



UNIVERSITY OF
GOTHENBURG

How do simulations of intense laser-matter interactions work?

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ExHILP 21

*with thanks to Arkady Gonoskov, Mattias Marklund, Stepan Bulanov, Anton Ilderton, Ben King, Daniel Seipt, Chris Ridgers, Stuart Mangles and many more



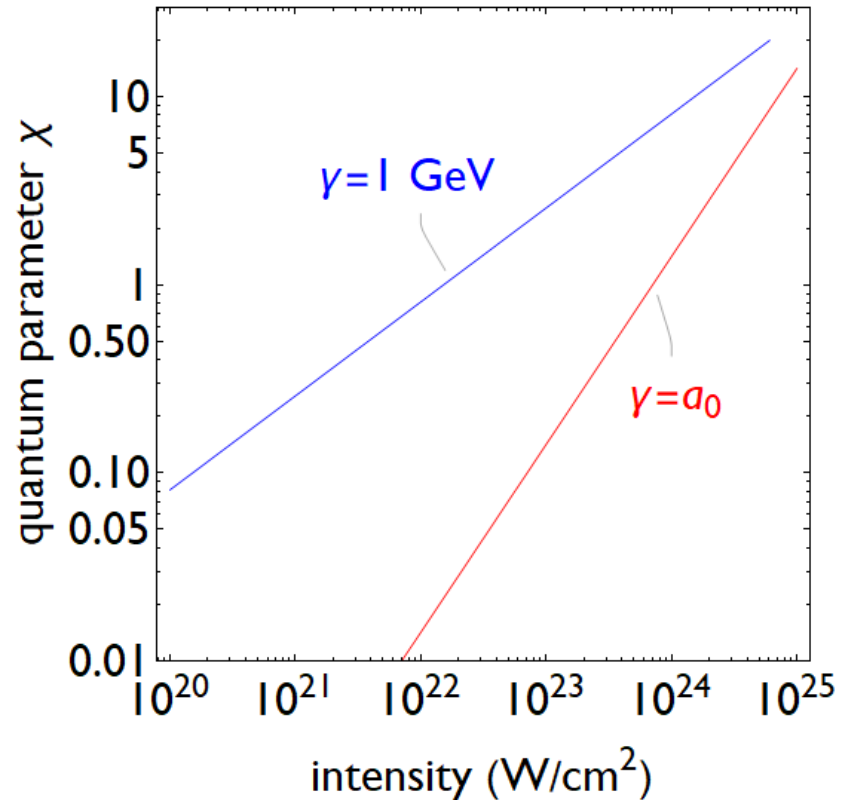
Overview

- **Motivation.** What are we interested in? And why simulate at all?
- **What does the theory tell us?** From strong-field QED to probability rates
- **How is this implemented?** The Monte Carlo approach, particle-in-cell codes and more, physics processes, numerical challenges
- **How are we doing?** Benchmarking, new approaches, experiments
- **What's next?**

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What is “high-intensity” anyway?



$$E_{\text{crit}} = 1.3 \times 10^{18} \text{ V/m}$$

$$B_{\text{crit}} = 4 \times 10^9 \text{ T} / 4 \times 10^{13} \text{ G}$$

- Classical nonlinearity

$$a_0 = \frac{eE}{m\omega_0} = 85 \left(\frac{I\lambda^2}{10^{22} \text{ Wcm}^{-2} \mu\text{m}^2} \right)^{1/2}$$

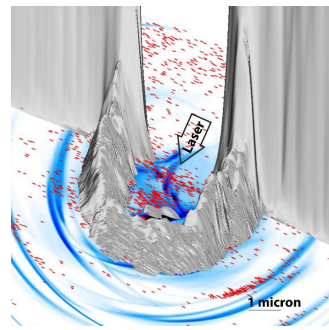
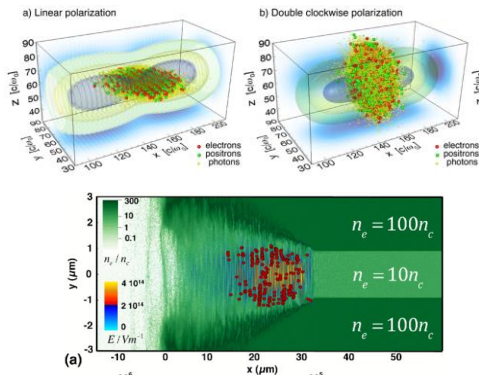
- Quantum nonlinearity

$$\chi = \frac{\gamma E}{E_{\text{crit}}} = 0.4 \left(\frac{\varepsilon}{\text{GeV}} \right) \left(\frac{I}{10^{22} \text{ Wcm}^{-2}} \right)^{1/2}$$

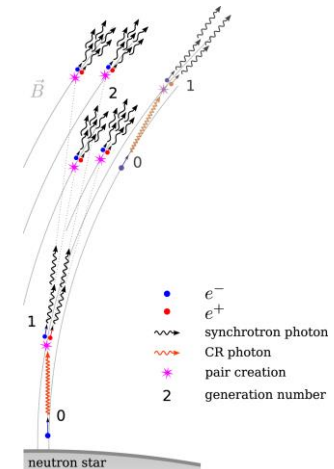
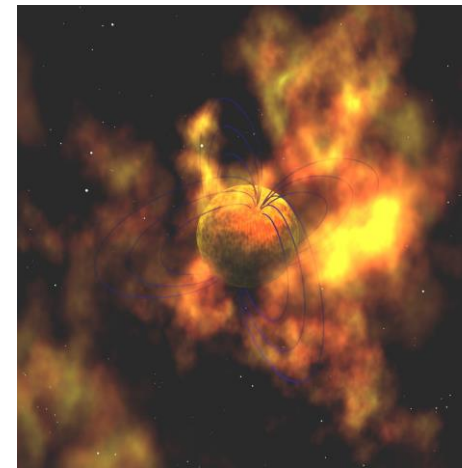
- Vacuum nonlinearity

$$f = \frac{E^2 - B^2}{E_{\text{crit}}^2} = \frac{I}{10^{29} \text{ Wcm}^{-2}}, \quad g = \frac{E \cdot B}{E_{\text{crit}}^2}$$

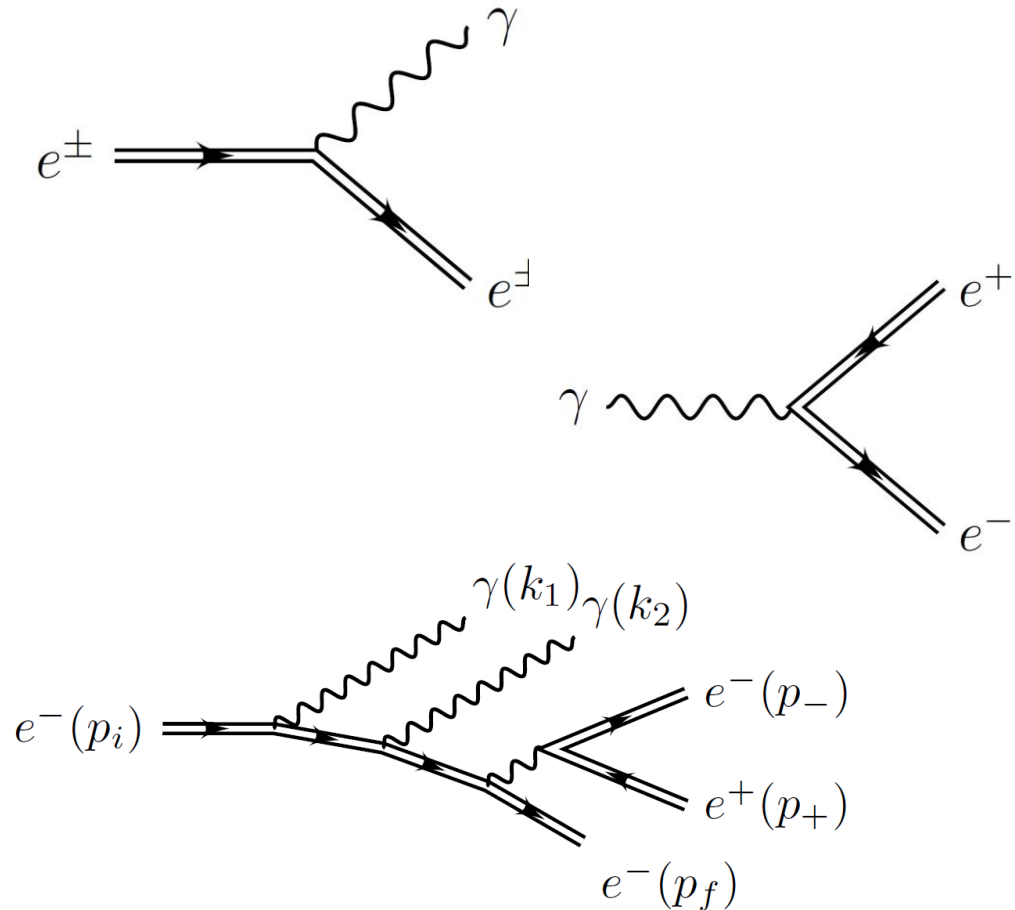
- **High-intensity lasers:** strong fields because a_0 is large, but also large χ because of accelerated electrons. Field is also spatially macroscopically large (even if characteristic scales are 10s micron/10s fs)



