

From Theory to Experiment: Challenges and Opportunities in SFQED

Tutorial/Plenary talk, ExHILP 2021

In collaboration with: many people (especially FACET and E-320 collaboration), in particular: David Reis, Vitaly Yakimenko, Phil Bucksbaum, Mark Hogan, Elias Gerstmayr, Zhijiang Chen, Doug Storey, Erik Isele, Rafi Mir-Ali Hessami, Christian Flohr Nielsen, Robert Holtzapple, Brian Naranjo, James Rosenzweig; as well as Roger Blandford & Michael Peskin; Nat Fisch & Kenan Qu

Sebastian Meuren

Sep 15, 2021



- Provide intuitive (some might say hand-waving) arguments for the relevant scales
- 1st part: phenomena that are accessible, e.g., to E-320 or LUXE
more details on **E-320**: talk by Elias Gerstmayr afterwards
more details on **LUXE**: talks by Beate Heinemann, Noam Tal Hod
- 2nd part: challenges, both theoretical and experimental, that might be relevant in the future

This should be a tutorial, so please interrupt and ask questions during the talk

Natural Units used (sometimes re-instated)

- $c = 1$ ($\approx 0.2998 \mu\text{m} / \text{fs}$), convert time \leftrightarrow length
- $\hbar = 1$ ($\approx 0.6582 \text{ eV fs}$), convert energy \leftrightarrow time
- $\epsilon_0 = 1$ (re-instate using α)
$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$$

Mostly focused on order-of-magnitude estimates

$$\pi \approx 3 \sim 1$$

$$2\pi \sim 1$$

$$\omega \sim 1/T$$

**Thank you very much for inviting me for this talk and for organizing ExHILP '21
I am grateful to DOE Office of Science for making this research possible**

Strong-field frontier

Where do we encounter extreme electromagnetic fields?

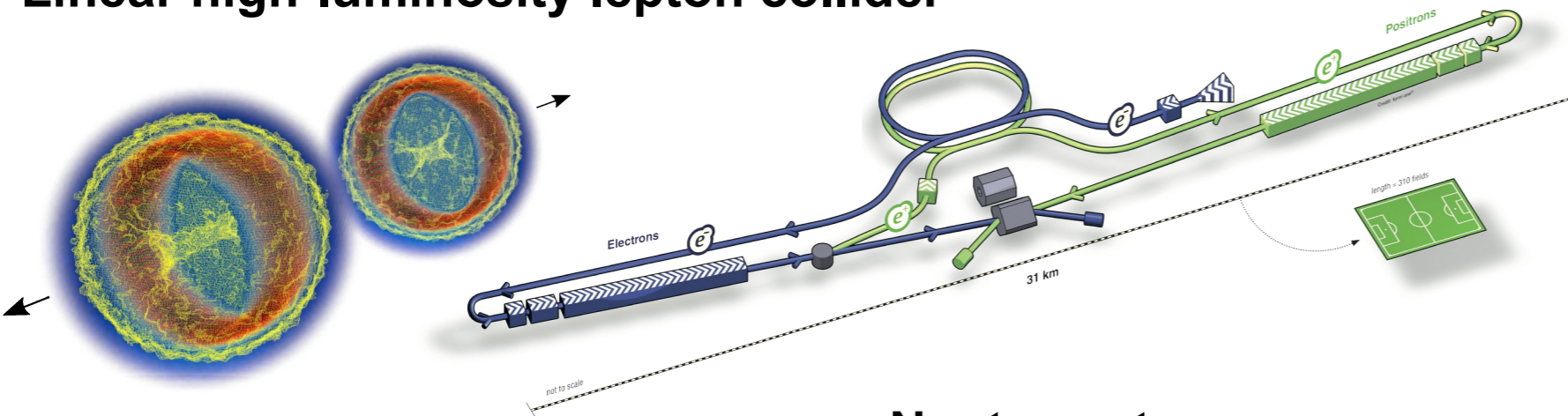
See talks by Alexander Philippov (15:00, Monday)

Tom Blackburn (12:30, Tuesday)

Chris Ridgers (12:30, Friday)



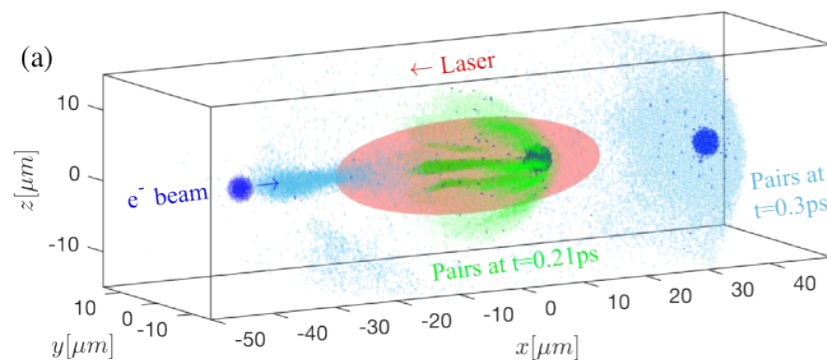
Linear high-luminosity lepton collider



Linear lepton collider are envisioned to produce extremely dense beams, which have an extremely high space charge

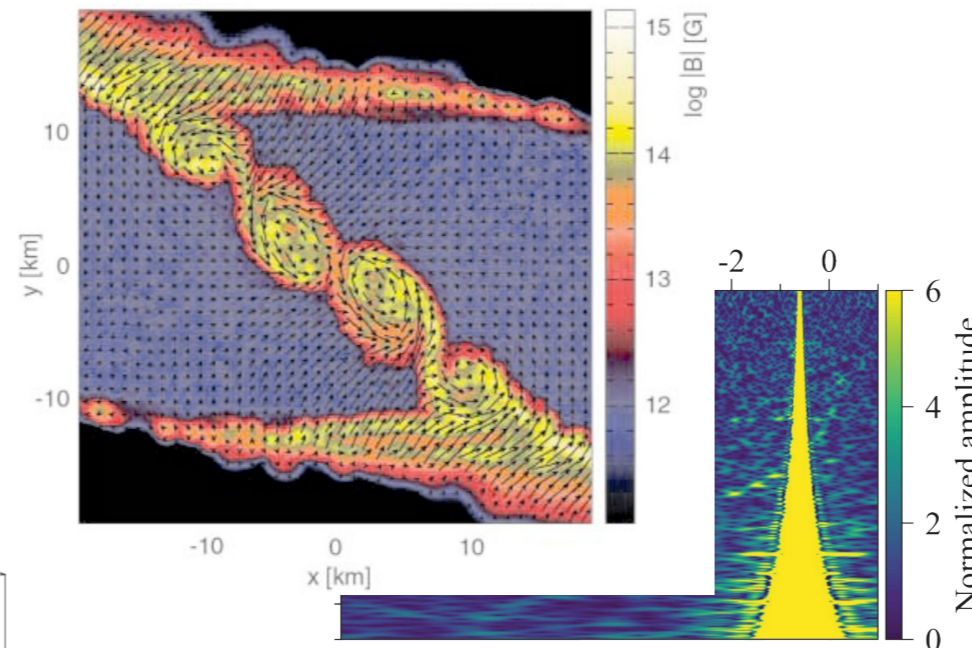
Yakimenko et al., PRL 122, 190404 (2019)
Esberg et al., PRSTAB 17, 051003 (2014)

Laser-beam collisions



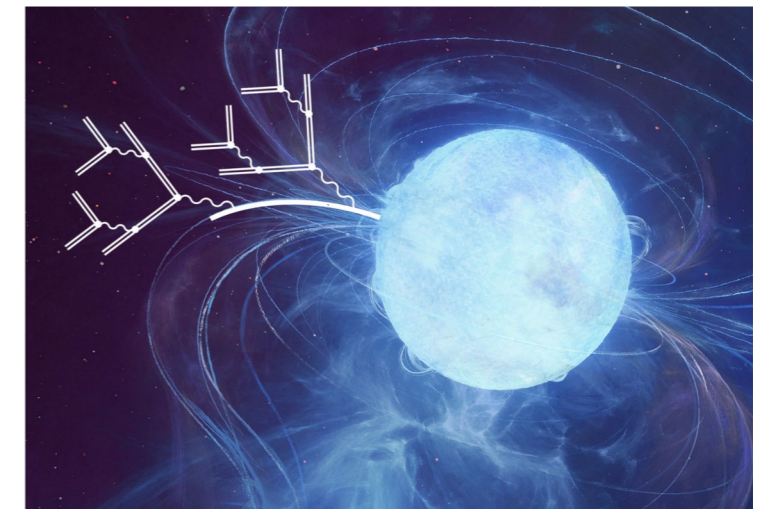
Qu et al., PRL 127, 095001 (2021)
Magnusson et al., PRL 122, 254801 (2019)

Neutron-star mergers



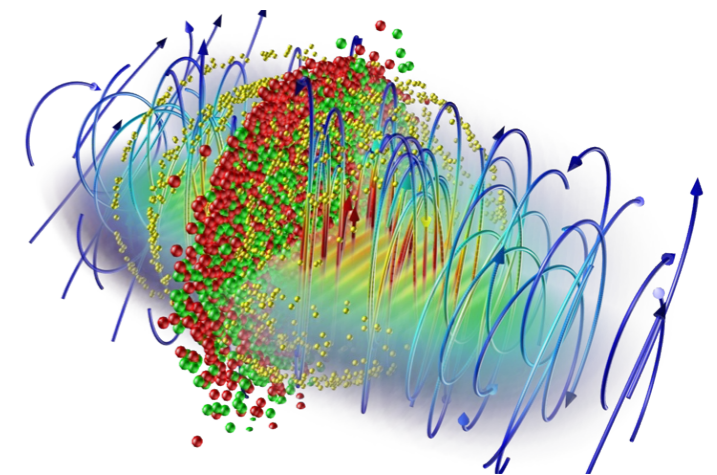
Gravitational waves & exotic matter in supercritical magnetic fields
Price & Rosswog, Science 312, 719 (2006)
LIGO/VIRGO, PRL 119, 161101 (2017)

Magnetars & Pulsars



QED plasma emerged in supercritical magnetic fields
Philippov et al., PRL 124, 245101 (2020)
Chen et al., APJ 889, 69 (2020)
Timokhin & Harding, APJ 871, 12 (2019)

Seeded laser-laser collisions



Bell & Kirk, PRL 101, 200403 (2008)
Grismayer et al., PoP 23, 056706 (2016)

Synchrotron radiation

Relativistic kinematics: Lorentz force

Basic notation: trajectory, momentum, force

- A particle is described by it's trajectory: $\vec{x}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$ velocity: $\vec{v}(t) = \frac{d\vec{x}}{dt} = \begin{bmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{bmatrix}$
- Relativistic kinematics: gamma factor (c=1): $\gamma = \frac{1}{\sqrt{1-v^2}}$ momentum: $\vec{p}(t) = \gamma m \vec{v}(t) = \begin{bmatrix} p_x(t) \\ p_y(t) \\ p_z(t) \end{bmatrix}$
- Lorentz force: $\frac{d\vec{p}}{dt} = \vec{F}_L = q (\vec{v} \times \vec{B} + \vec{E})$

Relativistic regime: nonlinear relationship between velocity and momentum

Relativistic charge ($v \approx c$) moving in a static magnetic field without drift

§ 21. Motion in a constant uniform magnetic field

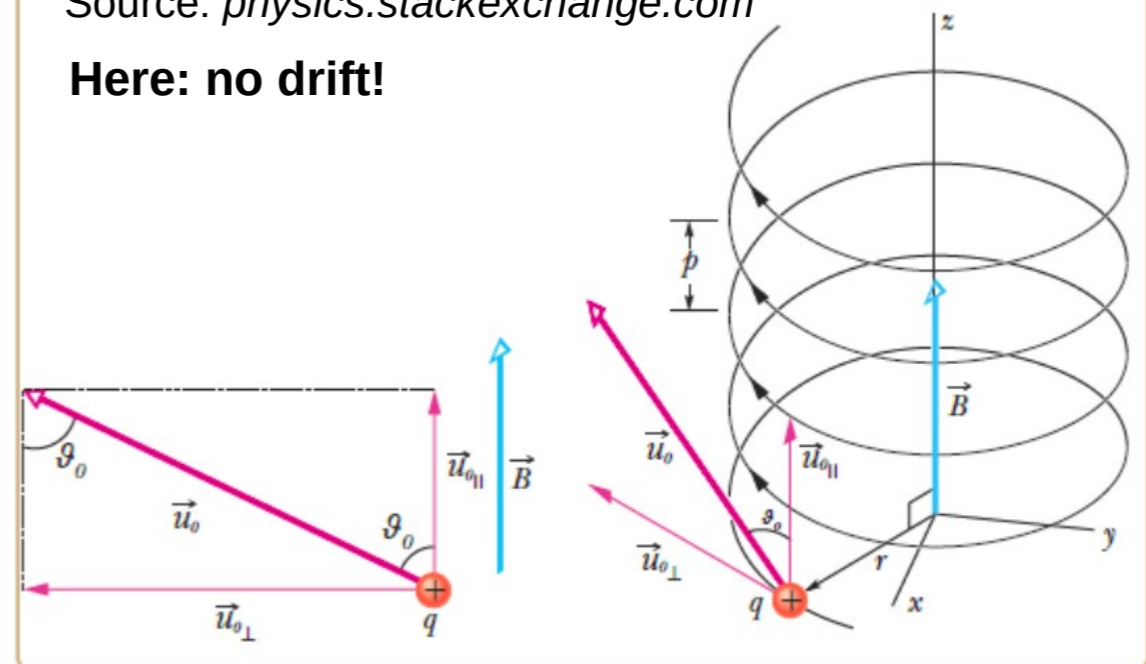
We now consider the motion of a charge e in a uniform magnetic field \mathbf{H} . We choose the direction of the field as the Z axis. We rewrite the equation of motion

$$\epsilon \omega \begin{bmatrix} -\sin(\omega t) \\ \cos(\omega t) \\ 0 \end{bmatrix} = \frac{d\vec{p}}{dt} = q\vec{v} \times \vec{B} \approx q \begin{bmatrix} \cos(\omega t) \\ \sin(\omega t) \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ B_0 \end{bmatrix} = qB_0 \begin{bmatrix} \sin(\omega t) \\ -\cos(\omega t) \\ 0 \end{bmatrix}$$

- Circular orbit with radius (no drift): $\rho = 1/\omega$
- Angular frequency: $\omega = eB/\epsilon$

Source: physics.stackexchange.com

Here: no drift!



Radiation by an accelerated charge: Angular and spectral distribution

$$\begin{aligned} \nabla \cdot \mathbf{D} &= \rho & \nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 & \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} &= 0 \end{aligned}$$

Maxwell's equations: a charge/current is a source for an electromagnetic field

$$\mathbf{B} = [\mathbf{n} \times \mathbf{E}]_{\text{ret}}$$

$$\mathbf{E}(\mathbf{x}, t) = e \left[\frac{\mathbf{n} - \boldsymbol{\beta}}{\gamma^2 (1 - \boldsymbol{\beta} \cdot \mathbf{n})^3 R^2} \right]_{\text{ret}} + \frac{e}{c} \left[\frac{\mathbf{n} \times \{(\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\}}{(1 - \boldsymbol{\beta} \cdot \mathbf{n})^3 R} \right]_{\text{ret}}$$

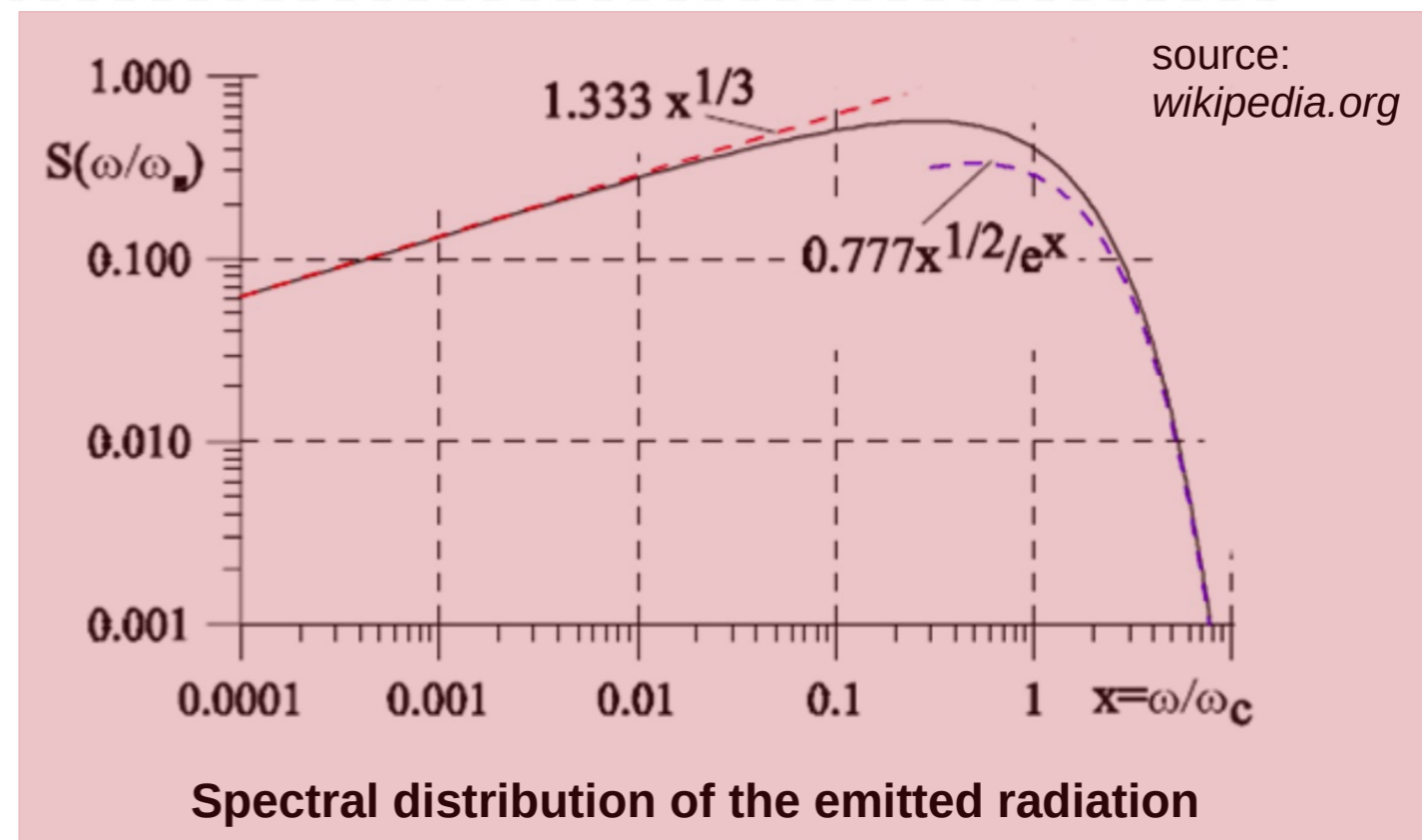
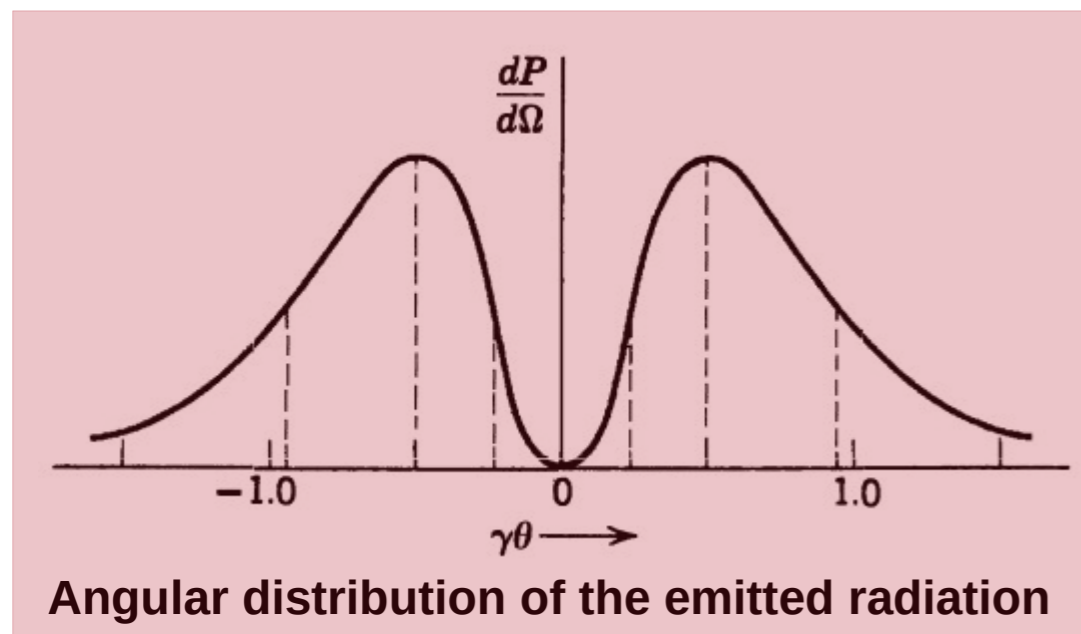
Fields of a moving charge

$$\mathbf{S} = \frac{c}{4\pi} \mathbf{E} \times \mathbf{B}$$

$$P = \frac{2}{3} \frac{e^2}{c^3} |\dot{\mathbf{v}}|^2$$

Pointing vector (energy flux)

