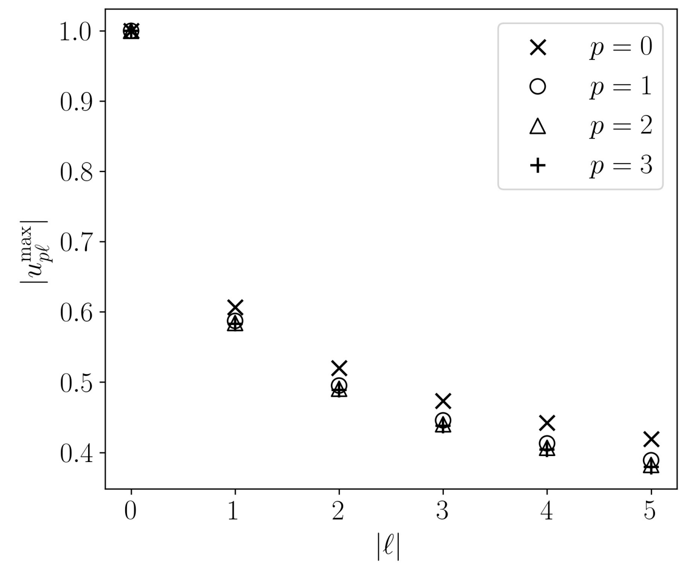
Comparison with Extended Gaussian (EG)

$a_0 = 2000, \gamma_\gamma = 400, \tau_{\mathrm{FWHM}} = 5\tau$, $\chi_0 = 4.85$



Extended Gaussian (EG) are constructed in order to have the same total energy and the same peak value as the LG of the corresponding order