- **Compact objects:** magnetic fields around pulsars/magnetars $>10^8$ T (10% of B_{crit}), with TeV accelerated particles. (χ depends on pitch angle.)

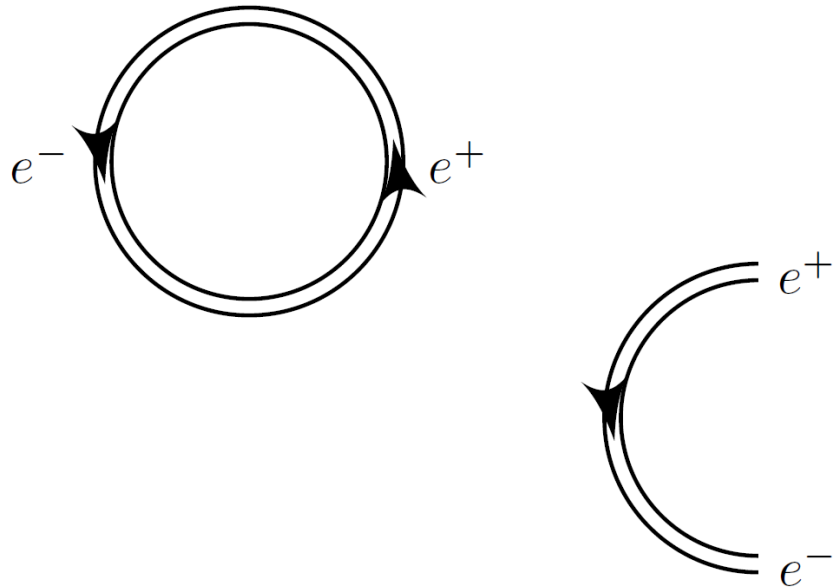


How do particles and fields interact?



- Photon emission (nonlinear Compton)
- Electron-positron pair creation (nonlinear Breit-Wheeler)
- Putting these together: radiation reaction, pair cascades
- Energy transfer to particles and radiation, collective dynamics: high fields creating their own plasma environment

How do fields interact?

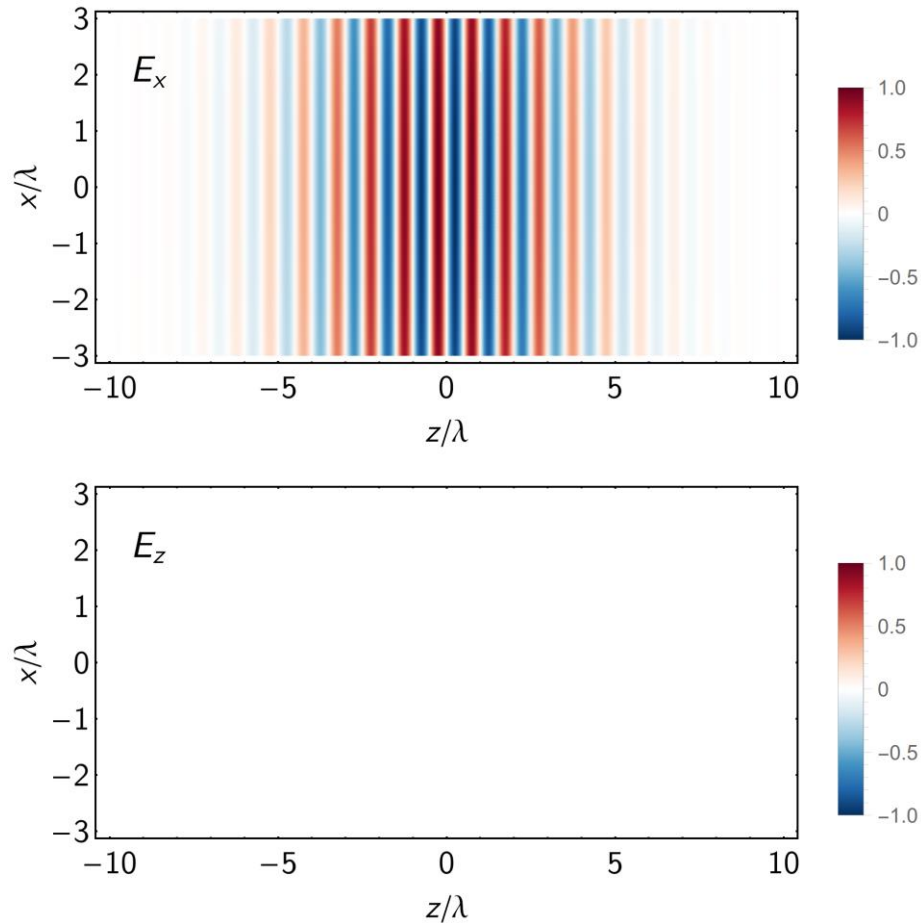


- Maxwell's equations are linear, at but ultrahigh field strengths this is not an adequate description anymore.
- Vacuum polarisation (photon-photon scattering, birefringence, dichroism)
- Schwinger pair creation/vacuum pair creation



Motivation

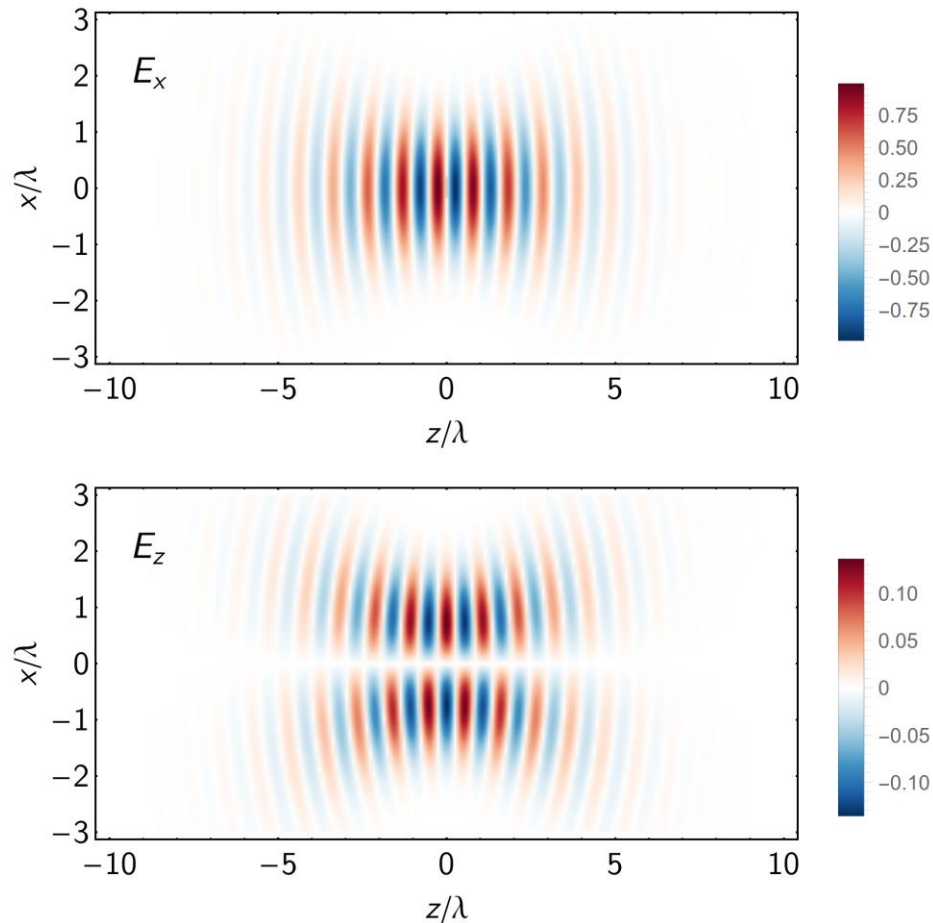
Why simulate at all?