Radiated total power



Relativistic kinematics: Lorentz transformations

Four-vector notation

$$\frac{dp^\mu}{d\tau} = \frac{q}{m} F^{\mu\nu} p_\nu$$

Lorentz force

$$x^\mu(t) = \begin{bmatrix} t \\ x(t) \\ y(t) \\ z(t) \end{bmatrix}$$

position
four-vector

$$p^\mu = \begin{bmatrix} \epsilon \\ p_x \\ p_y \\ p_z \end{bmatrix} = \gamma m \begin{bmatrix} 1 \\ v_x \\ v_y \\ v_z \end{bmatrix}$$

momentum
four-vector

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

field tensor

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

metric tensor

$$a^\mu = g^{\mu\nu} a_\nu$$

$$a_\mu = g_{\mu\nu} a^\nu$$

upper vs. lower indices

$$d\tau = \sqrt{dx^\mu dx_\mu} = dt/\gamma$$

proper time

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

gamma factor

Lorentz Boost

Fields enhancement by γ

$$\vec{E}' = \gamma(\vec{E} + \vec{v} \times \vec{B}) - \frac{\gamma^2}{(\gamma + 1)} \vec{v}(\vec{v} \cdot \vec{E})$$

$$\vec{B}' = \gamma(\vec{B} - \vec{v} \times \vec{E}) - \frac{\gamma^2}{(\gamma + 1)} \vec{v}(\vec{v} \cdot \vec{B})$$

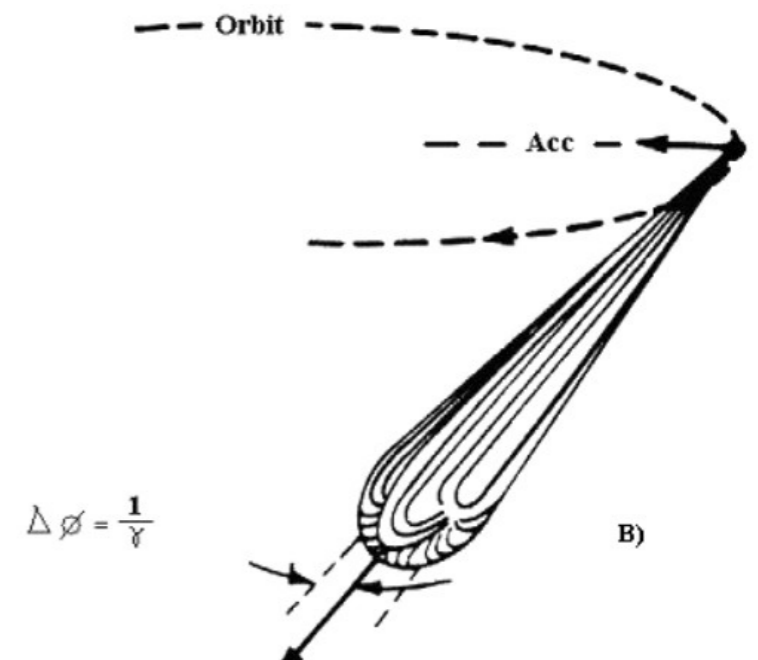
Longitudinal enhancement

$$\omega' = \gamma(\omega - vk_z)$$

$$k'_z = \gamma(k_z - v\omega)$$

$$k'_x = k_x, \quad k'_y = k_y$$

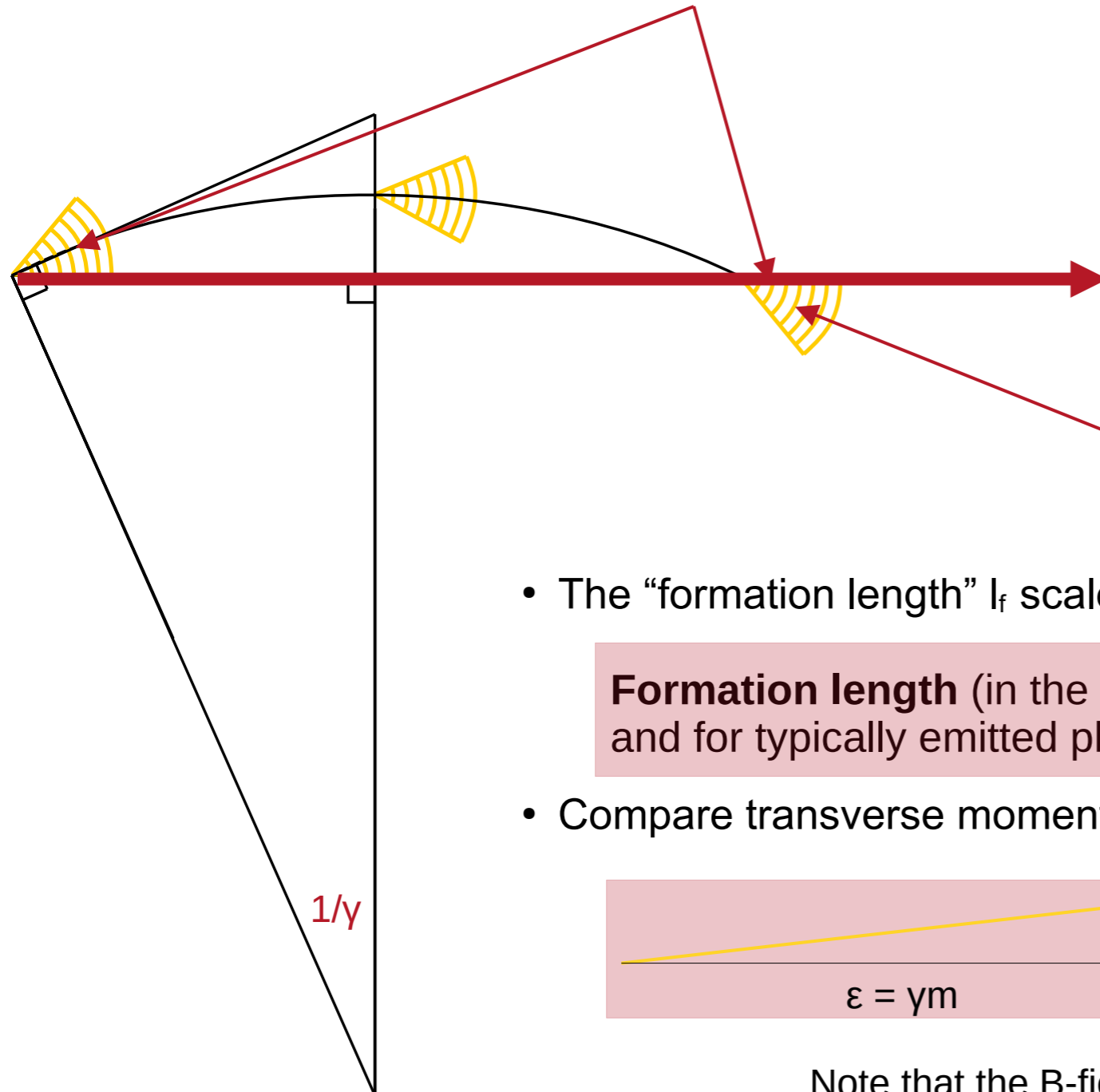
- Rest-frame fields are enhanced by γ
→ only way to reach extremely strong electromagnetic fields
- Angle $k_x/k_z, k_y/k_z \sim 1/\gamma$ (small-angle approximation)
→ radiation is emitted into a narrow cone into the forward direction



Synchrotron radiation: Formation length

Radiation can only reach the detector if it originates from a small arc segment

There are several terms out there with (slightly) different meaning depending on the context (formation length, coherence length, etc.)



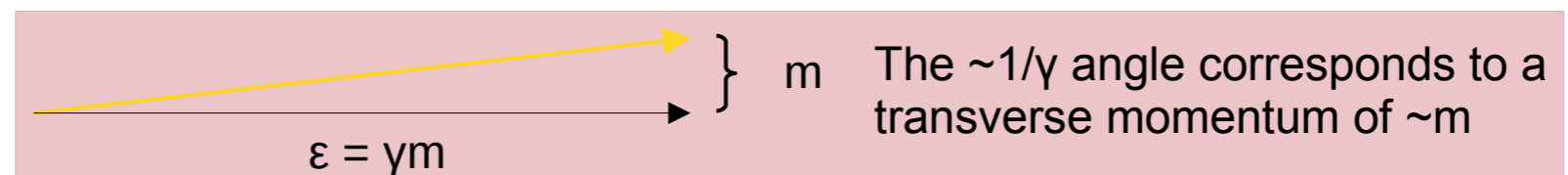
Light observed in this direction is predominantly emitted from the small arc shown

A relativistic particle emits (mostly) into a narrow cone ($1/\gamma$), due to the Lorentz boost

- The “formation length” l_f scales as $l_f \sim \rho/\gamma$ [radius: $\rho \sim \epsilon/(eB)$, particle energy: ϵ]

Formation length (in the classical regime and for typically emitted photon energies): $l_f \sim m / (eB)$

- Compare transverse momentum transfer ($\sim eB l_f$) with rest mass m



Note that the B-field is assumed to be orthogonal to the plane of motion

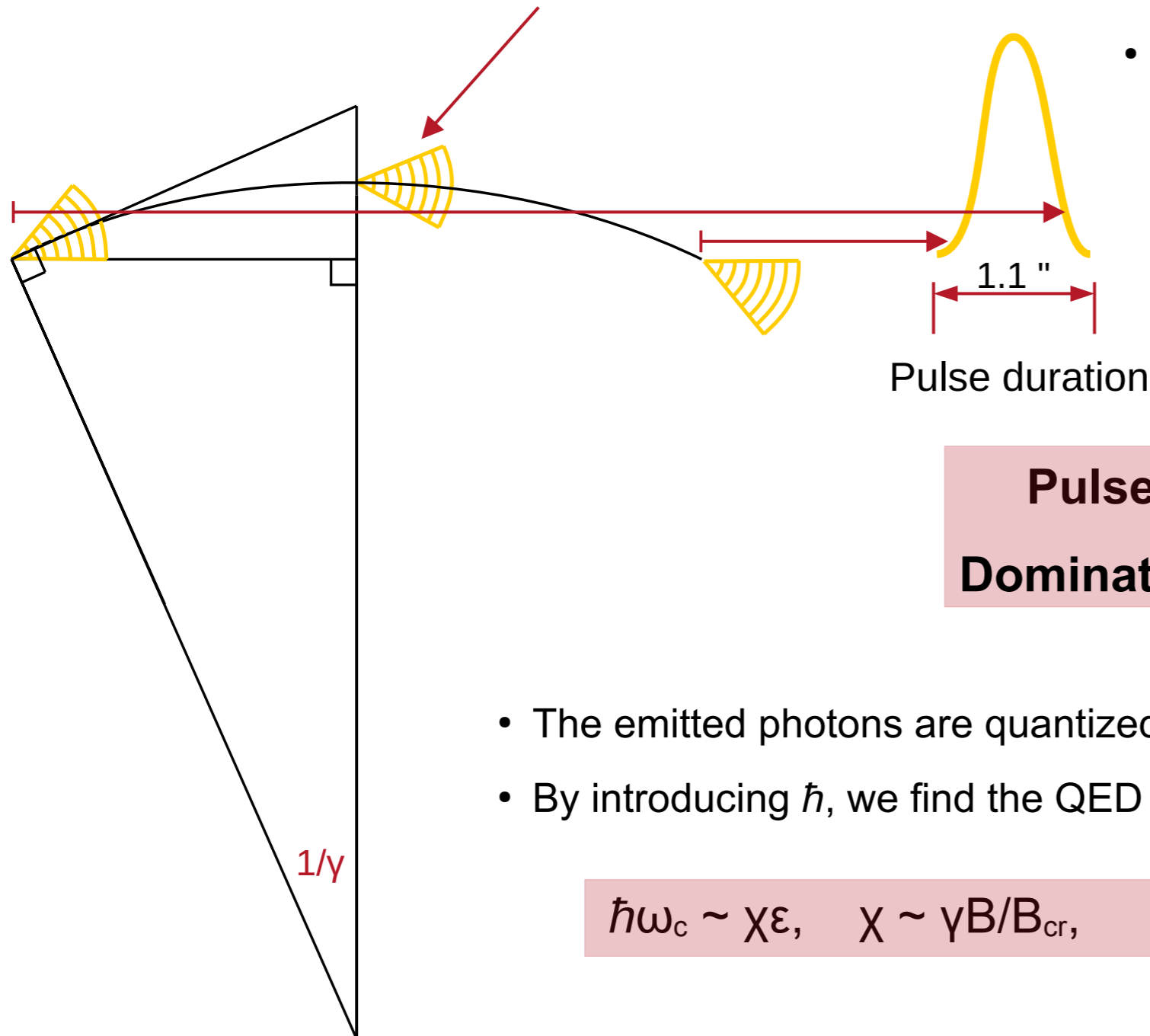
Blankenbecler & Drell, "Quantum treatment of beamstrahlung", PRD 36, 277, 1987

Jacob & Wu, "Quantum calculation of beamstrahlung" Nucl. Phys. B303, 373 (1988)

Synchrotron radiation:

Typically emitted photon energies (critical frequency)

The emitted light is faster (c vs. v) and can propagate straight (electron is bending)



- Path length difference: ρ/γ (true arc length) vs. $\rho \sin(1/\gamma) \approx \rho/\gamma - (\rho/6)(1/\gamma)^3$
- $1-v \approx (1-v^2)/2 \sim 1/\gamma^2$: both corrections result in the same scaling for the light-pulse duration

Pulse duration

Pulse duration: $T \sim \rho/\gamma^3 \sim m/(\gamma^2 eB)$

Dominating frequency: $\omega_c \sim 1/T \sim \gamma^2 eB/m$

[radius: $\rho \sim \epsilon/(eB)$, particle energy: ϵ]

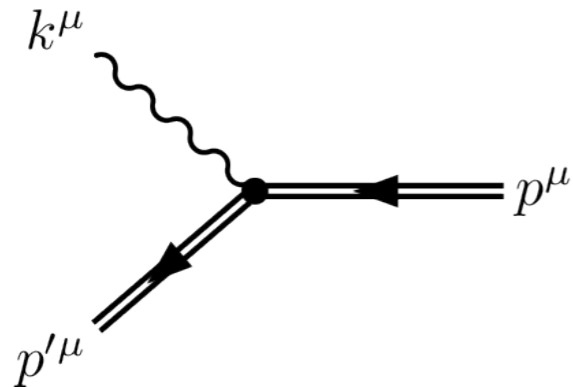
- The emitted photons are quantized, with energy $\hbar\omega$ ($\hbar=1$ most of the time)
- By introducing \hbar , we find the QED critical field / quantum parameter χ :

$$\hbar\omega_c \sim \chi\epsilon, \quad \chi \sim \gamma B/B_{cr}, \quad B_{cr} = m^2 c^2 / (\hbar e) \approx 4.4 \times 10^9 \text{ T}$$

Synchrotron radiation:

Typically emitted photon energies (critical frequency)

Quantum mechanical derivation (will be revisited later):



Photon emission (photon four-momentum: k^μ) by an electron with initial (final) four-momentum p^μ (p'^μ)

Using energy/time uncertainty, we can estimate the (maximum) formation time. For now, we require that m is the dominant “transverse” scale, i.e., that the transverse momentum transfer $eB l_f \lesssim m$ ($eB l_f \gg m$ is discussed later)

$$p^\mu - p'^\mu - k^\mu = \left. \begin{array}{l} \epsilon - \epsilon' - \omega \\ p_x - p'_x - k_x \\ p_y - p'_y - k_y \\ p_z - p'_z - k_z \end{array} \right\} \text{Assume momentum conservation}$$

To estimate the formation time, we assume momentum conservation and calculate the energy mismatch:

$$\begin{aligned} \epsilon &= \sqrt{m^2 + \vec{p}^2} \approx p + m^2/(2p) \\ \epsilon' &= \sqrt{m^2 + \vec{p}'^2} \approx p' + m^2/(2p') \end{aligned}$$

For ultra-relativistic particles and “weak” fields:
rest-mass dominant correction

$$\delta T \sim \frac{1}{\delta\epsilon}, \quad \delta\epsilon = \epsilon - \epsilon' - \omega \approx \frac{m^2}{2p} - \frac{m^2}{2p'} \approx \frac{\omega m^2}{2\epsilon\epsilon'}$$

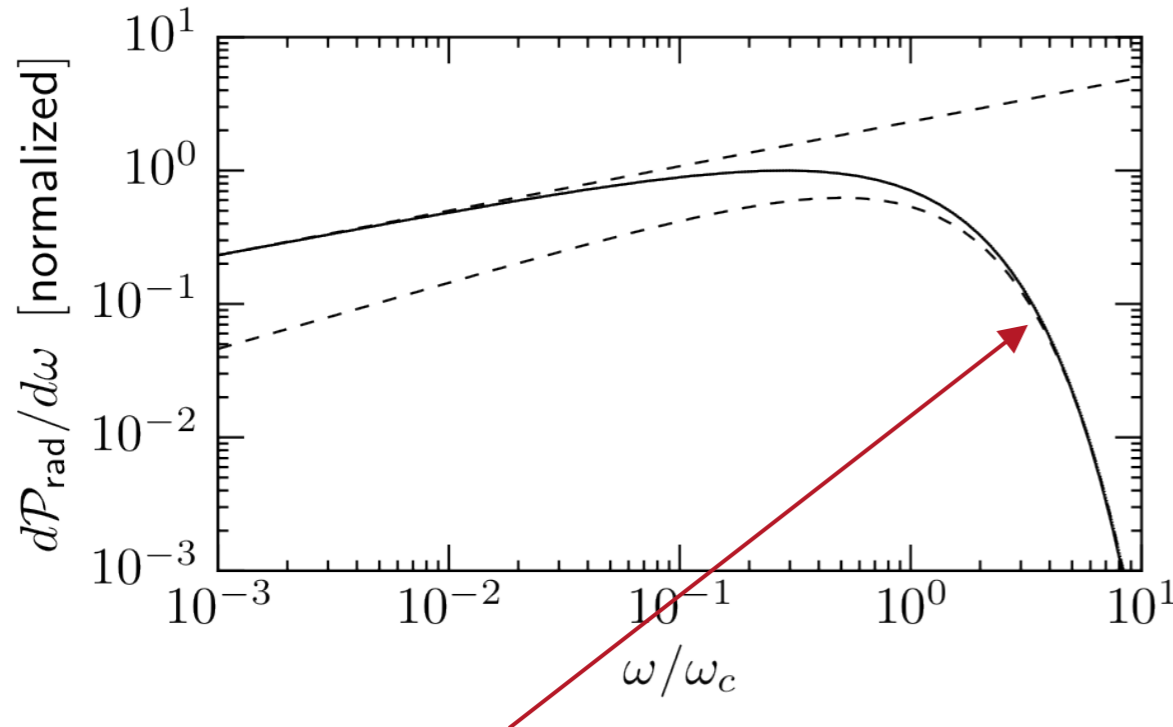
Energy uncertainty: $l_f \sim 2\epsilon\epsilon'/(\omega m^2)$ Transverse dynamics: $l_f \sim m/(eB)$

Combining both ($\epsilon' \approx \epsilon$): $\hbar\omega_c \sim \chi\epsilon$, $\chi \sim \gamma B/B_{cr}$, $B_{cr} = m^2 c^2/(\hbar e) \approx 4.4 \times 10^9 \text{ T}$

QED critical frequency:

Qualitative changes in the synchrotron spectrum

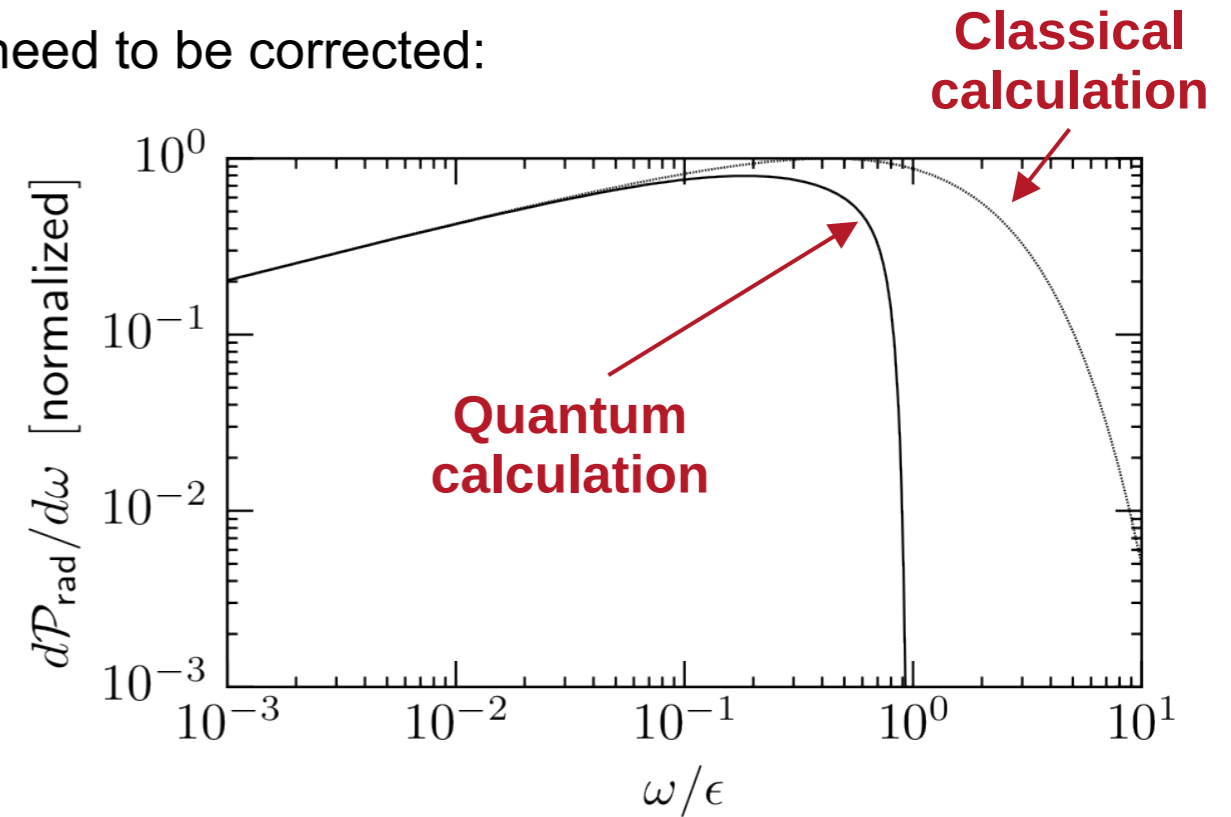
In the QED critical regime ($\chi \gtrsim 1$) classical predictions need to be corrected:



Classical prediction: exponential decay after the critical frequency

$$\exp(-\omega/\omega_c) \sim \exp[-\omega m/(\gamma^2 e B)]$$

“Tunneling exponent”, non-perturbative in the elementary charge e



For $\chi \gtrsim 0.1$ a substantial part of the spectrum is “unphysical”: emitted photon energy exceeds the energy of the radiating particle

$$\hbar\omega_c \sim \chi\epsilon, \quad \chi \sim \gamma B/B_{cr}, \quad B_{cr} = m^2 c^2 / (\hbar e) \approx 4.4 \times 10^9 \text{ T}$$

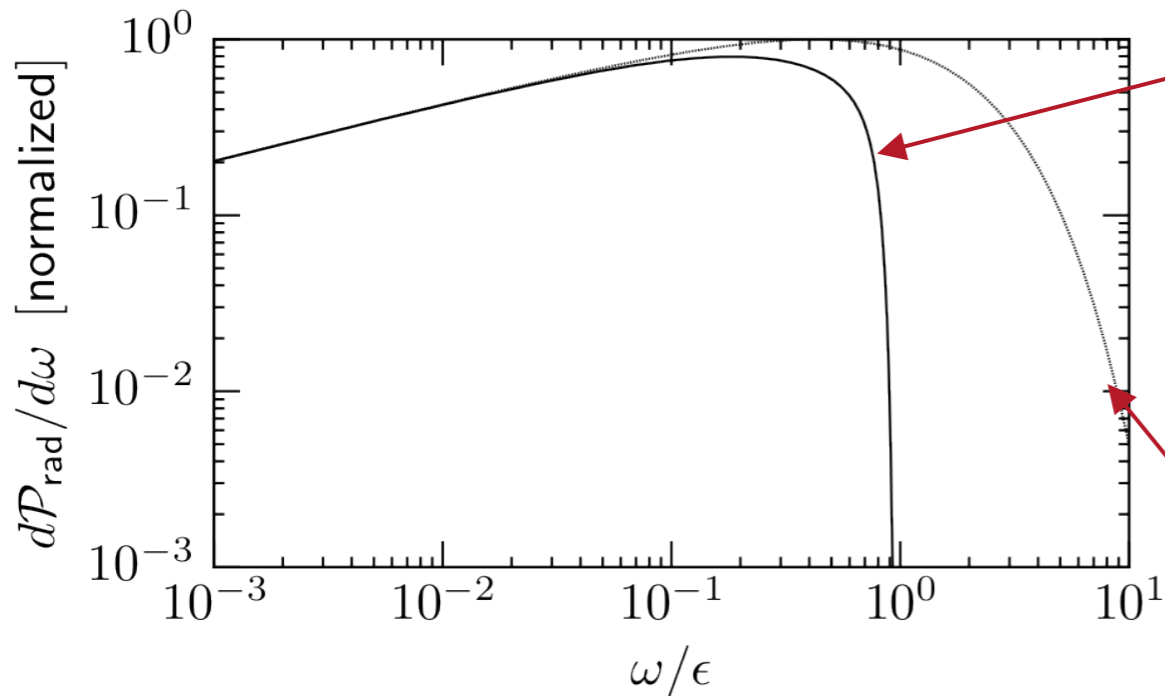
- Due to energy conservation an electron cannot emit a photon with energy higher than it's own (a magnetic field cannot transfer energy)

• Thus, quantum corrections become important around $\hbar\omega \gtrsim \epsilon$, which are relevant if $\chi \gtrsim 0.1$

• The QED critical regime is characterized by $\chi \gtrsim 1$, i.e., that in the boosted frame $B' \sim \gamma B \gtrsim B_{cr}$

Total emitted power:

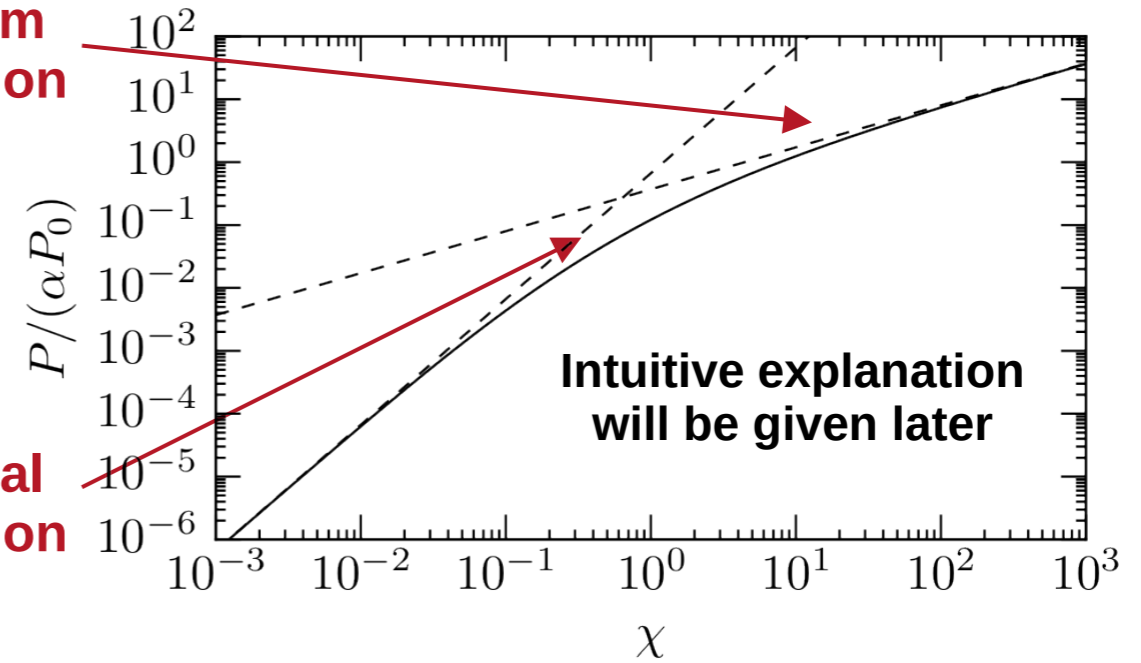
Quantum parameter determines particle acceleration



Quantum calculation

Classical calculation

Quantum corrections reduce the expected probabilities at high energies



Intuitive explanation will be given later

Total emitted power is reduced if χ approaches and exceeds unity

$$P = \frac{2}{3} \frac{\alpha \lambda_C m c^2 |a^2|}{c^3} = \frac{2}{3} \alpha \chi^2 \frac{m c^2}{\tau_C}, \quad a^2 = \frac{1}{m^2} \frac{dp_\mu}{d\tau} \frac{dp^\mu}{d\tau}$$

Larmor formula, total radiated power

$$|a^2| = \frac{e^2}{m^4} p_\mu F^{\mu\nu} F_{\nu\rho} p^\rho = \frac{m^2 c^6}{\hbar^2} \chi^2$$

Proper acceleration vs. quantum parameter

$$\chi = \frac{1}{m E_{cr}} \sqrt{p_\mu F^{\mu\nu} F_{\nu\rho} p^\rho} = \frac{\gamma}{E_{cr}} \sqrt{(\mathbf{E} + \mathbf{v} \times \mathbf{B})^2 - (\mathbf{E} \cdot \mathbf{v})^2 / c^2} = E^* / E_{cr}$$

Quantum parameter

$$\chi \sim \gamma B / B_{cr}$$

For purely magnetic field ($E=0$) which is orthogonal to the velocity and $v \approx c$

The quantum parameter measures the acceleration and thus determines the total radiated power

(Non-) linear Compton scattering

Laser fields (E-320): Achievable intensities & field strengths

See talk by Elias Gerstmayr
(after this one)



$$I(r, z, t) = I(r, z) \exp \left[-4 \ln(2) \frac{(z - ct)^2}{c^2 \tau_0^2} \right]$$

$$I(r, z) = I_0 \left[\frac{w_0}{w(z)} \right]^2 \exp \left[-\frac{2r^2}{w^2(z)} \right]$$

Gaussian intensity distribution in time and space

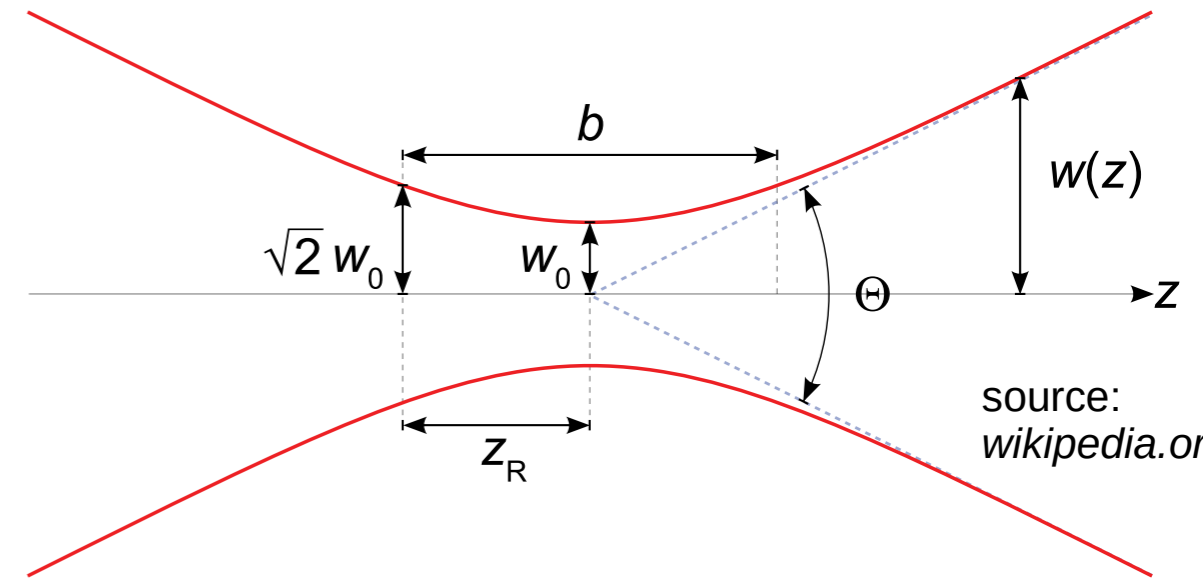
$$I_0 = \frac{n \mathcal{E}_L}{(\tau_0 \pi w_0^2)}, \quad n = 4 \sqrt{\frac{\ln 2}{\pi}} \approx \frac{3\pi}{5} \approx 1.88.$$

Peak intensity I_0 vs. total energy \mathcal{E}_L , FWHM pulse duration τ_0 , and focal waist w_0

E-320 parameters (example):

- OAP: focal length $\approx 3'' \approx 76$ mm
Laser diameter: 40mm (flat top profile); $\lambda_L \approx 0.8 \mu\text{m}$
- $w(z \approx 3'') \approx z \lambda_L / (\pi w_0) \approx 40 \text{mm} / \pi$ [this condition ensures that most of the beam is transmitted]
- $w_0 \approx 76 \text{mm} \cdot 0.8 / (40 \text{mm}) \approx 2 \lambda_L \approx 1.6 \mu\text{m}$
- Effectively (including Strehl): $\mathcal{E}_L \approx 0.4 \text{J}$, $\tau_0 \approx 40 \text{fs}$

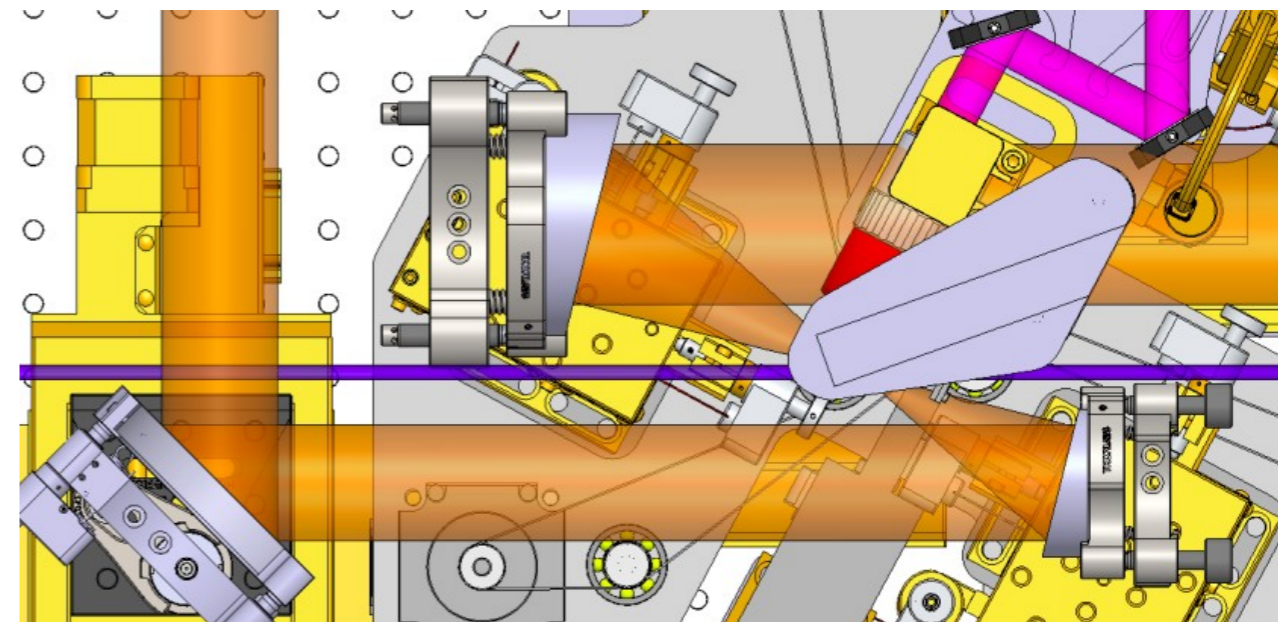
$$I_0 \approx 1.88 \times 0.4 \text{J} / (45 \text{fs} \pi (1.8 \mu\text{m})^2) \approx 1.6 \times 10^{20} \text{ W/cm}^2$$



source:
wikipedia.org

$$w(z) = w_0 \sqrt{1 + (z/z_R)^2}, \quad z_R = \pi w_0^2 / \lambda_L$$

Gaussian focusing, z_R : Rayleigh length



source: E-320 IP design

Electron beam (E-320): Achievable quantum parameter

See talk by Elias Gerstmayr
(after this one)



Main objectives for E-320

- Highest possible energy: 13 GeV (~0.1% rms deviation)
- Low backgrounds, clean beam → small divergence, large spot size

E-320 beam parameters

Energy (E) [GeV]	13.0
dE/E [%]	$\lesssim 0.1$
Charge [nC]	2.0
σ_x [μm]	24.4
σ_y [μm]	29.6
L [μm]	250
$\gamma\epsilon_x$ [$\mu\text{m} \cdot \text{rad}$]	3.7
$\gamma\epsilon_y$ [$\mu\text{m} \cdot \text{rad}$]	4.0
$\sigma_{x'}^* = \epsilon_x/\sigma_x$ [μrad]	6.1
$\sigma_{y'}^* = \epsilon_y/\sigma_y$ [μrad]	5.4

$$\chi \approx \frac{E^*}{E_{cr}} \approx 0.6 \frac{\mathcal{E}}{10 \text{ GeV}} \sqrt{\frac{2I}{10^{20} \text{ W/cm}^2}}$$

Quantum parameter for head-on laser-electron collisions

Aim of E-320: exceed the “threshold” $\chi = 1$
for the first time in laser-electron collisions

Electron-Laser collisions:

Electron dynamics in plane-wave laser fields

- A plane-wave laser field depends (non-trivially) only on one scalar coordinate, the laser phase $\phi = k_\mu x^\mu$

$$\frac{dp^\mu}{d\tau} = \frac{q}{m} F^{\mu\nu} p_\nu$$

Lorentz force

$$F^{\mu\nu} = F^{\mu\nu}(\phi), \quad \phi = kx$$

Plane-wave laser field tensor

$$k_\mu F^{\mu\nu} = 0 \quad d(\tilde{k}p)/d\tau = 0$$

Conservation of kp

- As kp is conserved, $d\phi/d\tau$ is constant, i.e. the mapping $\phi \leftrightarrow \tau$ is unique and we can parametrize the electron/positron trajectories using the laser phase:

$$\phi(\tau) = k_\mu x^\mu(\tau) = t(\tau)\omega_L - \vec{k}\vec{x}(\tau)$$

$$\frac{d\phi}{d\tau} = \frac{d(kx)}{d\tau} = \frac{kp_0}{m}, \quad \frac{d}{d\tau} = \frac{kp_0}{m} \frac{d}{d\phi}$$

$$\frac{dp^\mu(\phi)}{d\phi} = \frac{q}{kp_0} F^{\mu\nu}(\phi) p_\nu(\phi)$$

Lorentz force

$$\mathfrak{F}^{\mu\nu}(\phi, \phi_0) = \int_{\phi_0}^{\phi} d\phi' F^{\mu\nu}(\phi'), \quad \frac{\partial \mathfrak{F}^{\mu\nu}(\phi, \phi_0)}{\partial \phi} = F^{\mu\nu}(\phi)$$

Integrated field tensor

- Momentum of a particle with mass m and charge q inside a plane-wave laser field:

$$p^\mu(\phi) = p_0^\mu + \frac{q\mathfrak{F}^{\mu\nu}(\phi, \phi_0)p_{0\nu}}{kp_0} + \frac{q^2\mathfrak{F}^{2\mu\nu}(\phi, \phi_0)p_{0\nu}}{2(kp_0)^2}$$

four-momentum

$$x^\mu(\phi) = x_0^\mu + \int_{\phi_0}^{\phi} d\phi' \frac{p^\mu(\phi')}{kp_0}$$

four-position

Electron-Laser collisions:

Electron dynamics in plane-wave laser fields

four-momentum

$$p^\mu(\phi) = p_0^\mu + \frac{q\mathfrak{F}^{\mu\nu}(\phi, \phi_0)p_{0\nu}}{kp_0} + \frac{q^2\mathfrak{F}^{2\mu\nu}(\phi, \phi_0)p_{0\nu}}{2(kp_0)^2}$$

Initial four-momentum

transverse momentum transferred by the field

longitudinal momentum transferred by the field

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, \quad \mathfrak{F}^{\mu\nu} \sim k^\mu A^\nu - k^\nu A^\mu$$

- Transverse momentum: $a_0 mc \sim eE\lambda$ ($\lambda \sim 1/\omega$)
- Longitudinal momentum: $(a_0)^2 mc / \gamma$

four-position

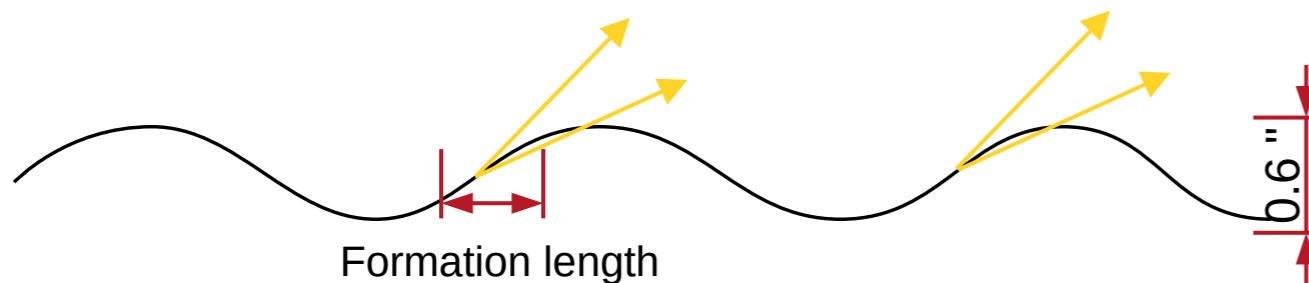
$$x^\mu(\phi) = x_0^\mu + \int_{\phi_0}^{\phi} d\phi' \frac{p^\mu(\phi')}{kp_0}$$

$$\vec{B} = \nabla \times \vec{A}, \quad \vec{E} = -\partial_t \vec{A}$$

$$\vec{E} \sim E \sin(\phi) \quad \vec{A} \sim E \cos(\phi)/\omega \sim E/\omega$$

$$a_0 = \frac{eE}{mc\omega} \sim \frac{e}{m} \sqrt{-A_\mu A^\mu}$$

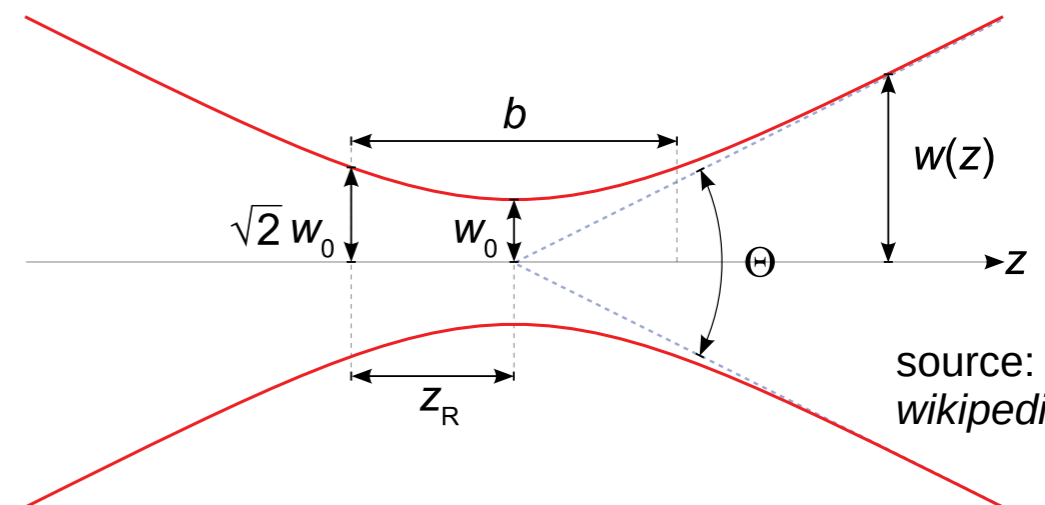
Classical intensity parameter



Important qualitative change between $a_0 \lesssim 1$ and $a_0 \gg 1$:

- Radiation is emitted into a small cone with angle $1/\gamma$
- Electron trajectory, transverse angle: a_0/γ
- $a_0 \lesssim 1$: electron “sees” an oscillatory field, radiation is coherent over many laser cycles
- $a_0 \gg 1$: only a small fraction of the trajectory is relevant for typically emitted photon frequencies

Transverse extend of the trajectory: $\lambda a_0/\gamma$



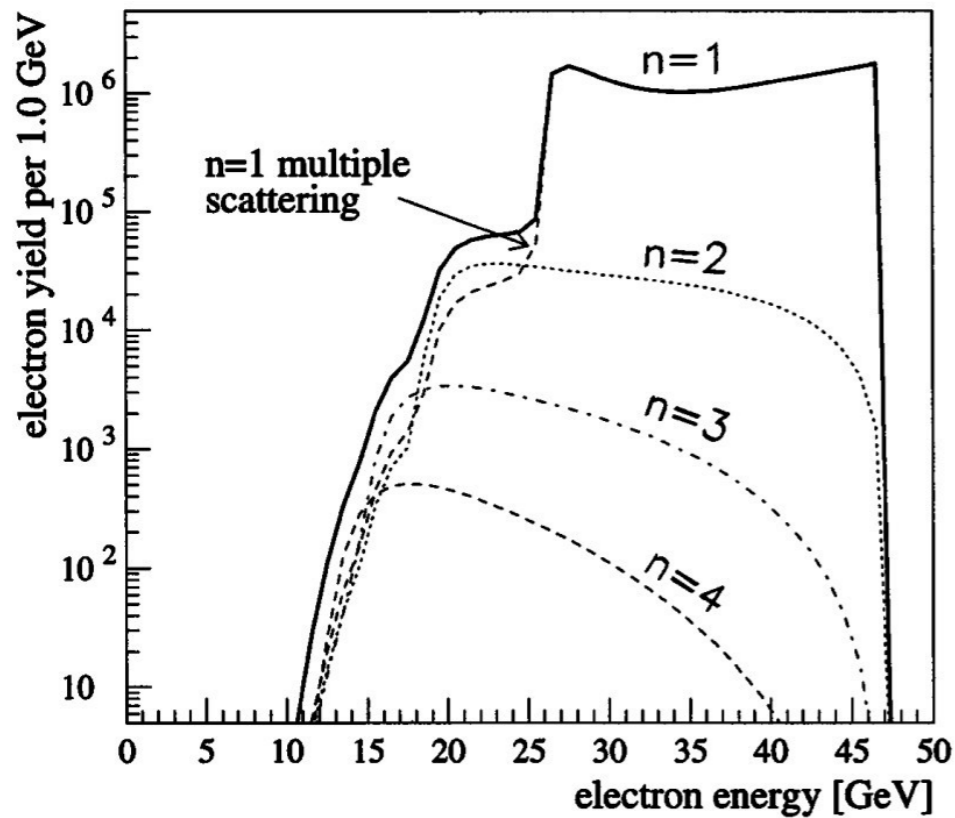
plane-wave approximation possible if $\gamma \gg a_0$

E-144 vs E-320:

From perturbative to non-perturbative laser-electron interactions

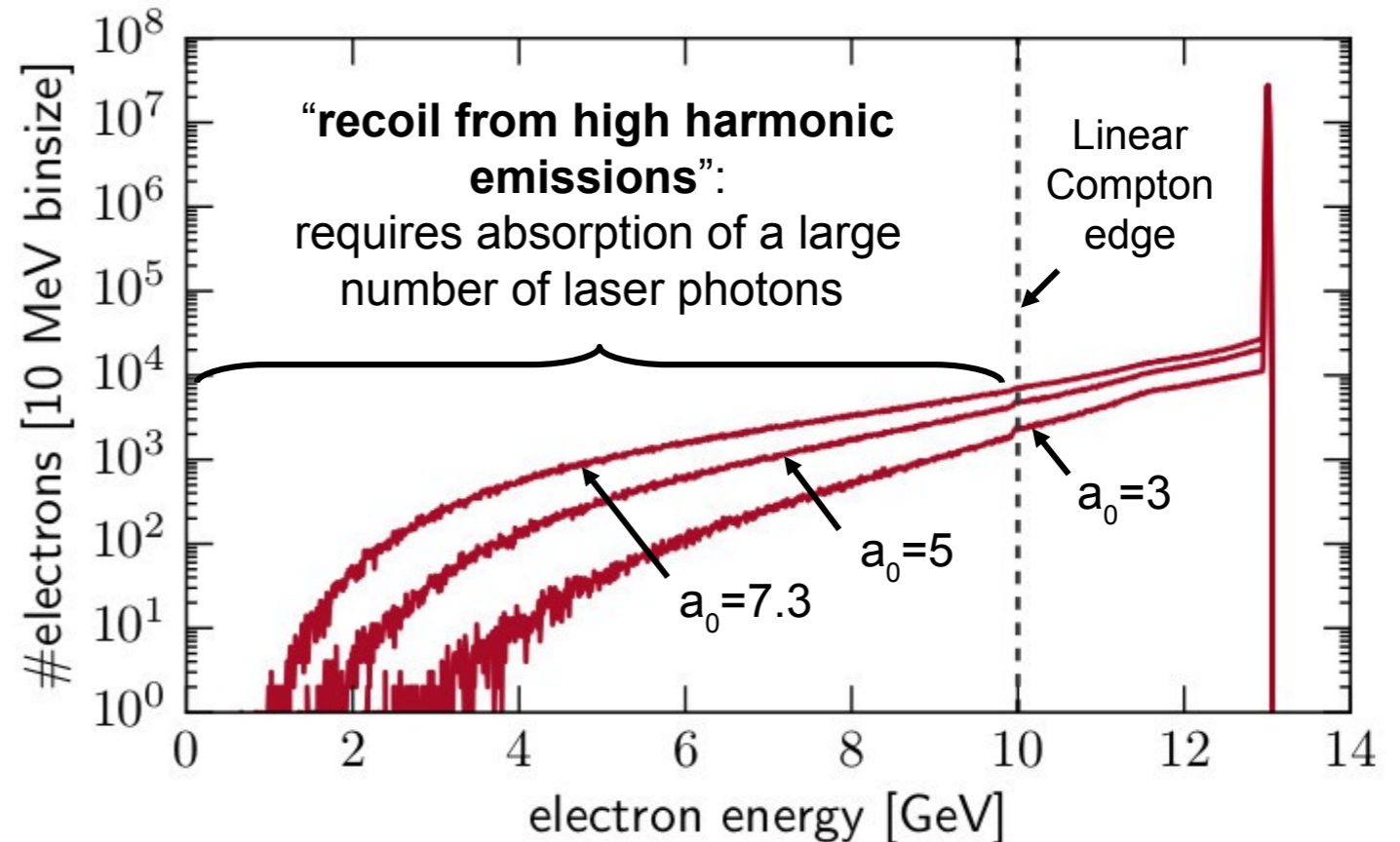


E-144: perturbative multi-photon regime ($a_0 \lesssim 1, \chi \lesssim 1$: 1990s)



E-144 PRL 76, 3116 (1996)

E-320: nonperturbative quantum regime ($a_0 \gg 1, \chi \gtrsim 1$: 2021/2022)



Interaction with $n \sim 100$ laser photons

- $a_0 \lesssim 1$: electron “sees” an oscillatory field, radiation is coherent over many laser cycles
- $a_0 \gg 1$: only a small fraction of the trajectory is relevant for typically emitted photon frequencies

$$a_0 = \frac{eE_0}{mc\omega} \approx 0.60 (\lambda_L [\mu\text{m}]) \sqrt{2I_0 [10^{18} \text{ W/cm}^2]}$$

E-320: $I_0 \sim 10^{20} \text{ W/cm}^2$ (existing laser), $\lambda_L \approx 0.8 \mu\text{m}$, $a_0 \sim 10$

Simulations: Nielsen / Tamburini / Vranic

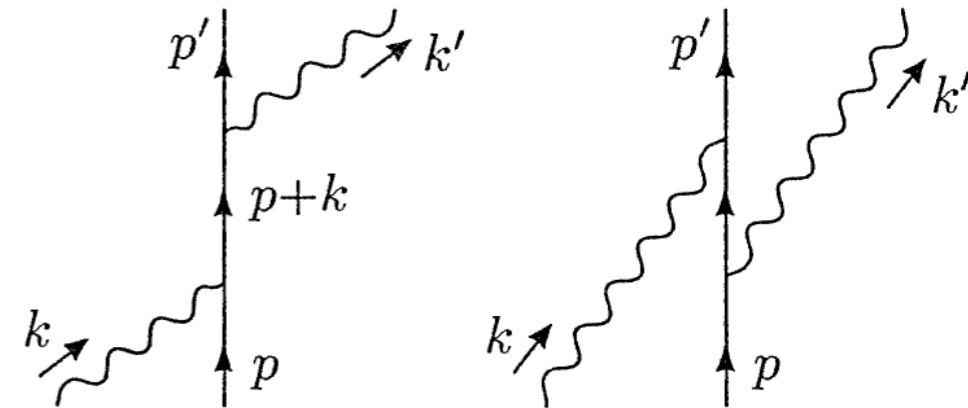
Linear Compton/Thomson scattering: Scattering probability per unit time

The leading terms in the expansion for $x \ll 1$ (the non-relativistic case) are

$$\sigma = \frac{8\pi r_e^2}{3} (1 - x). \quad (86.17)$$

The first term is the classical Thomson cross-section. In the opposite, ultra-relativistic, case ($x \gg 1$), the expansion of (86.16) gives

$$\sigma = 2\pi r_e^2 \frac{1}{x} (\log x + \frac{1}{2}). \quad (86.18)$$



$$x = 2kp/m^2$$

- Relativistic invariant for E-320: $x \approx 4 \times 1.55 \text{ eV} \times 13 \text{ GeV} / (0.511 \text{ MeV}) \approx 0.3$
- Probability (per unit time) that a single electron scatters from a laser is $dP/dt \sim \sigma N / (A \tau_0) \sim \sigma I_0 / (\hbar\omega)$

Number of laser photons $N \sim \mathcal{E}_L / \hbar\omega$ (total laser energy: \mathcal{E}_L , laser photon energy: $\hbar\omega$) per unit area (focal spot area: A) and per unit time (laser duration: τ_0) is proportional to the laser intensity $I_0 \sim \mathcal{E}_L / (\tau_0 A)$

$$\frac{dP}{dt} \sim \frac{\sigma I_0}{\hbar\omega} \sim \frac{\alpha a_0^2 c}{\lambda_L}$$

Probability per unit time to undergo linear Compton scattering (classical limit)

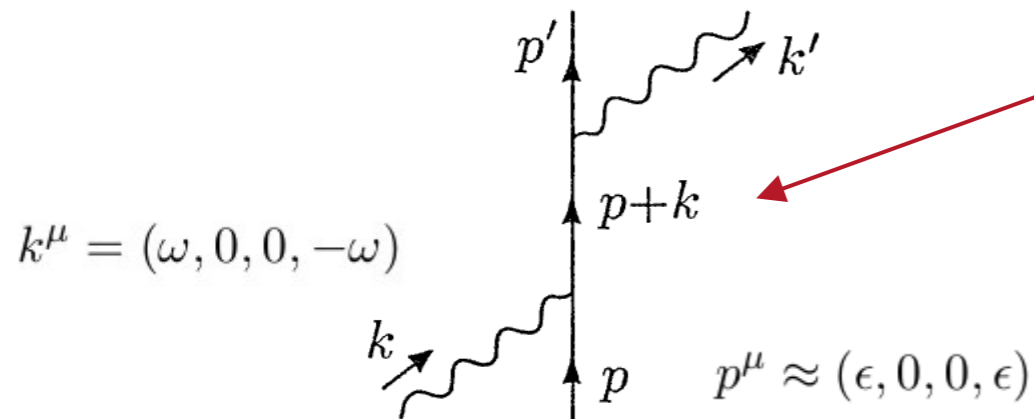
$$I_0 = \frac{\epsilon_0 c E^2}{2}, \quad a_0 = \frac{eE}{mc\omega}$$

$$r_e = \alpha \lambda_C, \quad \lambda_C = \frac{\hbar}{mc}$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \approx \frac{1}{137}$$

Nonlinear Compton scattering:

When does scattering with more than one photon become likely?



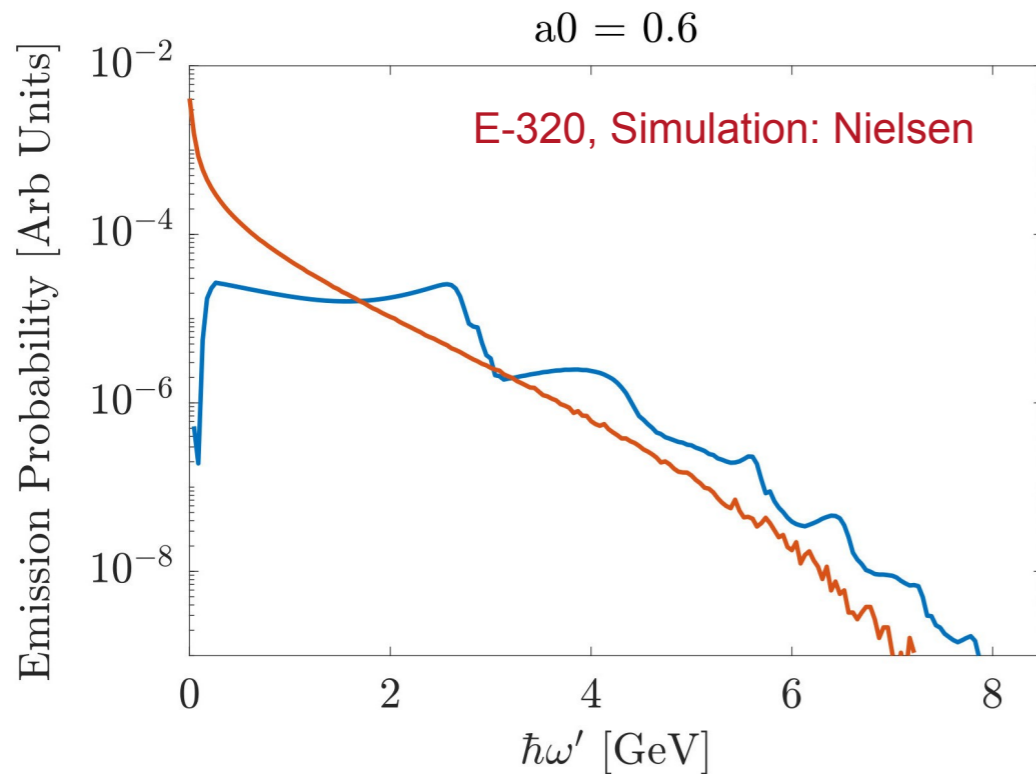
After the absorption the electron/positron is off-shell

$$\delta p_x \sim \delta p_y \sim m, \quad \delta p_z \sim \omega$$

Transverse momentum uncertainty is determined by the rest mass, longitudinal momentum uncertainty is determined by the absorbed laser photon energy

$$\delta V \sim \frac{\hbar}{mc} \times \frac{\hbar}{mc} \times \frac{c}{\omega}$$

Ultra-relativistic electron/positron absorbs a counter-propagating laser photon



Nonlinear effects become important if the photon density ρ is high enough such that a 2nd, 3rd, etc photon can be absorbed within the relevant volume δV

$$\frac{1}{\alpha} \sim \rho \delta V, \quad \rho \sim \frac{\mathcal{E}_L}{\hbar \omega A c \tau_0} \sim \frac{I_0}{\hbar \omega c}$$

The probability to interact with a photon is $\sim \alpha$ (once it is located within the relevant interaction volume)

$$a_0^2 = \left(\frac{eE}{mc\omega} \right)^2 \gtrsim 1$$

Condition to absorb more than one laser photon (non-linear threshold)

$$I_0 = \frac{\epsilon_0 c E^2}{2}, \quad a_0 = \frac{eE}{mc\omega} \quad \alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \approx \frac{1}{137}$$

Nonlinear Compton scattering:

Red shift of the kinematic edges (classical mass dressing)