$\ell \neq 0$, p=0

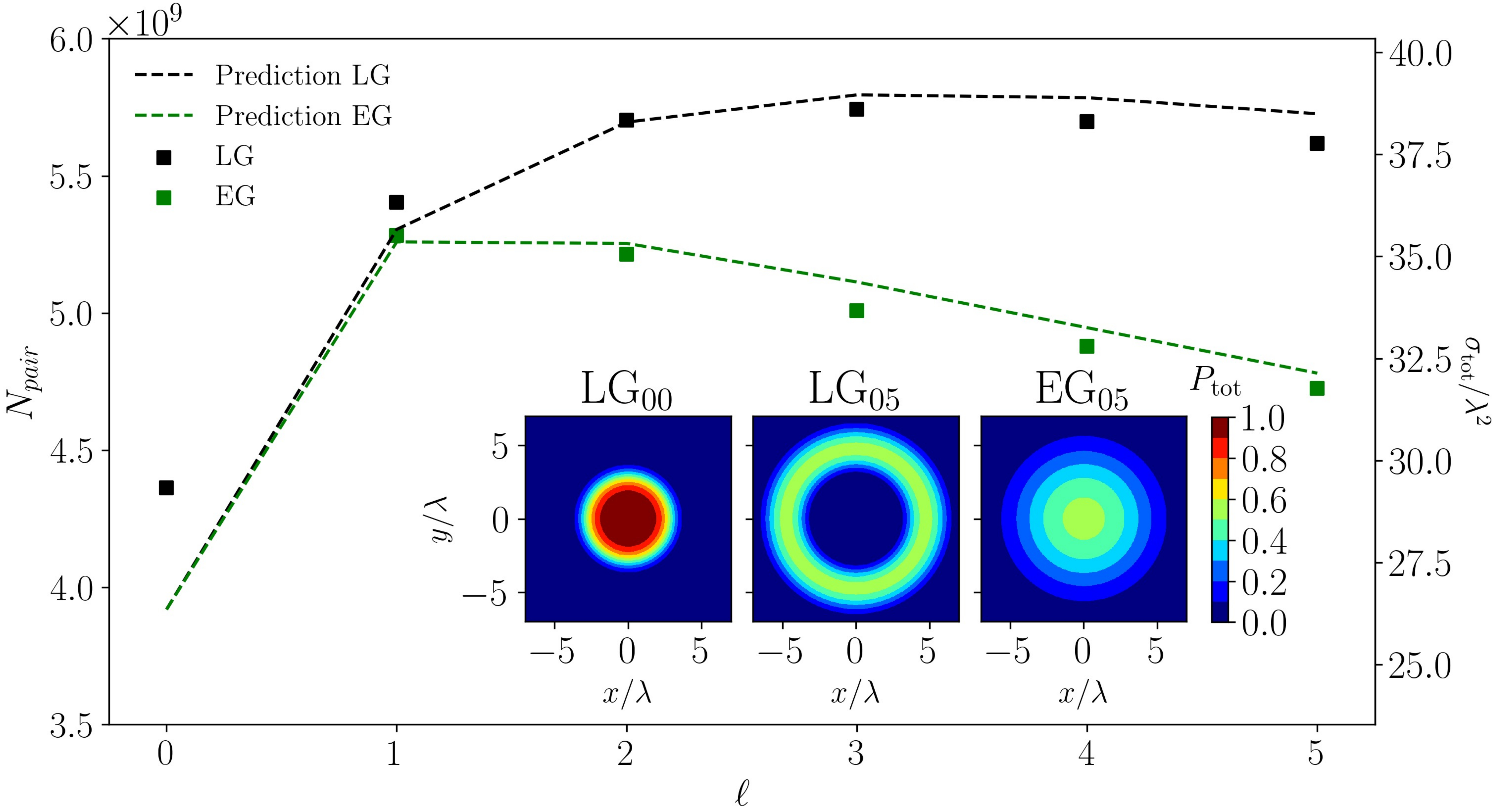


Number of produced pairs in EG higher than LG for ℓ <4 (bigger interaction area)

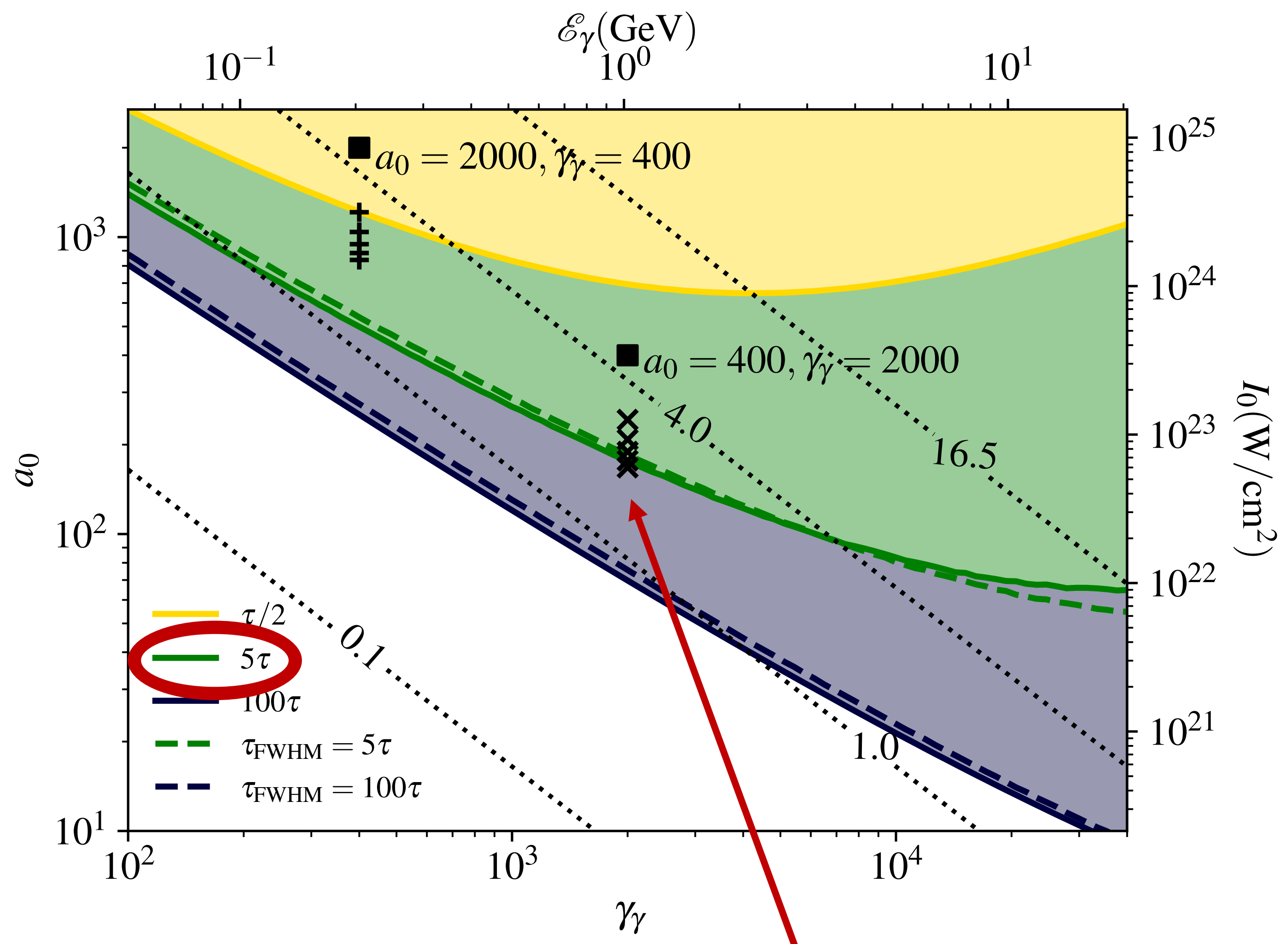
LG beams vs EG for lower intensities

Importance of a_0 versus γ_γ

Second set of PIC/MC Smilei) simulations
$a_0 = 400, \gamma_\gamma = 2000, \tau_{\mathrm{FWHM}} = 5\tau, \chi_0 = 4.85$