- The plane wave is the paradigmatic choice of background for calculations of nonlinear classical and quantum processes in strong electromagnetic fields.
- Classical and quantum dynamics of an electron in a plane-wave background are exactly solvable [see, for example, [Heinzl and Ilderton, PRL 118, 113202 \(2017\)](#)]

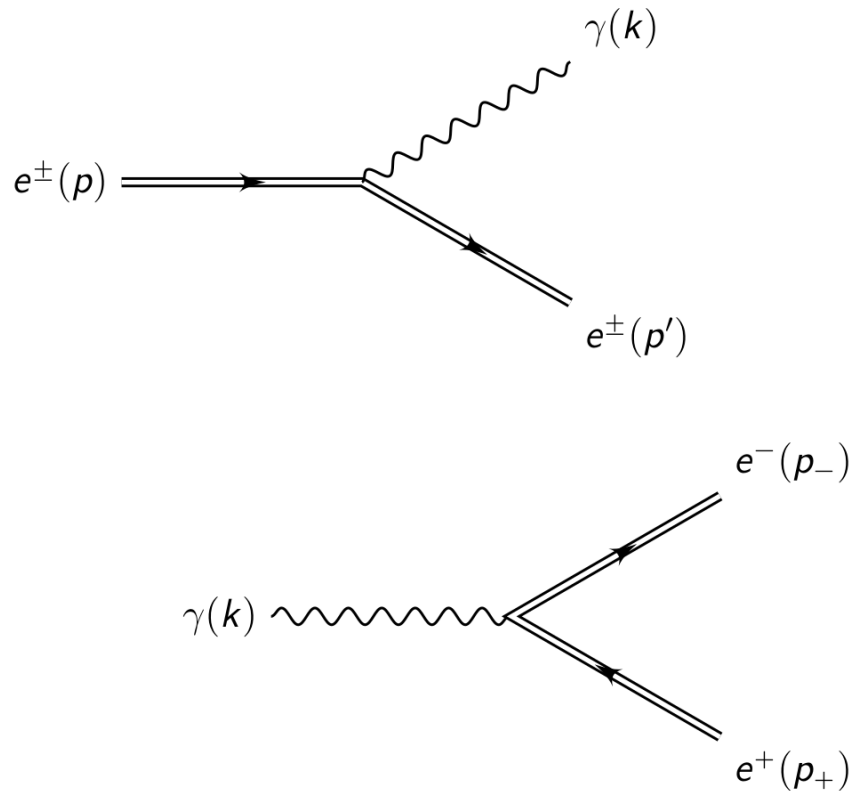


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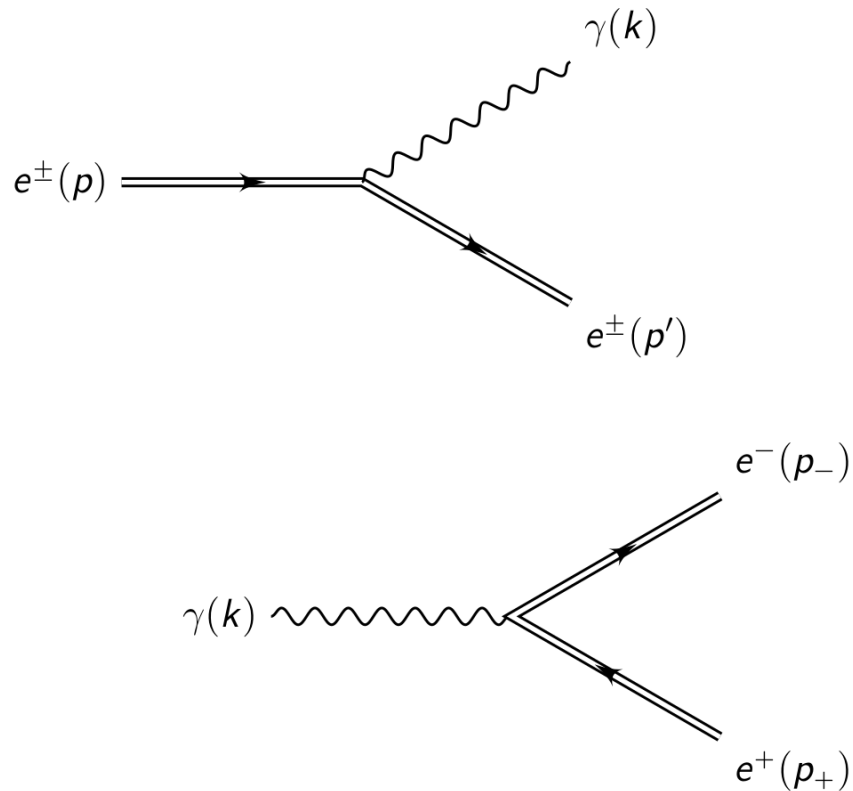
- Lasers reach high intensity by focusing – getting close to the diffraction limit
- A focusing electromagnetic pulse has to be described numerically (usually with a certain degree of approximation).
- No complete theory for QED interactions exists in this background. High-energy approximations possible [Di Piazza, PRL 113, 040402 (2014)]

Why simulate at all?



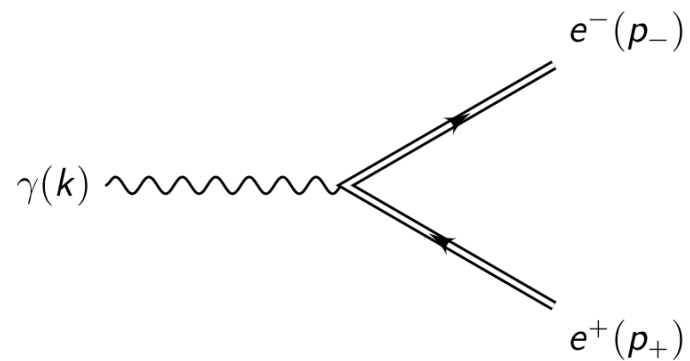
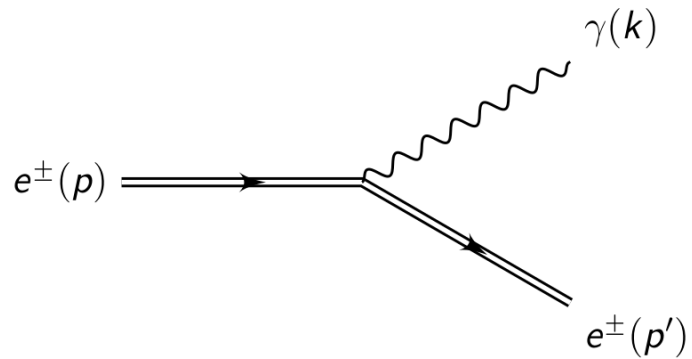
- Simplest two: photon emission and pair creation. Each first order in the fine-structure constant α .
- Expect that first order processes are more probable than second order, which are more probable than third order, etc.