The Green's function obtained in Appendix A shows that for large times the electron propagates in the beam with a mass $m^2 + \Delta m^2$, where Δm^2 is positive, and for a monochromatic beam is given by

$$\Delta m^2 = \frac{1}{2} e^2 (-\mathcal{Q} \cdot \mathcal{Q}^*) \quad (3.15)$$

As the electron propagates into the beam, its effective mass changes from m^2 to $m^2 + \Delta m^2$, and it is plausible that the only component of its momentum which can change during this process is that along the direction of k . Thus the effective momentum of the electron inside the beam is

$$\bar{p} = p + (\Delta m^2 / 2k \cdot p)k, \quad \bar{p}^2 = m^2 + \Delta m^2. \quad (3.16)$$

Since ζ may be written as

$$\zeta = (\Delta m^2 / 2k \cdot p') - (\Delta m^2 / 2k \cdot p),$$

the energy-momentum conservation equation may be written in the form

$$\bar{p}' + k' = \bar{p} + rk, \quad (3.17)$$

Brown & Kibble, Phys. Rev. 133 (1964)

E-320: $\epsilon \approx 10$ GeV, $\omega \approx 1.55$ eV, $r=1$ (first edge)

→ $\omega'_{\max} \approx 1.9$ GeV for $M_{\text{osc}} = 0$

→ $\omega'_{\max} \approx 1.1$ GeV for $M_{\text{osc}} = m$ [$a_0 \approx 1$]

$$p^\mu(\phi) - eA^\mu(\phi) = p_0^\mu - k^\mu \left[\frac{e p_0 A(\phi)}{k p_0} + \frac{e^2 A^2(\phi)}{2 k p_0} \right]$$

- The classical equations of motion keep the *instantaneous* four-momentum always on shell
- For $a_0 \lesssim 1$, however, the *average* four-momentum is the relevant quantity, which is off-shell

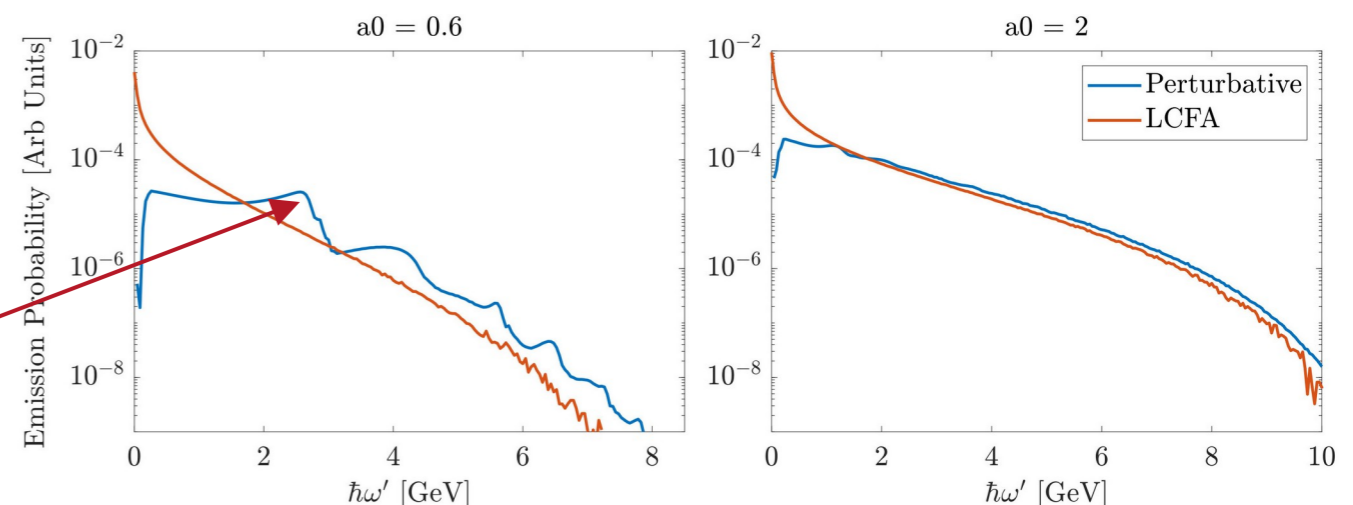
$$\langle p - eA \rangle^\mu = p_0^\mu + \frac{M_{\text{osc}}^2}{2k p_0} k^\mu, \quad M_{\text{osc}}^2 = e^2 \langle -A^2 \rangle$$

Yakimenko, Meuren, Del Gaudio, et al., PRL 122, 190404 (2019)

Kinematic limits:
head-on collision

$$\omega'_{\max} \approx \frac{4\epsilon^2 r \omega}{m^2 + M_{\text{osc}}^2 + 4r\omega\epsilon}$$

$$\omega' = \frac{(1 + \beta)\epsilon r \omega}{\epsilon[1 - \beta \cos(\theta)] + r\omega[1 + \cos(\theta)] + \frac{M_{\text{osc}}^2}{2(1 + \beta)\epsilon}[1 + \cos(\theta)]}$$

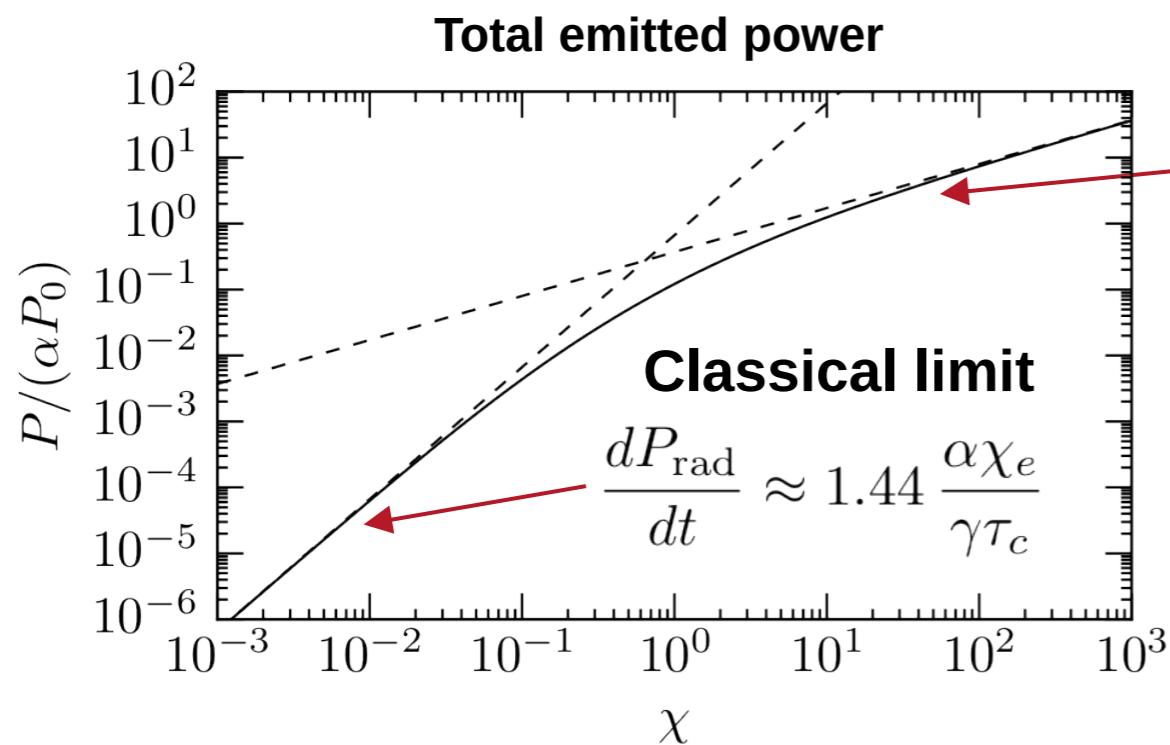


E-320, Simulation: Nielsen

**More details, deep quantum regime,
LCFA breakdown, recoil correlations,
Ritus-Narozhny conjecture, etc.**

Deep quantum regime:

Scaling of the radiation probability for $\chi \gg 1$



Radiation probability per unit time: $dP_{\text{rad}}/dt \sim \alpha/l_f$

Three important mass/energy scales:

- Electron/positron rest mass: m
- Lab-frame energy: ϵ
- Field-transferred energy: $M = eEl_f \sim \chi^{1/3}m$

As $\chi \sim m^{-3}$, $\chi^{1/3}m$ is independent of m !

Deep quantum limit

$$\frac{dP_{\text{rad}}}{dt} \approx 1.46 \frac{\alpha \chi_e^{2/3}}{\gamma \tau_c}$$

Particle propagating in z-direction

$$\epsilon = \sqrt{m^2 + (eEl_f)^2 + p_z^2} \approx p_z + \frac{m^2}{2p_z} + \frac{(eEl_f)^2}{2p_z}$$

$$\epsilon' = \sqrt{m^2 + (eEl_f)^2 + p_z'^2} \approx p_z' + \frac{m^2}{2p_z'} + \frac{(eEl_f)^2}{2p_z'}$$

So far we considered only $eEl_f \lesssim m$ (classical regime), now we consider $eEl_f \gg m$ (deep quantum regime)

$$\delta\epsilon = \epsilon - \epsilon' - \omega' \sim m^2 \frac{\omega'}{2\epsilon\epsilon'} + (eEl_f)^2 \frac{\omega'}{2\epsilon\epsilon'}$$

$$l_f \sim \frac{\epsilon}{m^2} \frac{\epsilon'}{\omega'} \sim \frac{\epsilon}{\chi m^2} \quad l_f \sim \frac{\epsilon}{\chi^{2/3} m^2} \left(\frac{\epsilon'}{\omega'} \right)^{1/3}$$

Classical formation length Quantum formation length

In the deep quantum regime the field-induced mass scale dominates over the rest-mass

Photon emission:

Formation length, local constant field approximation (LCFA)



General formation length (depends on the emitted photon energy)

$$l_f \sim \frac{\epsilon}{m^2} \frac{\epsilon'}{\omega'} \sim \frac{\epsilon}{\chi m^2} \quad l_f \sim \frac{\epsilon}{\chi^{2/3} m^2} \left(\frac{\epsilon'}{\omega'} \right)^{1/3} \sim \frac{\epsilon}{\chi^{2/3} m^2}$$

Classical formation length Quantum formation length

Formation length for typically emitted photon energies

Typical formation length ($a_0 \gtrsim 1$):

$$l_f \sim \lambda_L/a_0 \text{ (classical)} \quad l_f \sim \lambda_L \chi^{1/3}/a_0 \text{ (quantum)}$$

- For $a_0 \gg 1$: $l_f \ll \lambda_L$ (typical photon energies)
→ laser field can be considered constant during the emission (LCFA); important for numerical codes
- In general, however, the formation length depends on the energy of the emitted photon: diverges for $\omega' \rightarrow 0$
→ LCFA Breakdown for low photon energies

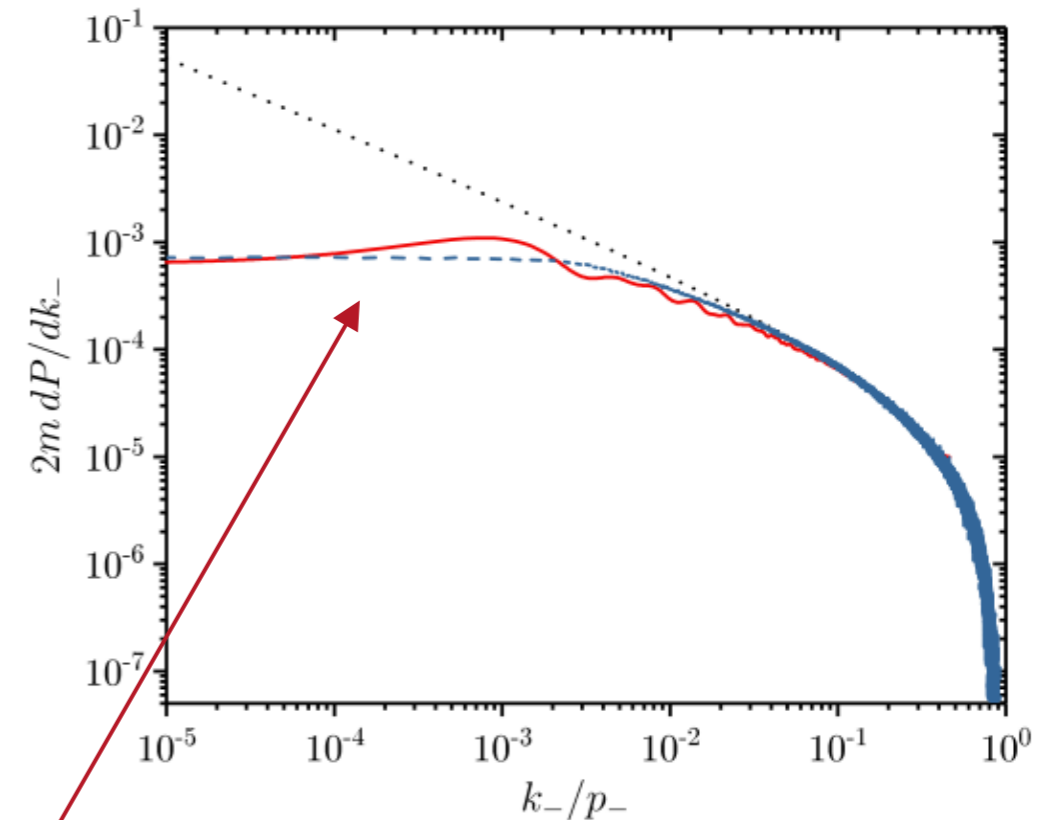


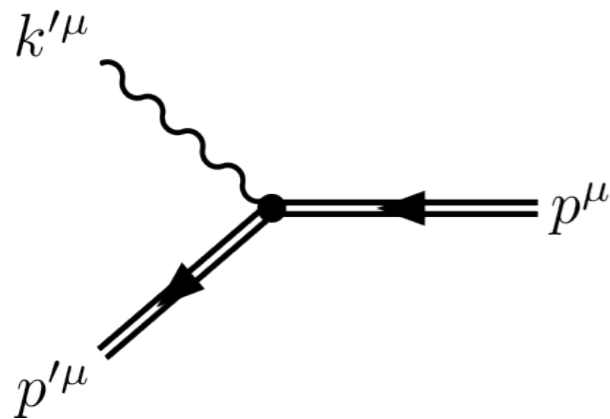
FIG. 3. Exact (solid red curve) vs local-constant-field approximated (dotted black curve) differential photon emission probability for an electron with initial energy of 10 GeV colliding head-on with a plane-wave pulse of 5 fs FWHM duration and 2.7×10^{20} W/cm² peak intensity. The dashed blue curve shows the same probability obtained via the numerical code presented in [54], with the improved emission model as described in the main text.

Photon emission:

The unitarity problem in strong-field QED

See talks by Tobias Podszus
(13:30, Thursday)

and by Greger Torgrimsson
(10:00, Friday)



Leading-order description of photon emission inside a background field

Classical limit

$$\frac{dP_{\text{rad}}}{dt} \approx 1.44 \frac{\alpha \chi_e}{\gamma \tau_c}$$

Deep quantum limit

$$\frac{dP_{\text{rad}}}{dt} \approx 1.46 \frac{\alpha \chi_e^{2/3}}{\gamma \tau_c}$$

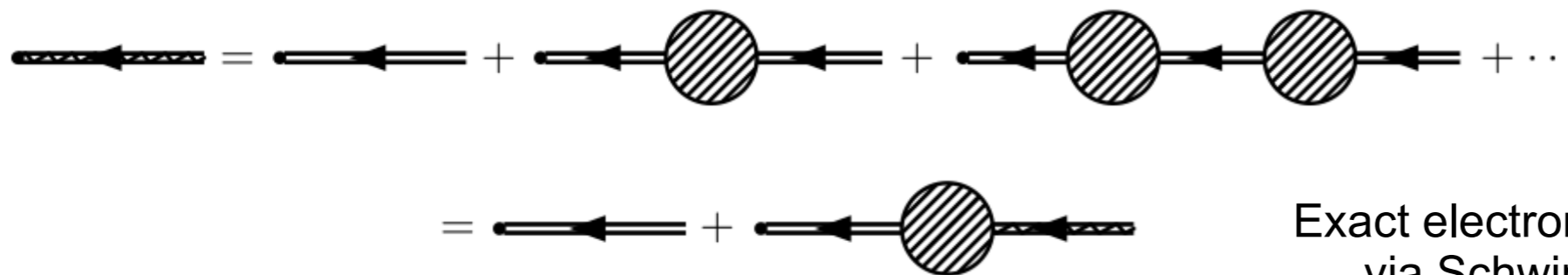
E-320 parameters: $\chi \gtrsim 1$, $\tau_0 \gtrsim 30$ fs,
 $\gamma \gtrsim 2 \times 10^4$, $\tau_c \approx 1.3 \times 10^{-21}$ s

$$\tau_0 \frac{dP_{\text{rad}}}{dt} \sim 8.5$$

The total radiation probability becomes (much) larger than unity

Radiative corrections are (normally) neglected, but (in general) they need to be considered for self-consistent analytical calculations

- Conventional interpretation: expectation value of the number of emitted photons
- However: this becomes problematic as soon as the recoil introduces non-trivial correlations
- Radiative corrections become important

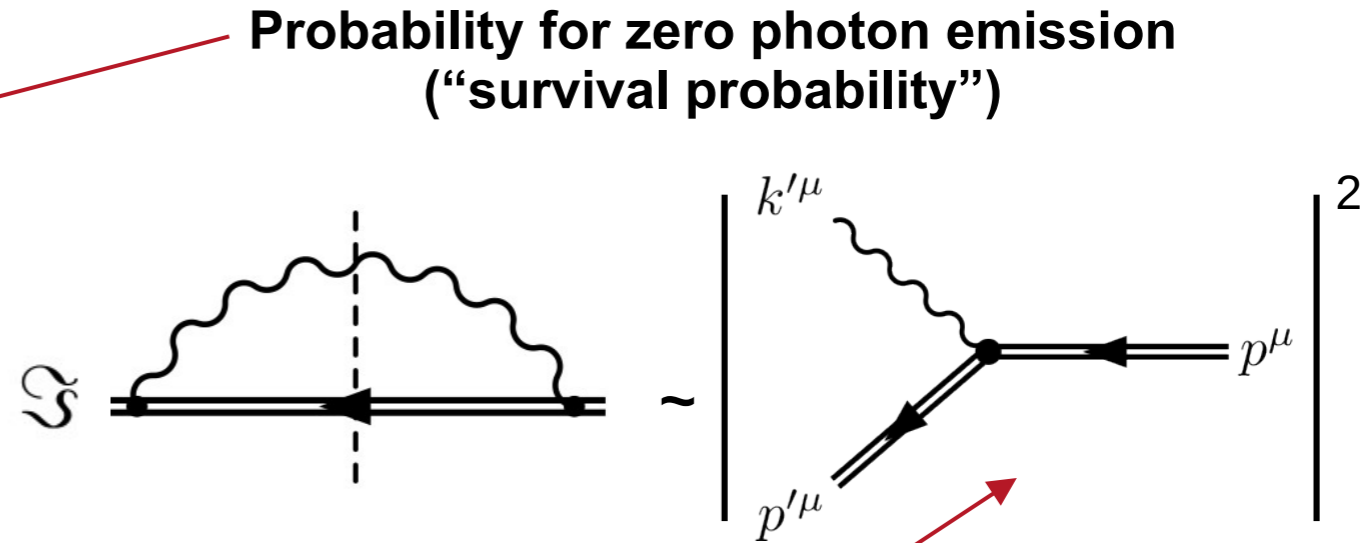
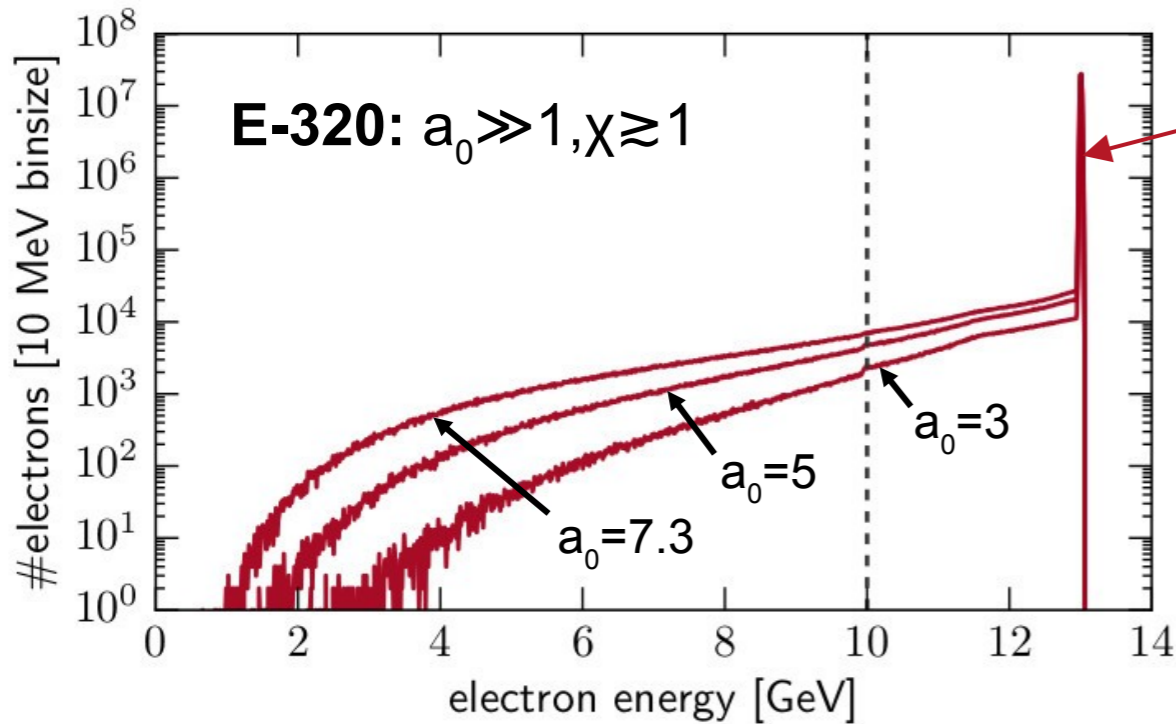