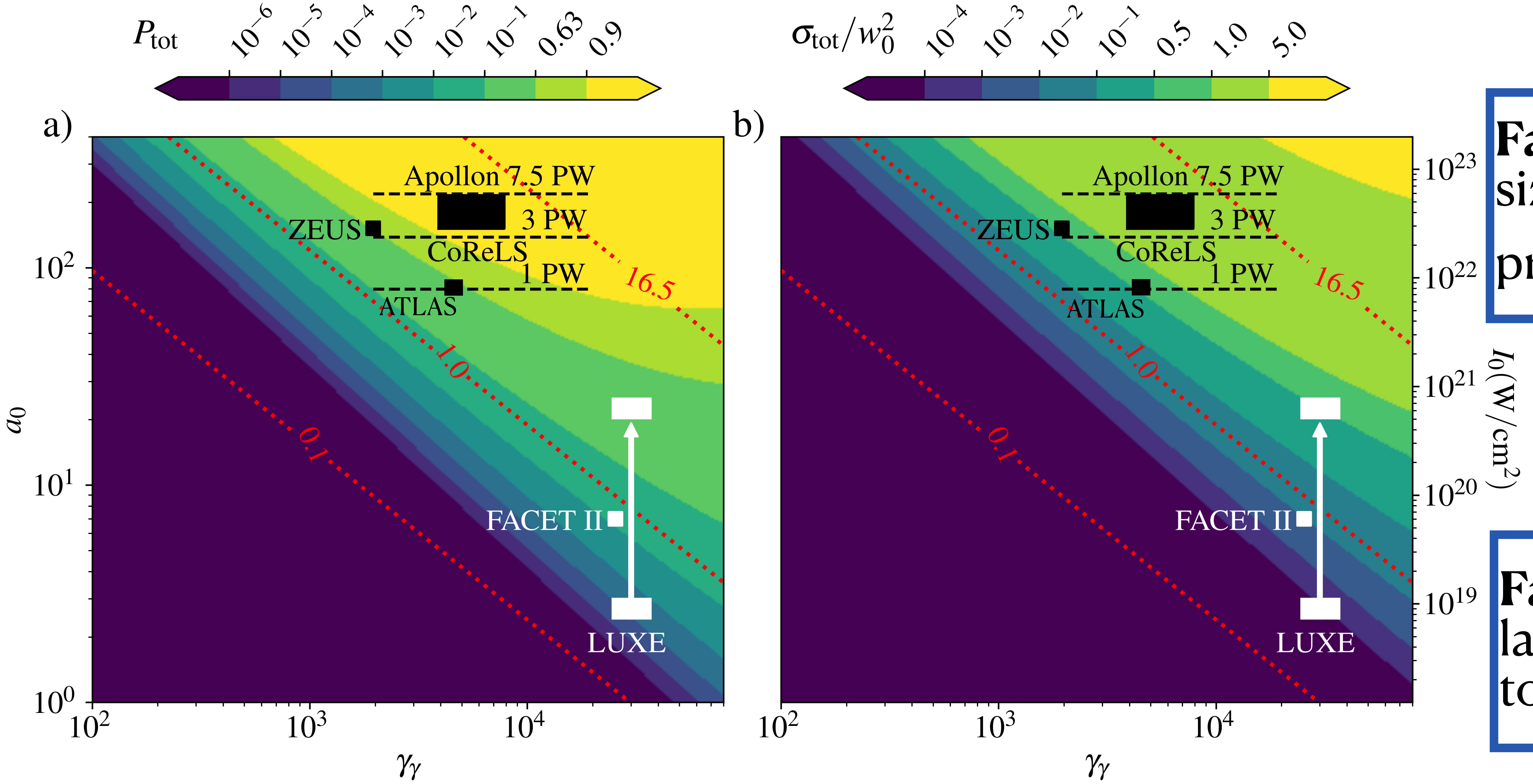
High probability isocontours (P>0.63)



Start from lower a_0 : increasing the size is not enough to compensate for the field's amplitude diminution, no improvement on pair production.

Guidelines for upcoming facilities

Model's prediction for $\tau_{FWHM} = 25\text{fs}, \lambda = 0.8\mu\text{m}$



Dotted red lines = constant χ_0 (0.1 ; 1 ; 16.5)

Strategy for optimisation

Facilities in black: increase focal spot size (no tight focus) as the maximum probability is already ~ 1

Facilities in white : need to increase laser amplitude (tight focus) in order to have significant pair creation

Thank you

More details : A. Mercuri-Baron *et al* 2021 *New J. Phys.* **23** 085006