Why simulate at all?



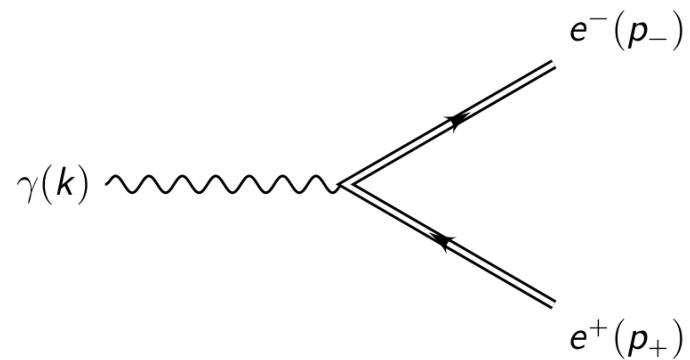
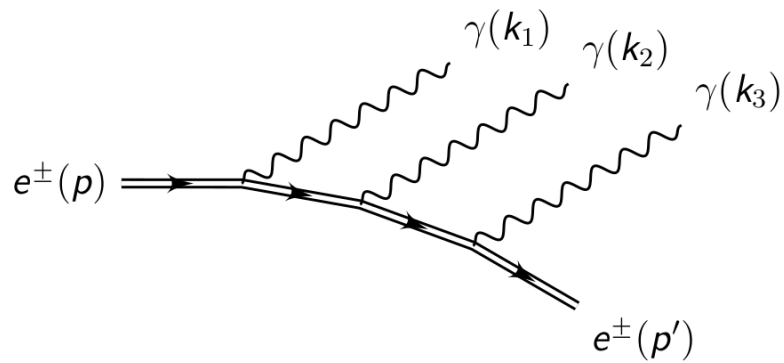
- Simplest two: photon emission and pair creation. Each first order in the fine-structure constant α .
- Expect that first order processes are more probable than second order, which are more probable than third order, etc.
- However, in strong (or long-duration) electromagnetic fields, the probability for only one event is suppressed.

Why simulate at all?



- A single high-energy electron emits many photons (radiation reaction).

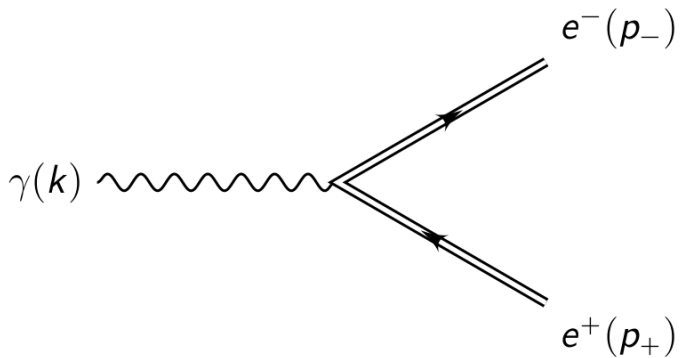
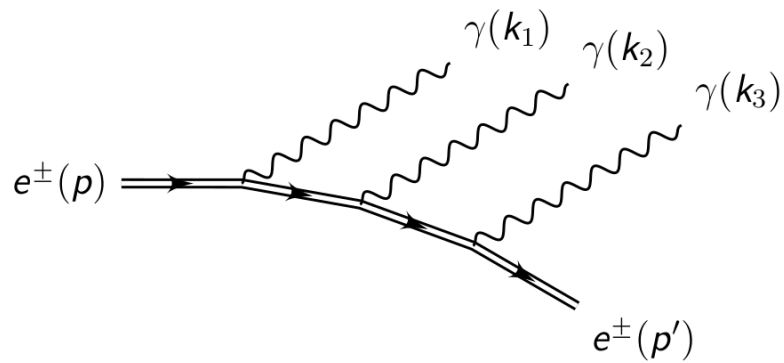
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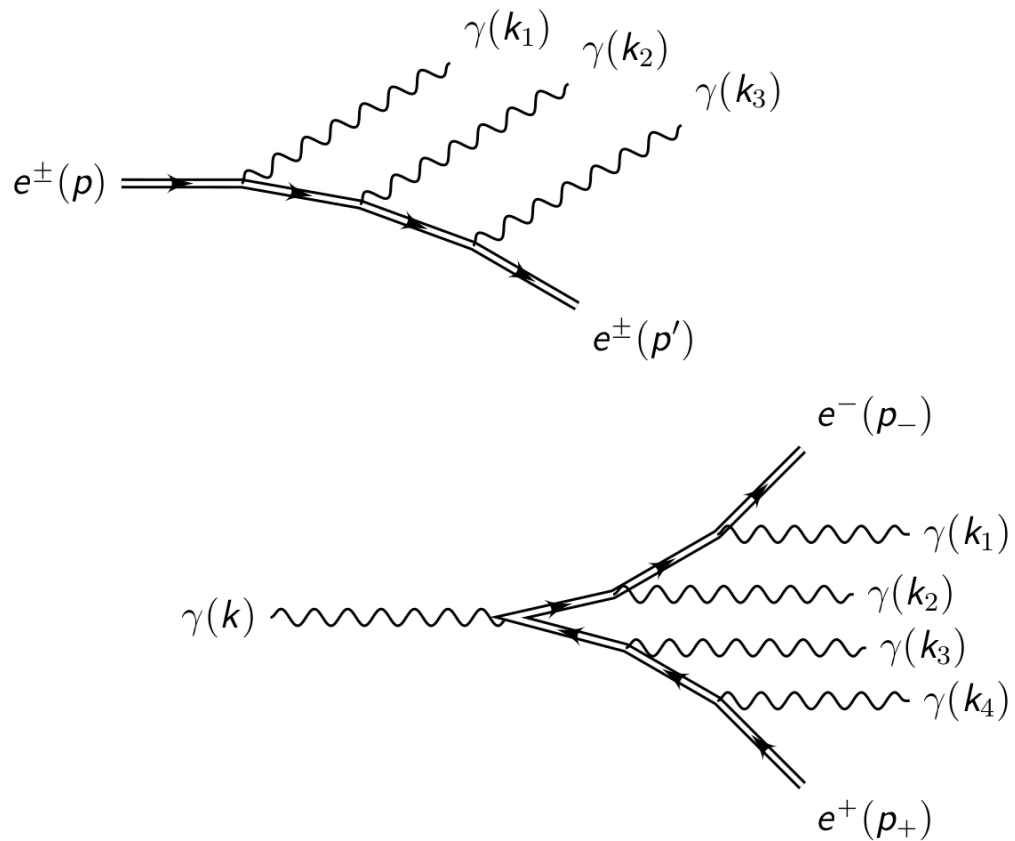
Why simulate at all?



- A single high-energy electron emits many photons (radiation reaction).
- A single photon creates an electron and positron that radiate additional photons.