Exact electron wave function, defined via Schwinger-Dyson equation



Self-consistent calculations: Electron/positron radiative life time

See talks by Tobias Podszus
(13:30, Thursday)
and by Greger Torgrimsson
(10:00, Friday)



Optical theorem: total radiation probability determines the imaginary part of the mass operator

What is the probability that an electron/positron will not be emitting a photon?

$$S(t, \tau; \varepsilon)S(\tau, t'; \varepsilon) = S(t, t'; \varepsilon), \quad S(t, t; \varepsilon) = 1$$

Total radiation probability

$$\frac{d}{dt} S(t, t'; \varepsilon) = -\frac{dW}{dt}(\varepsilon, t) S(t, t'; \varepsilon)$$

$$S(t, t'; \varepsilon) = \exp \left[-\int_{t'}^t d\tau \frac{dW}{d\tau}(\varepsilon, \tau) \right]$$

Exponential decay factor

Derivation based on fundamental properties of probabilities
Tamburini & Meuren, arXiv 1912.07508 (2019)

Derivation based on solving the Schwinger-Dyson equation (exact electron wavefunction)

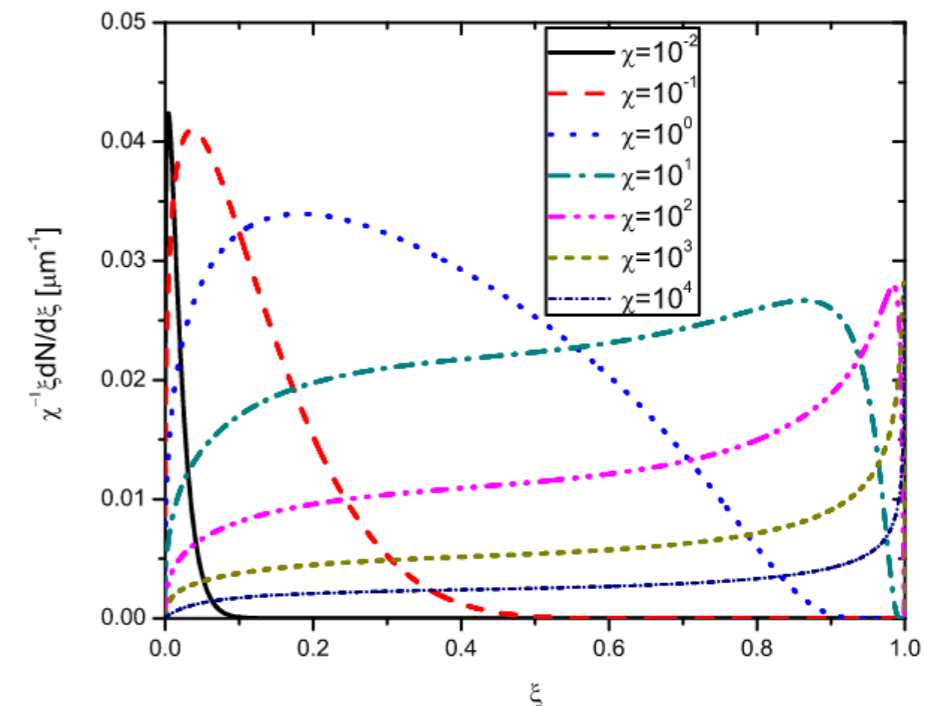
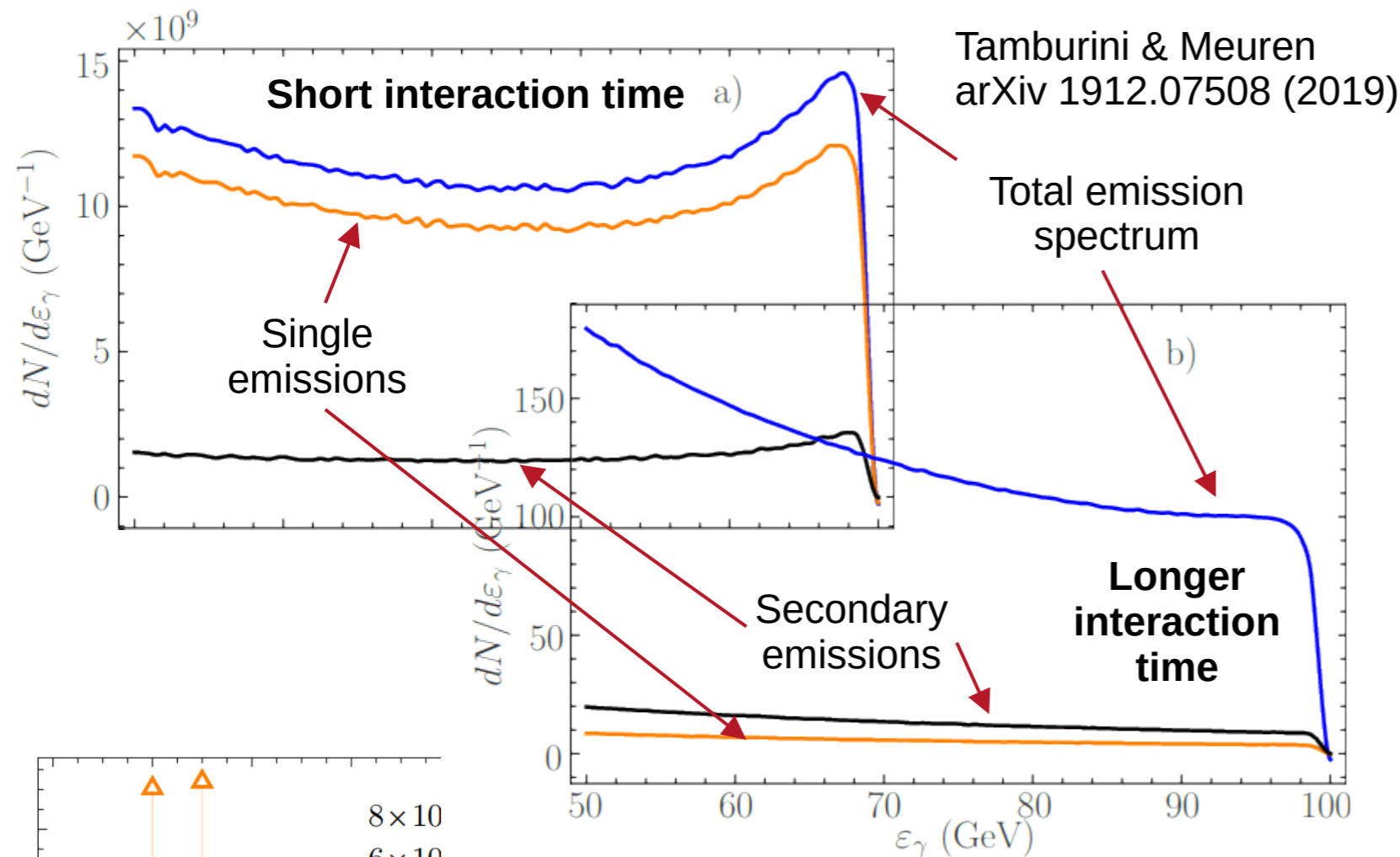
$$\Psi_{p,\sigma}^E(x) = \left[\mathbf{1} + \frac{e}{kp} \gamma f \gamma \psi(kx) - \frac{i}{4\xi} \frac{e}{kp} \gamma f \gamma \mu_B(\phi) \right] e^{iS_{p,\sigma}^E(x)} u_{p,\sigma}$$

Imaginary part of the mass operator induces a decay of the exact electron/positron wave function

Strong-field photon emission:

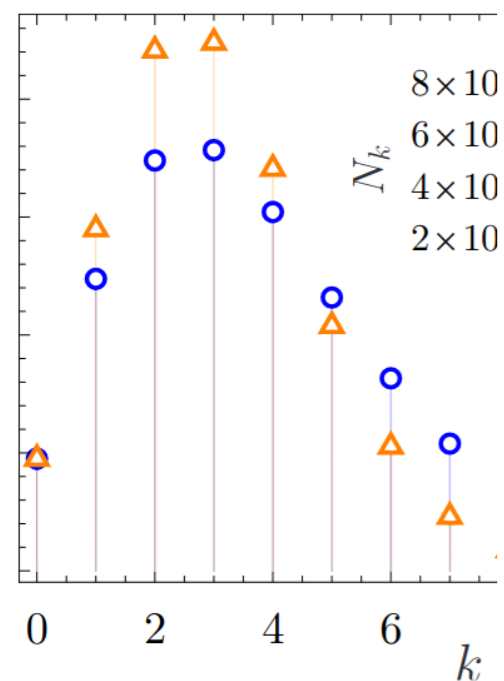
Spectrum for $\chi \gg 1$, recoil correlations, & non-Poissonian statistics

Peak at large photon energies is very hard to observe:

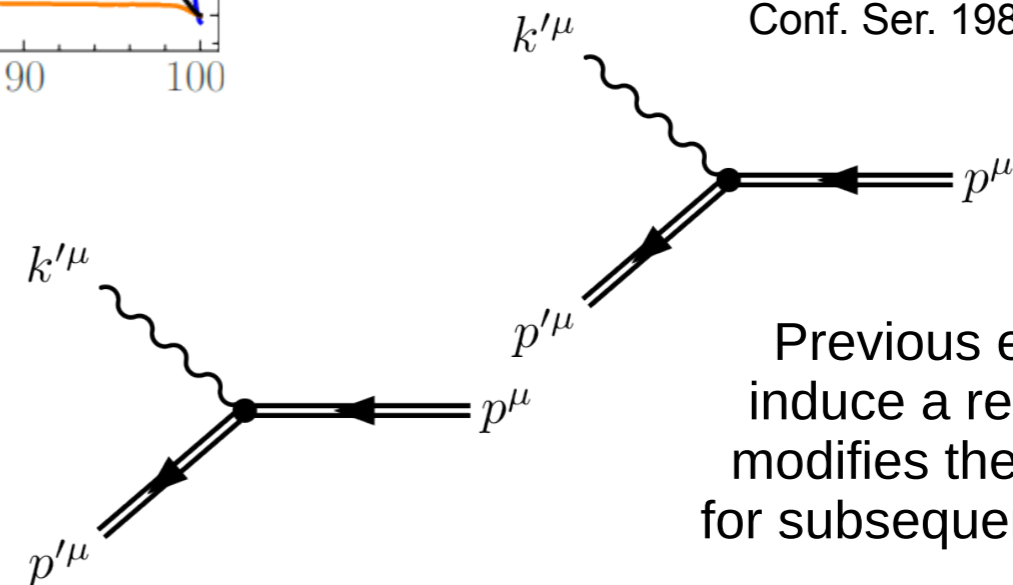


In the deep quantum regime ($\chi \gg 1$) the emitted photon spectrum/probability develops a peak close to the energy of the radiating lepton

Esberg & Uggerhoj J. Phys. Conf. Ser. 198, 012007 (2009)



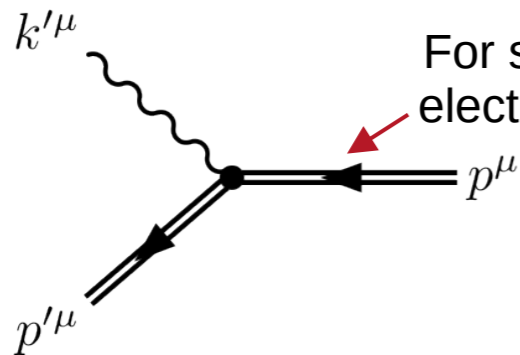
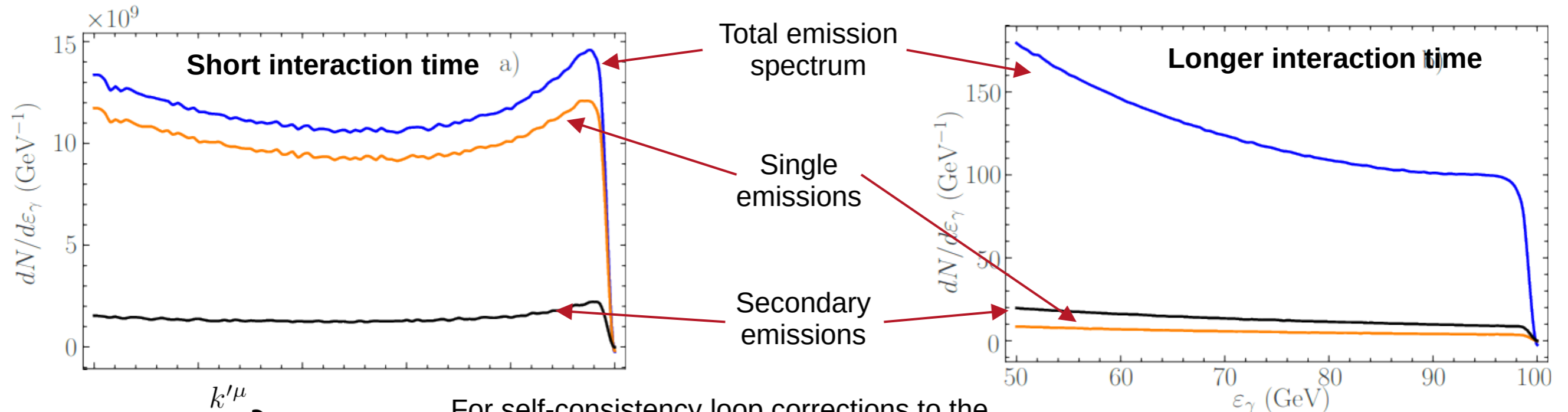
Recoil introduces non-trivial correlations between different emissions, which result in a deviation from Poisson statistics (Glauber result)



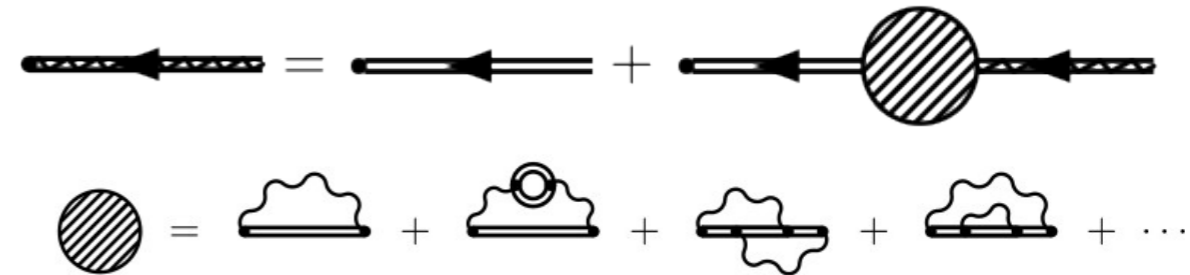
Previous emissions induce a recoil, which modifies the probability for subsequent emissions

Strong-field photon emission: Changes to the single-photon emission spectrum

Very drastic changes to the photon spectrum, in particular the single-emission spectrum:



For self-consistency loop corrections to the electron wave function need to be included



The emission of one (and only one) photon is *not* described by this diagram (in general)

$$\frac{dP_1}{d\varepsilon'}(\varepsilon', t) = \int_{-\infty}^t d\tau S(t, \tau; \varepsilon') \times \frac{d^2W}{d\tau d\varepsilon'}(\varepsilon', \varepsilon_i, \tau) S(\tau, -\infty; \varepsilon_i)$$

Due to the recoil, the decay exponent changes

To leading-order the loop expansion can be truncated at leading order, but this diagram has to be exponentiated, in general, as the propagation time can become “long”

Radiative corrections lead to radiation reaction effects in the single-photon emission spectrum (unlike some papers reporting that RR is only affecting two & more emissions)

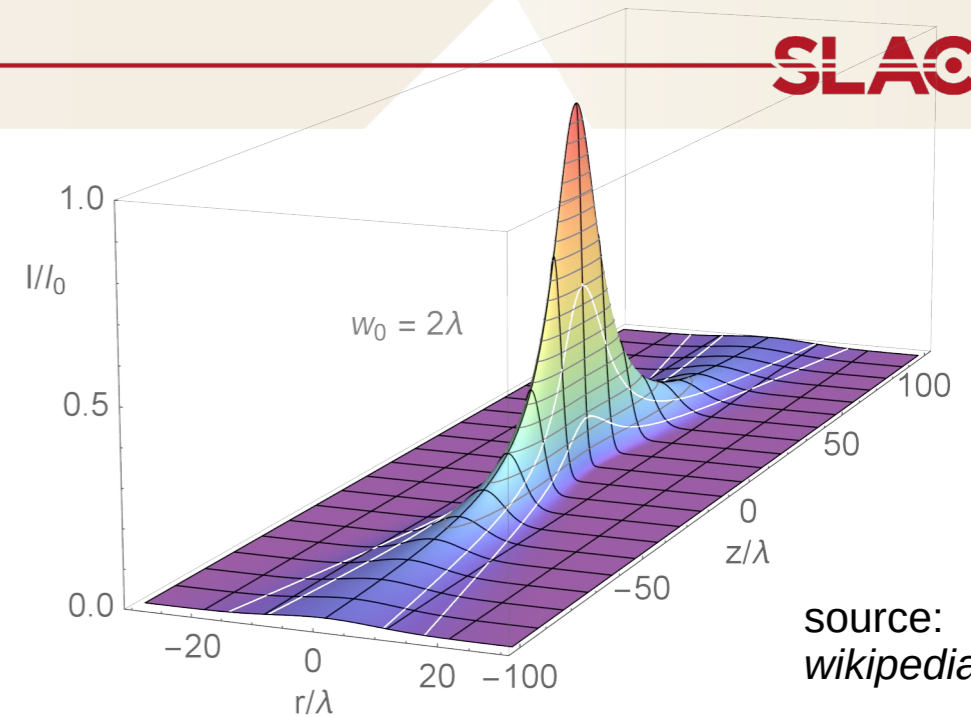
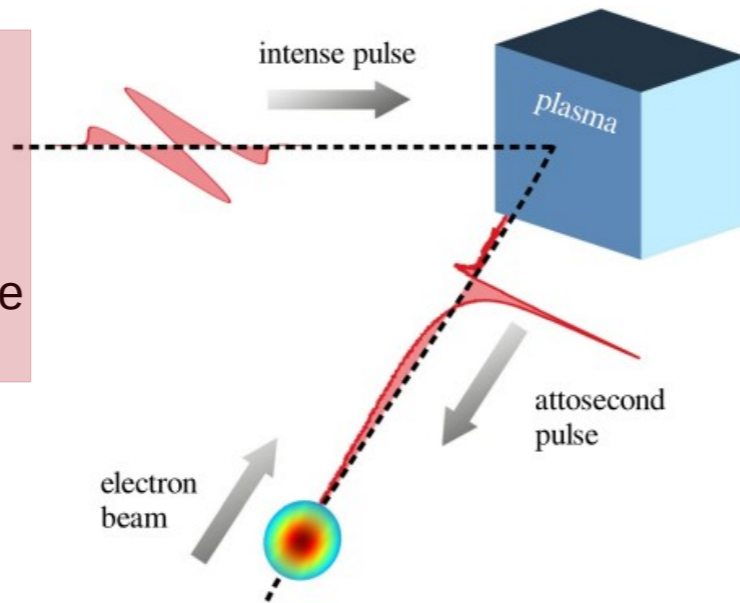
Single-photon spectrum: no emission in $[-\infty, \tau]$; emission at τ with recoil $\varepsilon_i \rightarrow \varepsilon'$; no further emission in $[\tau, \infty]$