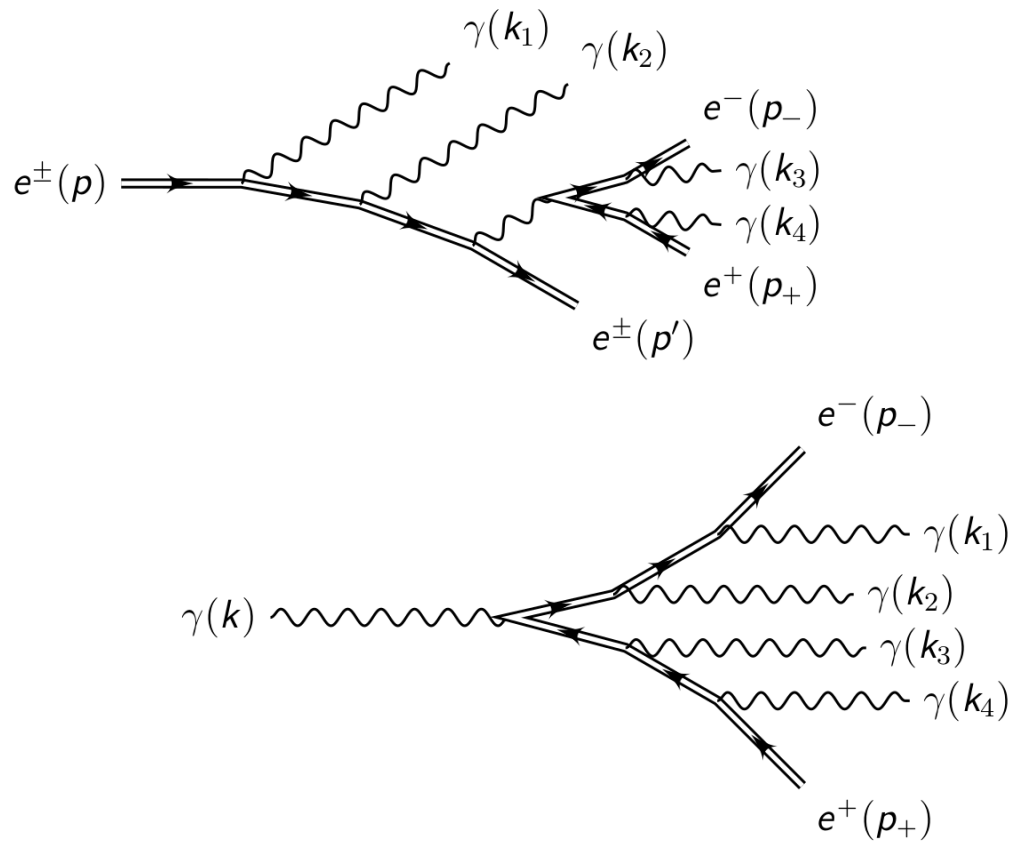
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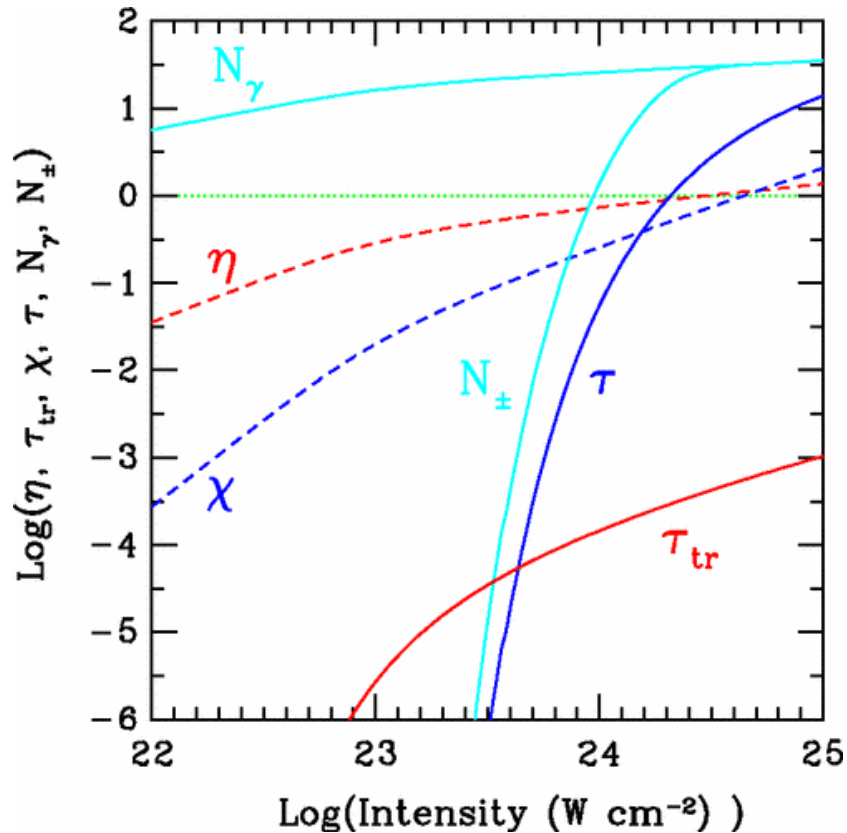
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Why simulate at all?



- A single high-energy electron emits many photons (radiation reaction).
- A single photon creates an electron and positron that radiate additional photons.
- ... and so on

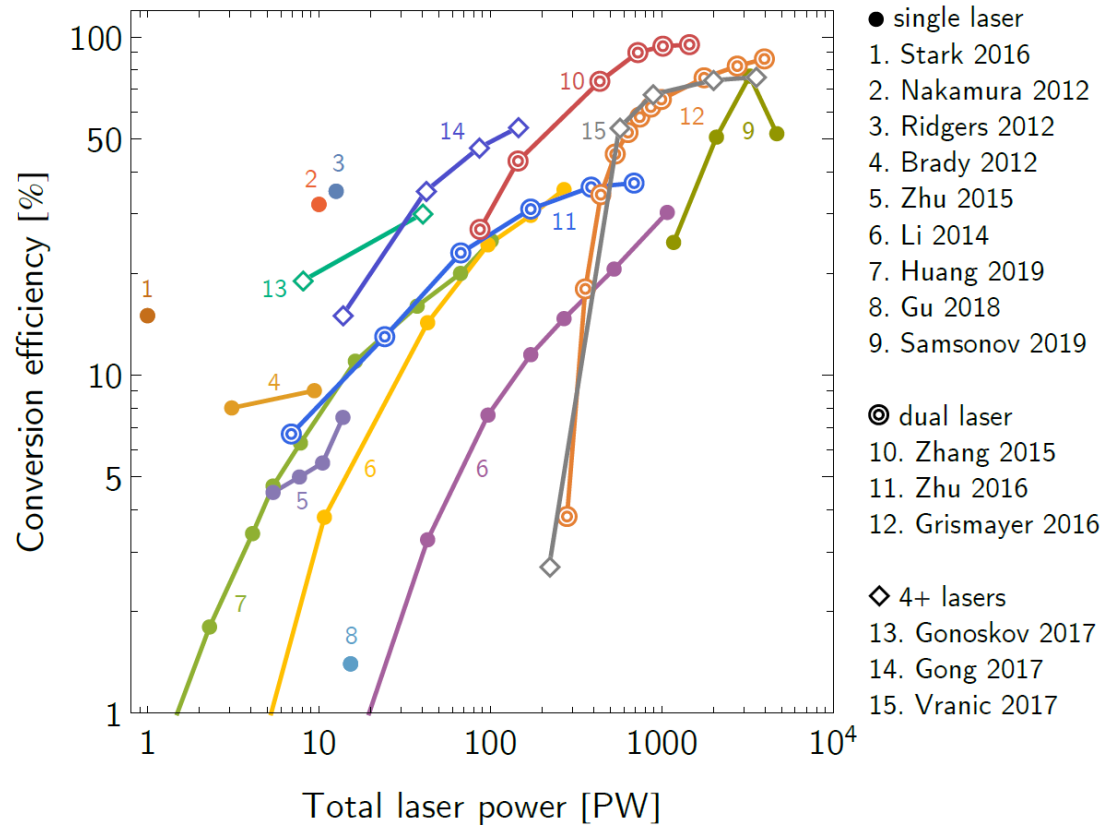
Why simulate at all?



Bell and Kirk, PRL 101, 200403 (2008)

- The possibility of driving an electron-positron pair avalanche purely with high-intensity optical lasers (seeded by even a single electron) was raised by Bell and Kirk, PRL 101, 200403 (2008).
- The transfer of energy from laser to pairs might set a limit on the maximum achievable intensity [Fedotov et al, PRL 105, 080402, (2010); Bulanov et al, PRL 105, 220407 (2010)].