Probing extremely strong fields: From beam-laser to beam-beam interactions

$$\frac{dP_{\text{rad}}}{dt} \approx 1.44 \frac{\alpha \chi_e}{\gamma \tau_c} \longleftrightarrow l_{\text{rad}} \approx \frac{\epsilon}{10 \text{ GeV}} \frac{0.7 \mu\text{m}}{\chi}$$

For typical beam energies and field strengths the radiation length becomes comparable to the laser wavelength

Unless the interaction time is short (compared to the radiation length), the gamma factor will be degraded by radiation reaction and extreme fields will not be attained.



source: wikipedia.org

- Spatial gradients are limited by the focusing power of the optical system (it is hard to reach $f_{\#} \lesssim 2$)
- Temporal gradients are limited by the bandwidth of the lasing medium, hard to get $\lesssim 10\text{fs}$ @ $0.8\mu\text{m}$

Loophole: frequency spectrum broadening via a highly nonlinear process [e.g., Baumann, Nerush, Pukhov, & Kostyukov; Sci. Rep. 9:9407 (2019)]

$$\chi \sim \alpha N \gamma \frac{\lambda_C^2}{(\sigma_x + \sigma_y) \sigma_z}$$

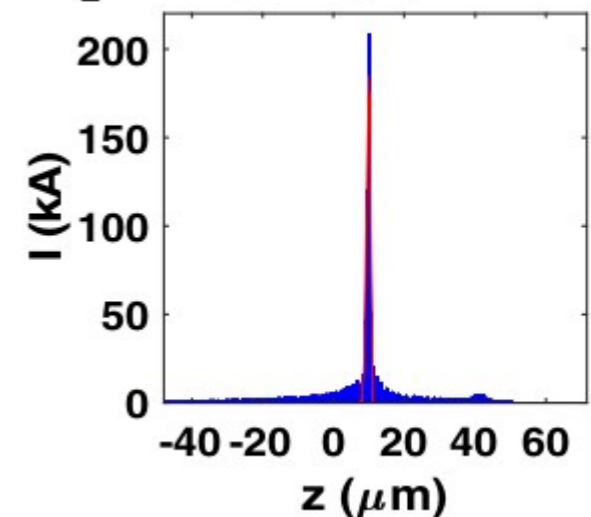
Result for Gaussian beams

$$\chi \sim \gamma \frac{E}{E_{\text{cr}}} \sim \alpha \lambda_C^2 \gamma \left(\frac{N}{\sigma_x \sigma_y \sigma_z} \right)^{2/3}$$

$$E \sim \frac{e}{4\pi \epsilon_0 \langle d \rangle^2}, \quad \langle d \rangle \sim \left(\frac{\sigma_x \sigma_y \sigma_z}{N} \right)^{1/3}$$

Naive estimate of the field strength using average distance & Coulomb field

$\sigma_z = 0.5 \mu\text{m}$ $I(\text{pk}) = 208.32 \text{ kA}$



Nonperturbative QED Collider

Beamstrahlung mitigation with ultra-short bunches

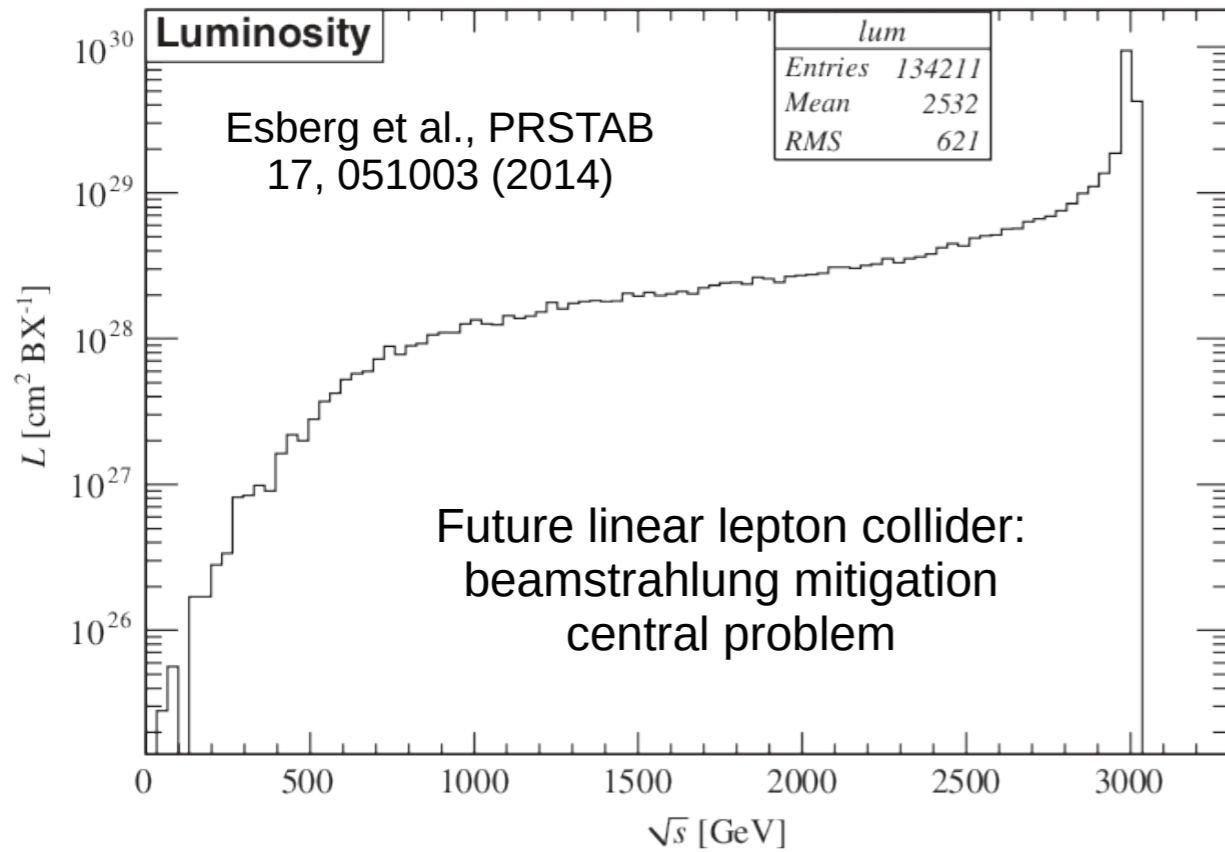


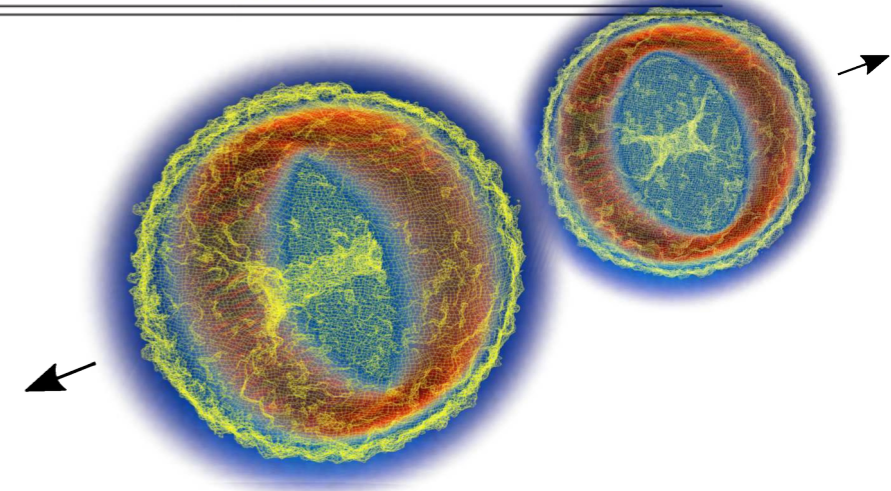
FIG. 13. CLIC nominal luminosity spectrum at ideal conditions; the spectrum includes contributions from coherent pairs and initial state radiation.

$$\frac{dP_{\text{rad}}}{dt} \approx 1.46 \frac{\alpha \chi_e^{2/3}}{\gamma \tau_c} \longleftrightarrow l_{\text{rad}} \approx \frac{\epsilon}{10 \text{ GeV}} \frac{0.7 \mu\text{m}}{\chi^{2/3}}$$

$$\epsilon \approx 100 \text{ GeV}, \chi \approx 1700 \rightarrow l_{\text{rad}} \approx 50 \text{ nm}$$

Alternative concept: collide bunches which are “too short to radiate”

Parameter	[Unit]	NPQED Collider	FACET-II	ILC	CLIC
Beam energy	[GeV]	125	10	250	1500
Bunch charge	[nC]	0.14–1.4	1.2	3.2	0.6
Peak current	[kA]	1700	300	1.3	12.1
Energy spread (rms)	[%]	0.1	0.85	0.12	0.34
Bunch length (rms)	[μm]	0.01–0.1	0.48	300	44
Bunch size (rms)	[μm]	0.01	3	0.47	0.045
Pulse rate × Bunches/pulse	[Hz] × N_{bunch}	100 × 1	30 × 1	5 × 1312	50 × 312
Beamstrahlung	χ_{av}	969		0.06	5
Parameter	χ_{max}	1721		0.15	12
Disruption Parameters	$D_{x,y}$	0.001–0.1		0.3	0.15
Peak electric field	[TV/m]	4500	3.2	0.2	2.7
Beam power	[MW]	0.002–0.02	10^{-4}	5	14
Luminosity	[cm ⁻² s ⁻¹]	6×10^{30} – 4×10^{32}		10^{34}	10^{34}



The Ritus-Narozhny conjecture:

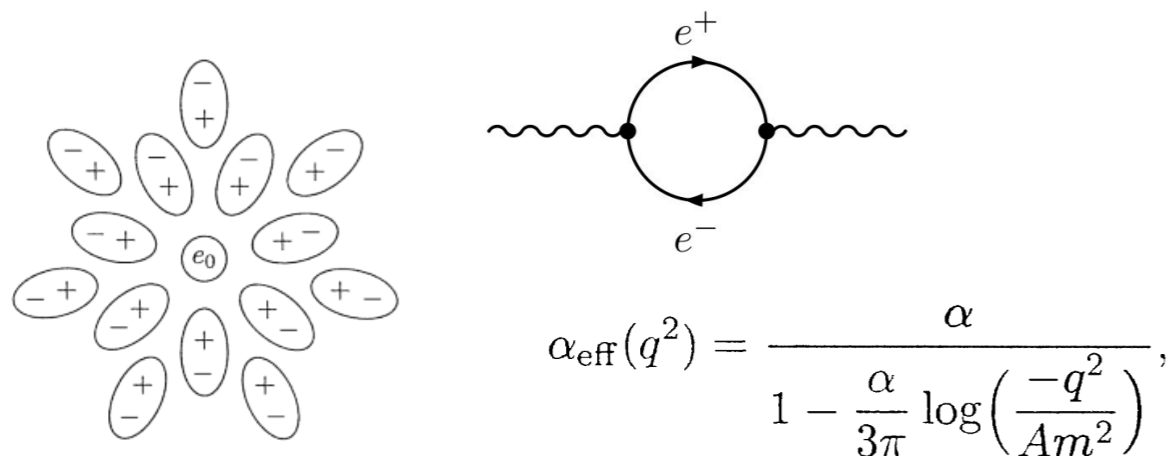
Fully nonperturbative regime of QED: almost unexplored

See talks by Anton Ilderton
(12:30, Thursday)

and Alexander Fedotov
(11:30, Friday)



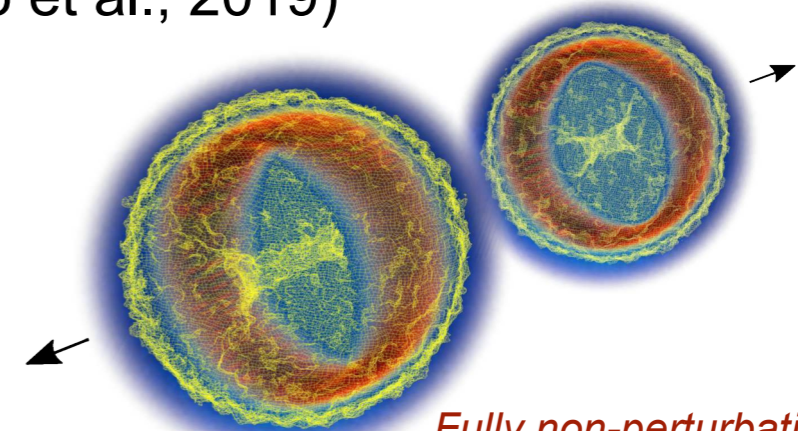
Radiative corrections in perturbative QED (Serber/Uehling 1935, Gell-Mann&Low 1954,...)



- In the absence of strong fields vacuum polarization scales logarithmically with energy (weak effect) → effective (renormalized) charge $\alpha \rightarrow \alpha_{\text{eff}}$
- Drastic changes in the presence of a strong background field: power-law scaling with χ → photon/lepton mass: $\delta m^2 \sim \alpha \chi^{2/3} m^2$ → **Ritus/Narozhny conjecture:** breakdown of perturbation theory if $\alpha \chi^{2/3} \gtrsim 1$

Probing QED in the fully non-perturbative regime (Yakimenko et al., 2019)

1 loop	
(1a)	$\alpha \chi^{2/3}$ [12]
(1b)	$\alpha \chi^{2/3}$ [13]
3 loops	
(3a)	$\alpha^3 \chi^{2/3} \log \chi$ [17]
(3b)	$\alpha^3 \chi^{2/3} \log \chi$ [17]
(3c)	$\alpha^3 \chi \log^2 \chi$ [18]
(3d)	$\alpha^3 \chi^{2/3} \log^2 \chi$ [17]
(3e)	$\alpha^3 \chi^{4/3}$ [17]
(3f)	$\alpha^3 \chi \log^2 \chi$ [18]
(3g)	$\alpha^3 \chi^{5/3}$ [18]



Fully non-perturbative QED collisions of two dense electron bunches

Ritus, Annals of Physics, 69, 555 (1972)
Narozhny, PRD 21, 1176 (1980)

Mironov & Fedotov, arXiv:2109.00634
Mironov et al., PRD 102, 053005 (2020) 31

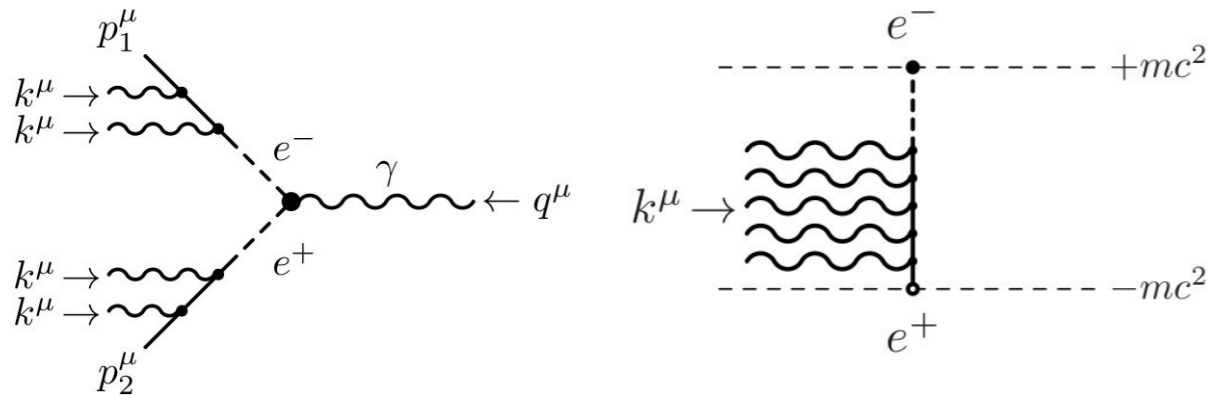
Requires fully self-consistent non-perturbative calculations

Thank you for your attention

Backup slides

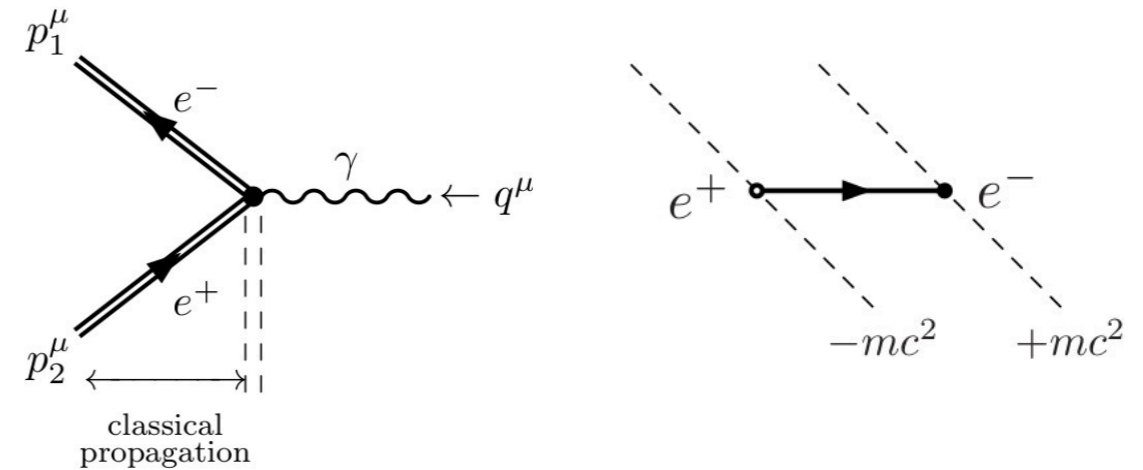
E-320: observing photon-induced vacuum decay

E-144: multi-photon pair production



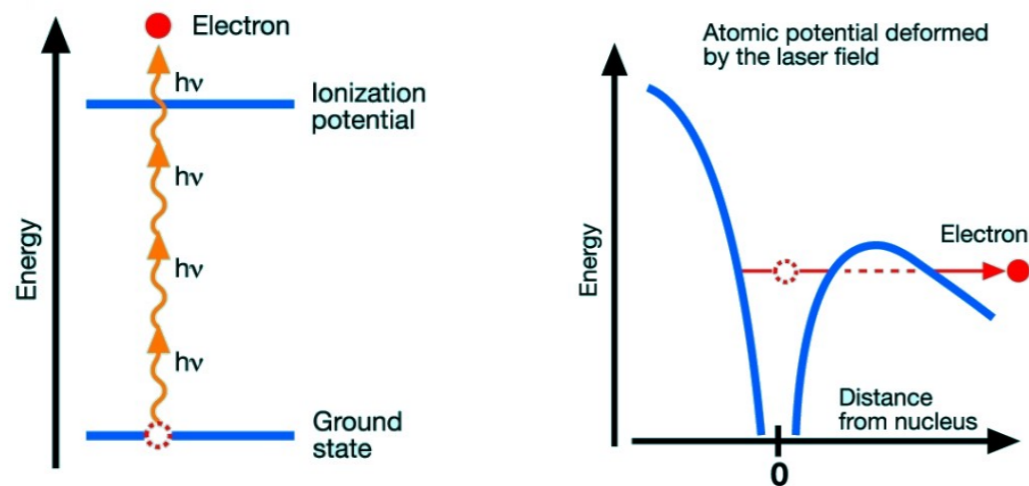
“Positron Production in Multiphoton Light-by-Light Scattering” E-144 PRL 79, 1626 (1997)