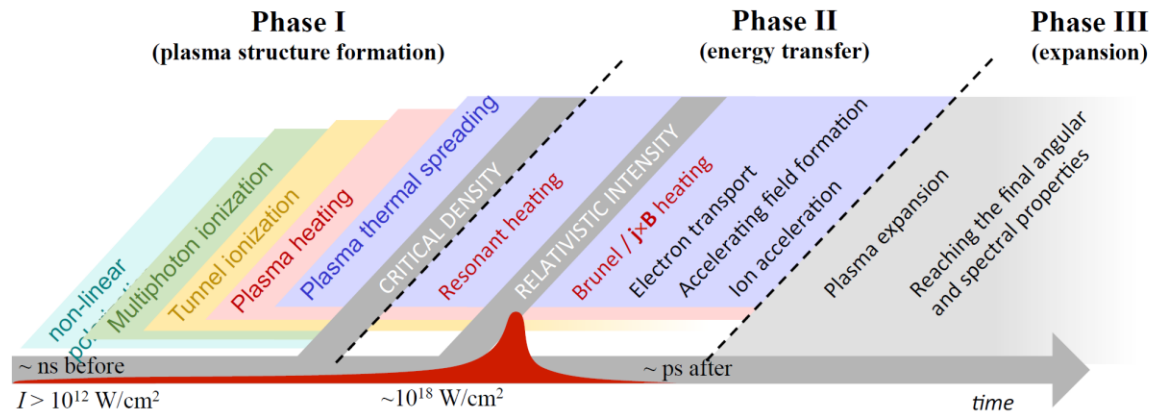
Why simulate at all?



- Spatiotemporal structure of the field is not known in advance – one reason to use simulations is to find out what this structure is.
- EM field could be depleted over the course of the interaction, as energy is transferred to particles and radiation.

Motivation

Why simulate at all?



Atoms: TDSE, ionization
theoretical/empirical rates

Plasma: Vlasov equation,
MHD, other kinetic
approaches

Plasma: Particle-In-Cell
- Maxwell's eq. (FFT, FDTD+)
- Particles motion (relativistic solvers)

Particles:
- Maxwell's eq. (FFT, FDTD+)
- Particles (ballistic propagation)

- Laser-plasma interactions contain rich dynamics over a wide range of timescales.
- “Proper treatment of systems where both the microscopic and macroscopic behaviour are important will undoubtedly challenge simulation physicists for many years to come.”
– J. M. Dawson

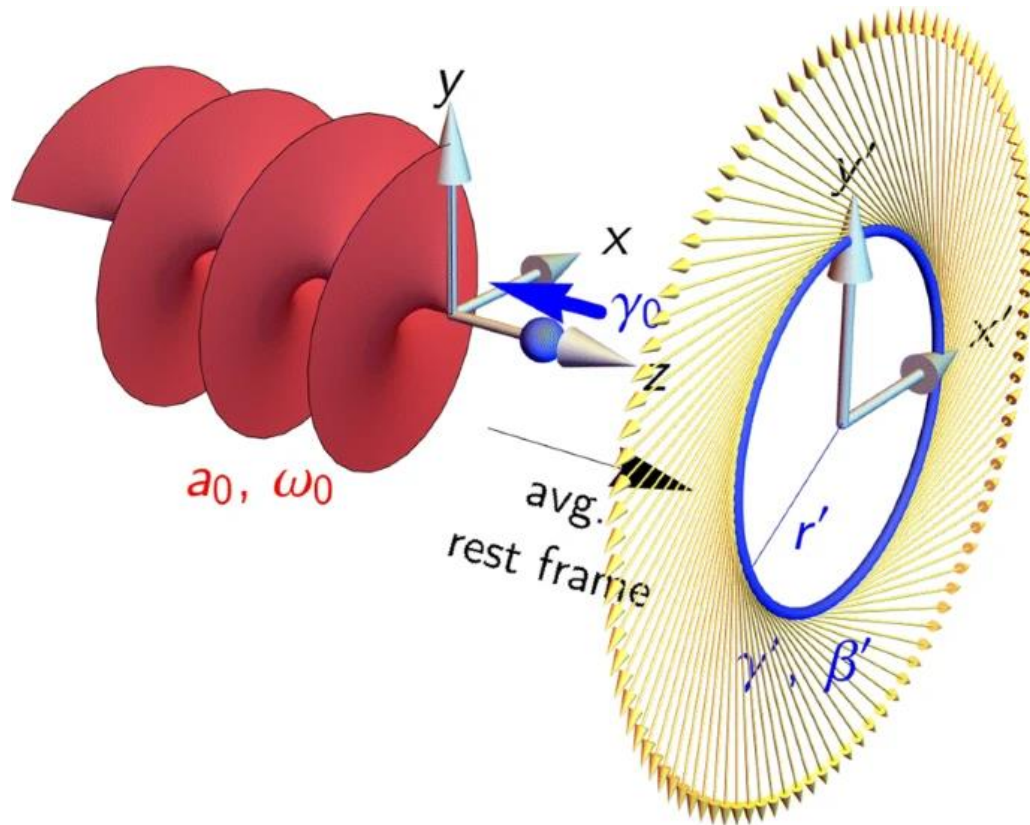


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Theory and simulations

Nonlinear QED



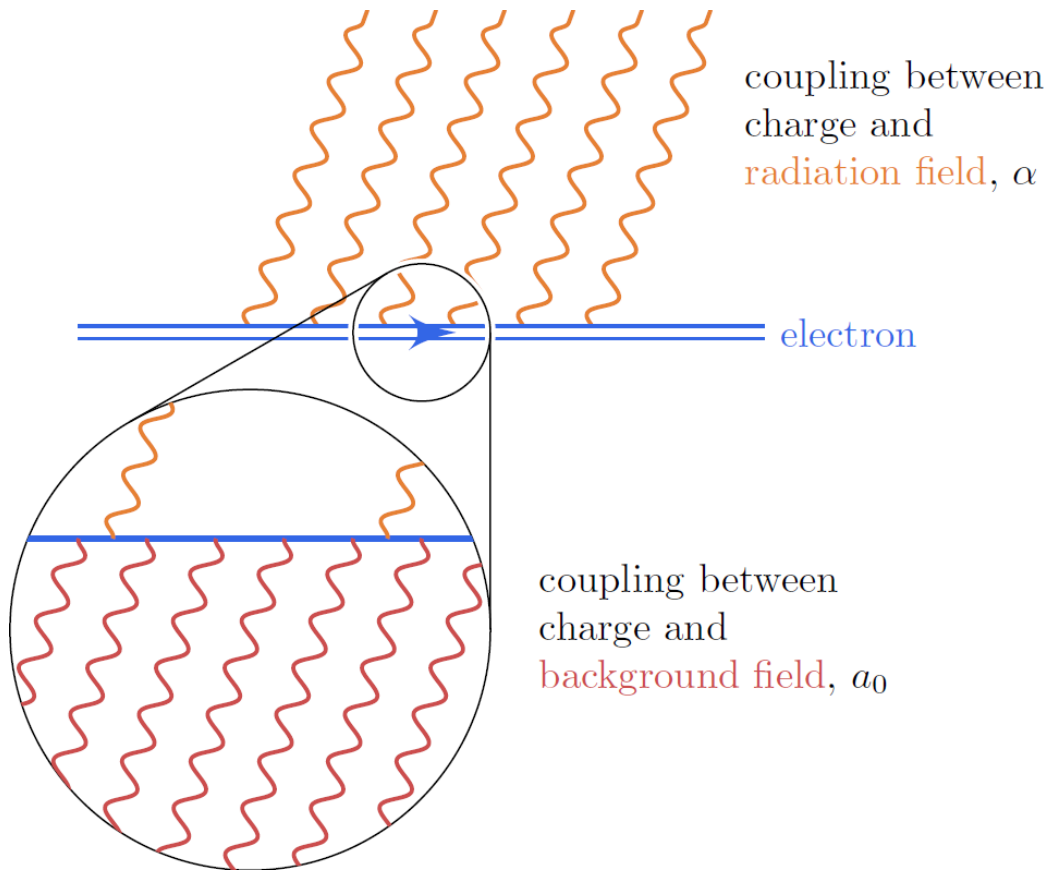
- Compare the characteristic frequency of the emitted radiation to the orbital (cyclotron) frequency:

$$\frac{\omega'}{\omega_c} \simeq a_0^3$$

- Also measures work done by the field over a Compton length, in units of the photon energy.
- Electron does not interact with one or two photons of the laser, but many.