E-320: tunneling pair production

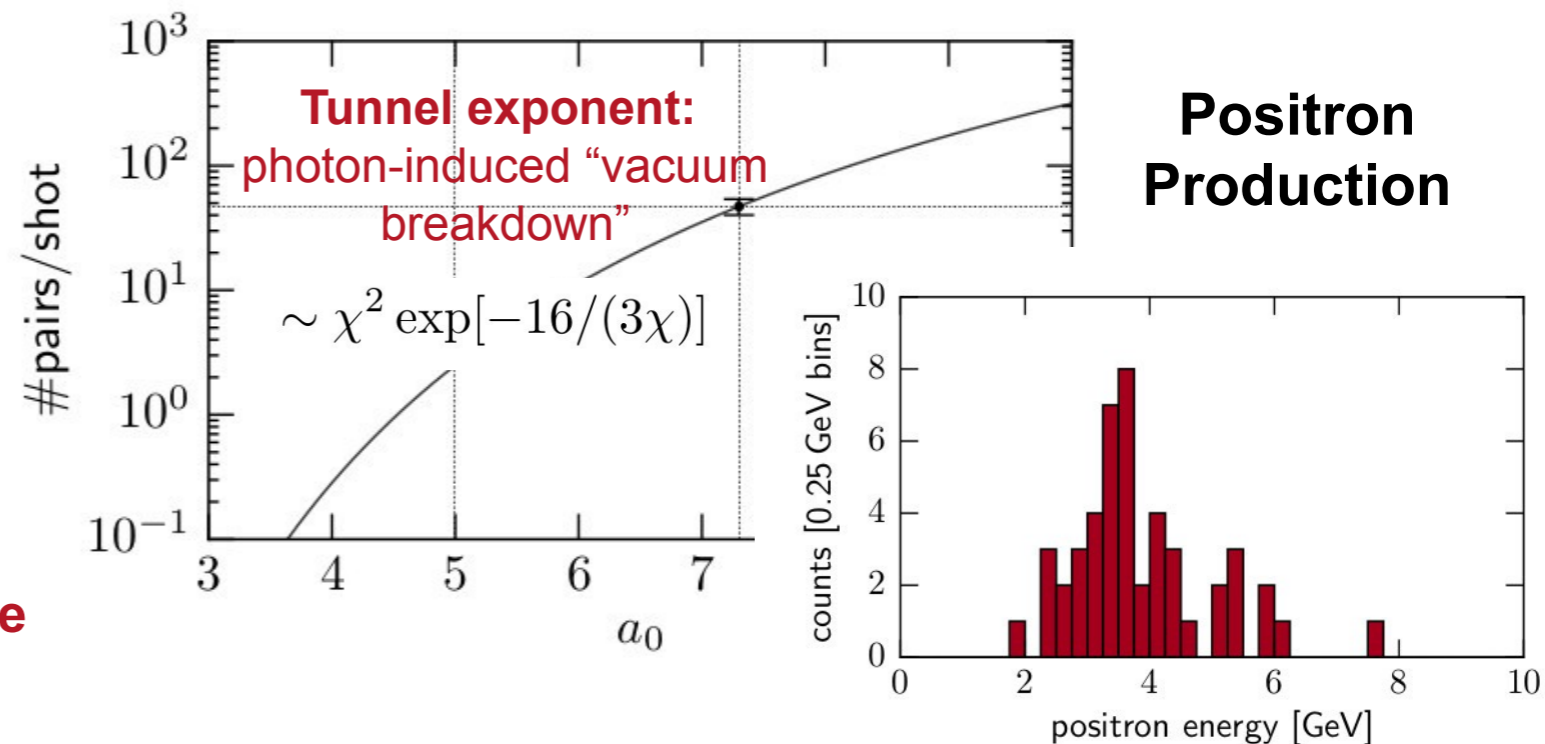


Photon → virtual pair → tunneling → real pair (local constant field approximation)

Completely analogous to tunnel ionization



E-320 will probe a qualitatively new regime of light-matter interaction

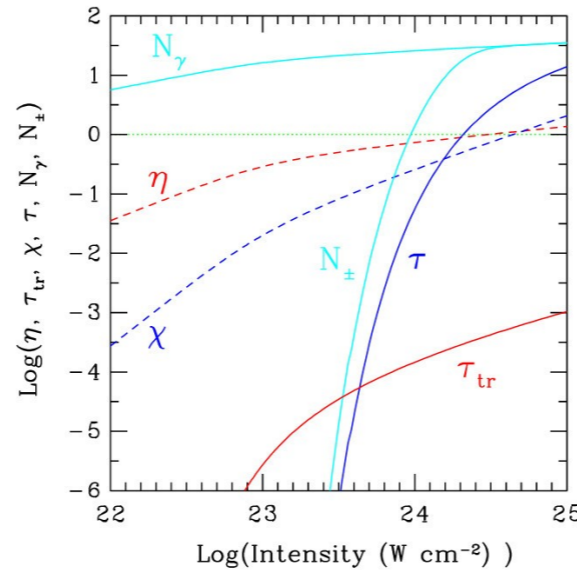
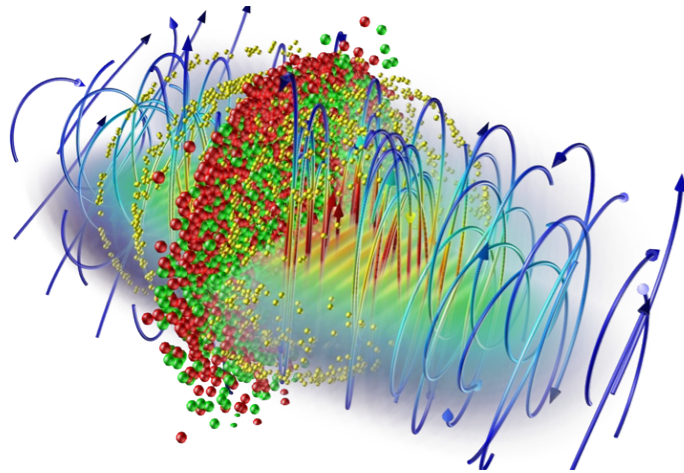


Probing the QED Plasma Regime for the first time

Observing the interplay between collective & strong-field quantum effects



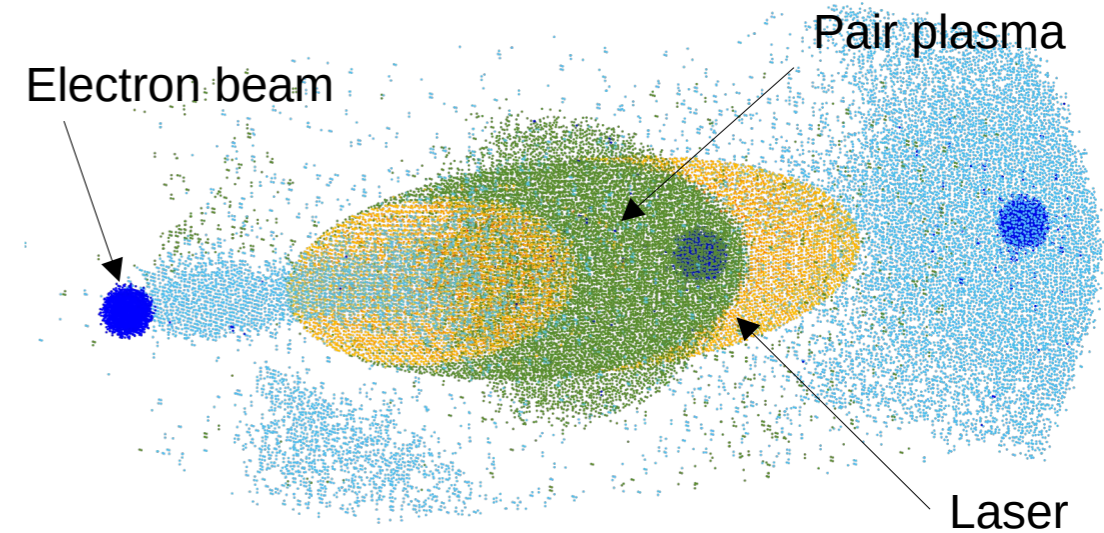
Seeded laser-laser collisions



Bell & Kirk, PRL 101, 200403 (2008)

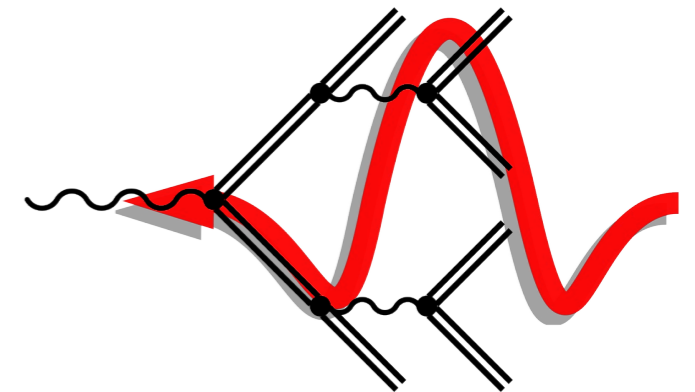
Grismayer et al., PoP 23, 056706 (2016)

Beam-laser collisions



Qu et al., PRL 127, 095001 (2021)

- Two complementary approaches to access the **QED plasma regime**: interplay between strong-field quantum and collective plasma effects
- Exponential growth: pair creation stops at $\chi \sim 1$: beam energy is transferred into plasma density, **multiplicity: $\sim \chi$ pairs per beam particle**
- Radiative energy loss inefficient at $\chi \lesssim 0.1$: laser stops and re-accelerates the plasma: **final gamma factor $\gamma \sim a_0$**



Exponential growth of the pair density

$$\left(\frac{\hbar \omega_{plasma}}{1 eV} \right)^2 \approx \frac{n_{plasma}}{10^{21} cm^{-3}} \frac{1}{\gamma}$$

Plasma frequency

$$a_0 = \frac{eE}{\omega_L mc} \approx 0.6 \frac{\lambda_L}{1 \mu m} \sqrt{\frac{2I}{10^{18} W/cm^2}}$$

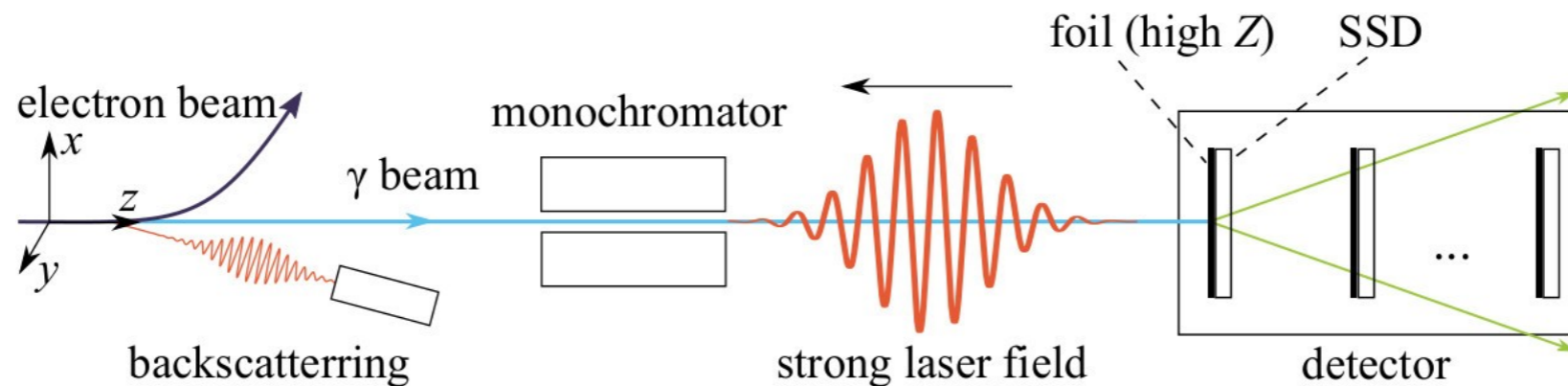
Classical intensity parameter

$$\frac{l_{rad}}{1 \mu m} \approx \left(\frac{\mathcal{E}}{10 GeV} \right)^{1/3} \left(\frac{10^{20} W/cm^2}{2I} \right)^{1/3}$$

Radiation length

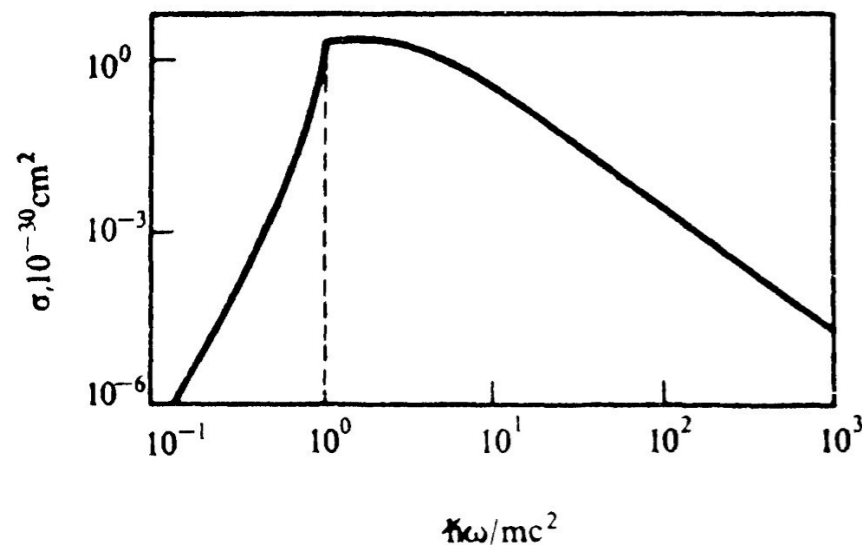
Measuring birefringence in the non-perturbative regime

Controlling and measuring the polarization of multi-GeV photons



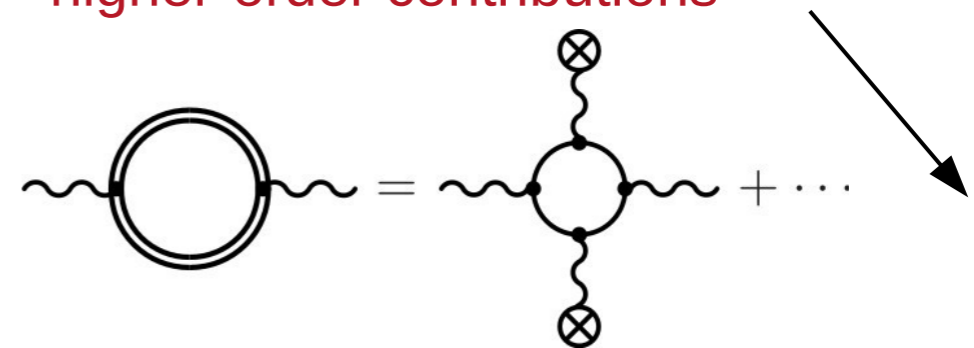
- ① Compton backscattering: highly polarized GeV photons
- ② High-intensity laser polarizes the quantum vacuum
- ③ Pair production (foil): polarization dependent distribution

Measuring the influence of quantum fluctuations below and above the pair-creation threshold



Light-by-light scattering cross section

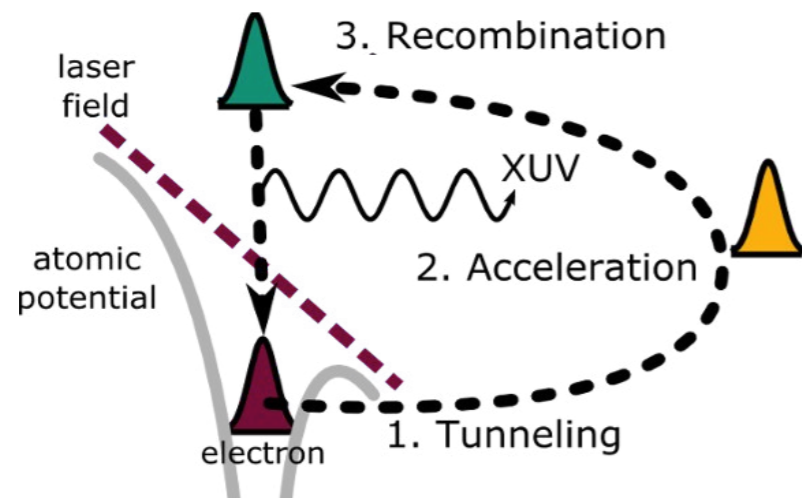
We are curious about the non-perturbative higher-order contributions



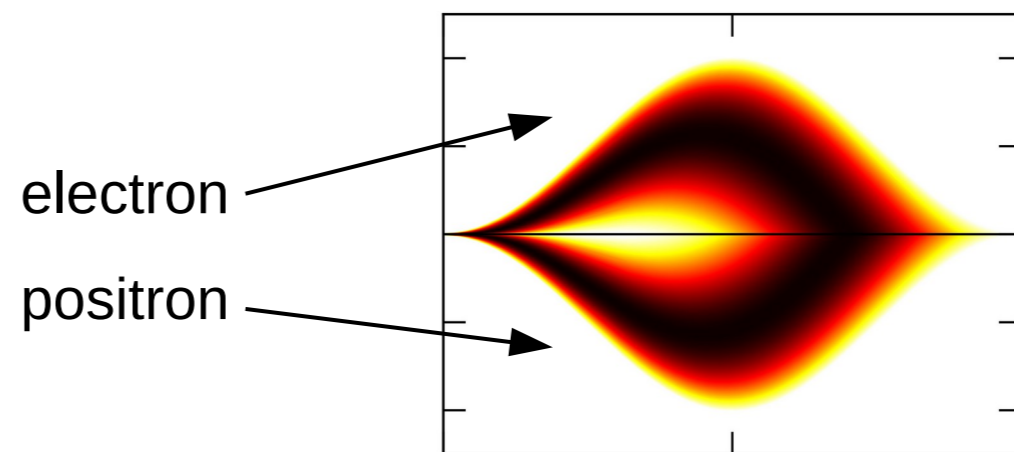
Leading-order perturbative contribution: light-by-light scattering, which corresponds to the leading-order term to the Euler-Heisenberg effective action

Observing coherent re-collisions

Non-relativistic recollisions in atoms



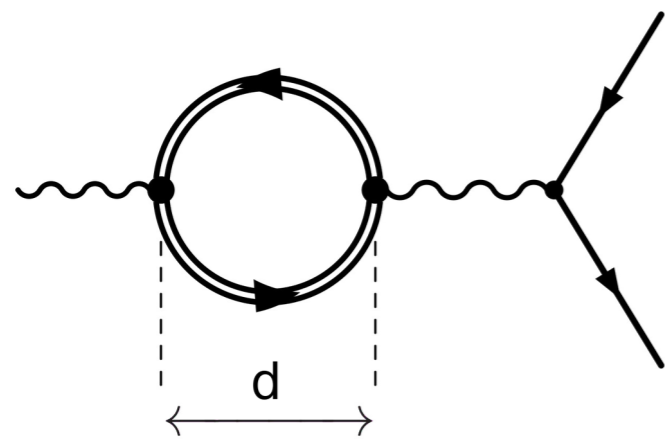
Classical electron/positron trajectories



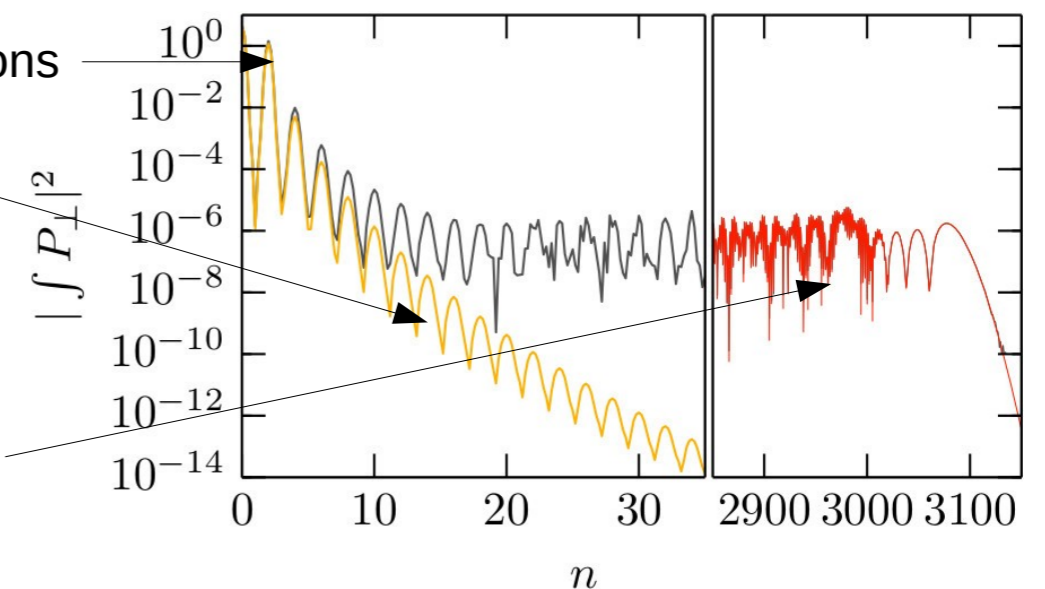
Due to proton/electron mass difference this mechanism becomes inefficient at relativistic light intensities

Color indicates the classical energy gain at the recollision point

Probing “macroscopic” quantum loops – does QED still work or is it only an effective theory?



Perturbative: 0+2 photons
 Non-perturbative, local-constant field approx.
 Stationary points with macroscopic separation between creation + annihilation of the pair



Vacuum QED: $d \sim \lambda_C = \hbar/(mc)$
 Recollisions: $d \sim \lambda \sim \mu\text{m} \gtrsim 10^3 \lambda_C$

Number of absorbed laser photons