Theory and simulations

Nonlinear QED

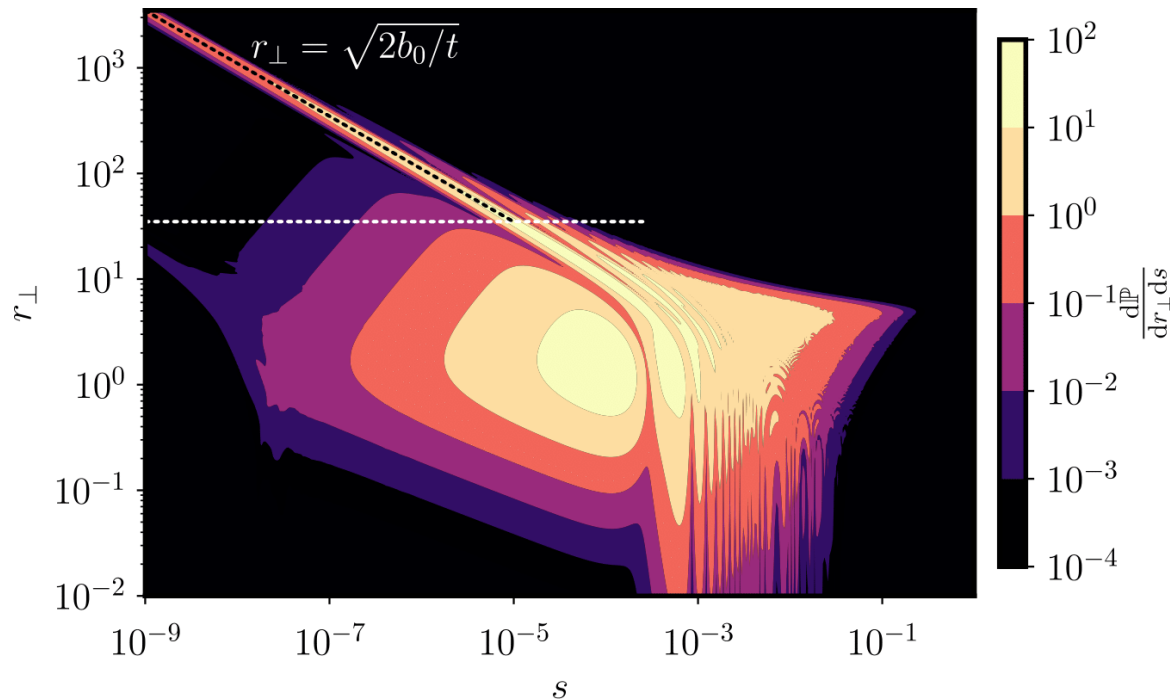


- Splits the EM field into a **fixed, classical background** and a **fluctuating, quantized radiation field**.
- Two kinds of nonperturbativity: absorbing many photons from the background (scaling as a_0^3) – need ‘all order’ solutions.
- Emitting many photons: depends on **strength** and **size/duration** of the field.

Theory and simulations

Scattering calculations

$$\frac{d^3\mathbb{P}}{df d^2\mathbf{r}_\perp} = \frac{\alpha m^2}{(4\pi\omega_0 p^+)^2} \frac{f}{1-f} \frac{1}{2} \sum_{\text{spin, pol}} \left| \sum_j \mathcal{F}_j \mathcal{C}_j \right|^2$$

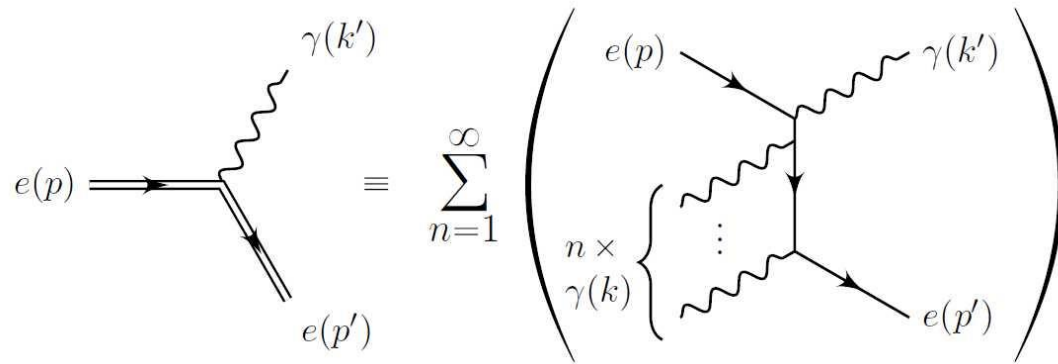


Ilderton et al, PRA 99, 042121 (2019)

- Fundamental approach: treat interaction with laser field exactly (i.e. nonperturbatively) and expand perturbatively in the dynamical EM field (i.e. the high-energy photons).
- Limitations: transition between asymptotic states \rightarrow complete knowledge of background field required, can't do arbitrary field configurations, backreaction neglected, multiplicity (# particles in final state).

Theory and simulations

Scattering calculations



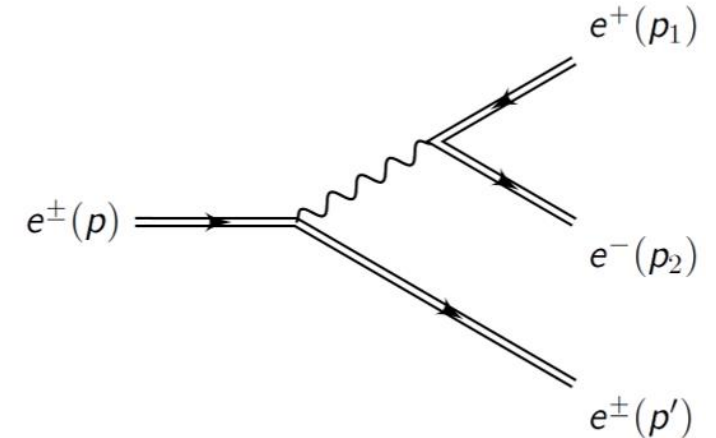
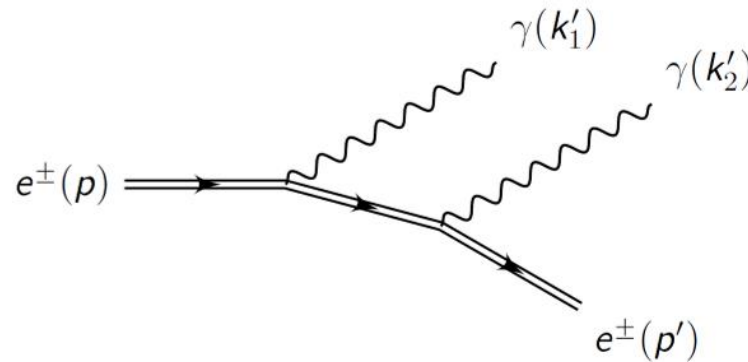
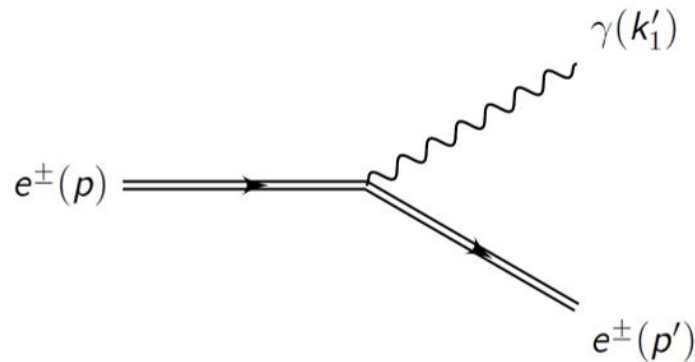
$$j^\mu = \frac{\bar{\Psi} \gamma^\mu \Psi}{\bar{\Psi} \Psi} = \frac{p^\mu(\phi)}{m} = \frac{p_0^\mu + eA^\mu}{m} - \left(\frac{eA \cdot p_0}{k \cdot p_0} + \frac{e^2 A^2}{2k \cdot p_0} \right)$$

- This is what the double lines in strong-field QED diagrams means – the interaction with the background field is accounted for exactly.
- In a plane-wave background, the wavefunctions ([Volkov states](#)) have a phase-dependent momentum.
- The probability current coincides with classical solution of the Lorentz force equation.

Theory and simulations

Scattering calculations

electron-seeded + pulsed plane wave:



Narozhnyi and Fofanov, Sov Phys JETP 83, 14 (1996)
 Boca and Florescu, PRA 80, 053403 (2009)
 Harvey, Heinzl and Ilderton, PRA 79, 063407 (2009)
 Mackenroth, Di Piazza and Keitel, PRL 105, 063903 (2010)
 Heinzl, Ilderton and Marklund, PLB 692, 250 (2010)
 Krajewska and Kaminski, PRA 85, 062102 (2012)
 ... and many more

Lötstedt and Jentschura, PRL 103, 110404 (2009)
 Seipt and Kämpfer, PRD 85, 101701 (2012)
 Mackenroth and Di Piazza, PRL 110, 070402 (2013)
 King, PRA 91, 033415 (2015)
 Dinu and Torgrimsson, PRD 99, 096018 (2019)

Hu, Muller and Keitel, PRL 105, 080401 (2010)
 Ilderton, PRL 106, 020404 (2011)
 King and Ruhl, PRD 88, 013005 (2013)
 Dinu and Torgrimsson, PRD 97, 036021 (2018)
 King and Fedotov, PRD 98, 16005 (2018)
 Mackenroth and Di Piazza, PRD 98, 116002 (2018)
 Dinu and Torgrimsson, PRD 102, 16018 (2020)

*hot off the presses: resummation techniques for very high-order processes

Theory and simulations

Probability rates

$$\frac{dP}{dk_-} = -i \frac{\alpha}{2\pi} \frac{1}{p_-} \frac{\xi_0}{\chi_0} \int \frac{d\varphi d\varphi'}{\varphi - \varphi' + i0} \left\{ 1 + \frac{p_-^2 + p_-'^2}{4p_- p_-'} [\xi_{\perp}(\varphi) - \xi_{\perp}(\varphi')]^2 \right\} \\ \times \exp \left\langle i \frac{1}{2} \frac{k_-}{p_-'} \frac{\xi_0}{\chi_0} \left\{ \varphi - \varphi' + \int_{\varphi'}^{\varphi} d\tilde{\varphi} \xi_{\perp}^2(\tilde{\varphi}) - \frac{1}{\varphi - \varphi'} \left[\int_{\varphi'}^{\varphi} d\tilde{\varphi} \xi_{\perp}(\tilde{\varphi}) \right]^2 \right\} \right\rangle$$

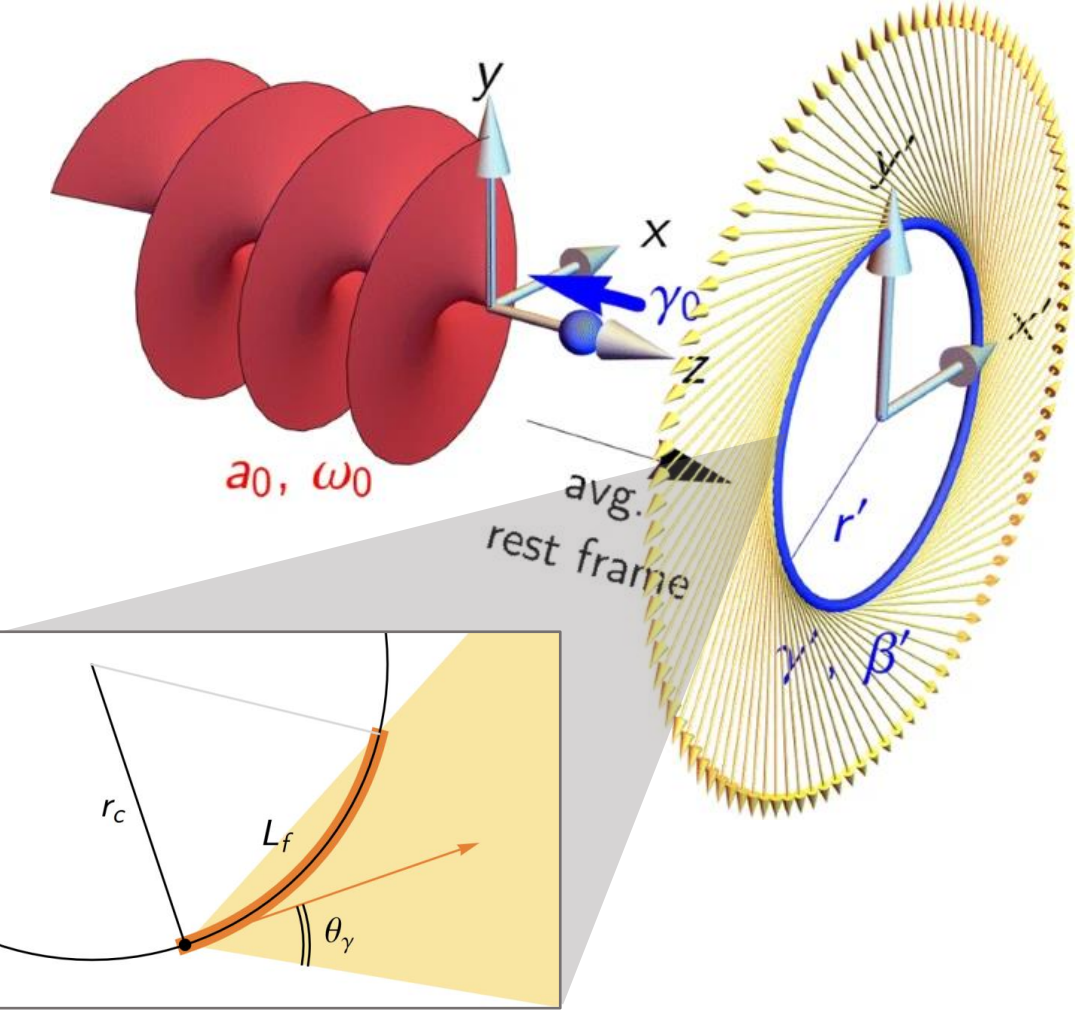


$$\frac{dP}{dk_- d\varphi_+} = \frac{\alpha}{2\pi} \frac{1}{p_-} \frac{\xi_0}{\chi_0} \text{Im} \int \frac{d\varphi_-}{\varphi_- + i0} \left\{ 1 + \frac{p_-^2 + (p_- - k_-)^2}{4p_- (p_- - k_-)} \left[\xi_{\perp} \left(\varphi_+ - \frac{\varphi_-}{2} \right) - \xi_{\perp} \left(\varphi_+ + \frac{\varphi_-}{2} \right) \right]^2 \right\} e^{i\Phi(k_-, \varphi_-, \varphi_+)},$$

$$\Phi(k_-, \varphi_-, \varphi_+) = \frac{1}{2} \frac{k_-}{p_- - k_-} \frac{\xi_0}{\chi_0} \left\{ \varphi_- + \int_{-\varphi_-/2}^{\varphi_-/2} d\tilde{\varphi} \xi_{\perp}^2(\varphi_+ + \tilde{\varphi}) - \frac{1}{\varphi_-} \left[\int_{-\varphi_-/2}^{\varphi_-/2} d\tilde{\varphi} \xi_{\perp}(\varphi_+ + \tilde{\varphi}) \right]^2 \right\},$$

- Probability for a single-vertex process is given by a double integral over phase variables φ_1 and φ_2 .
- Exchange for **average phase** $\varphi_{\text{av}} = (\varphi_1 + \varphi_2)/2$ and **interference phase** $\varphi = (\varphi_1 - \varphi_2)/2$.
- In the limit that the interference phase is small, the probability is a single integral over a **probability rate**.

Theory and simulations LCFA



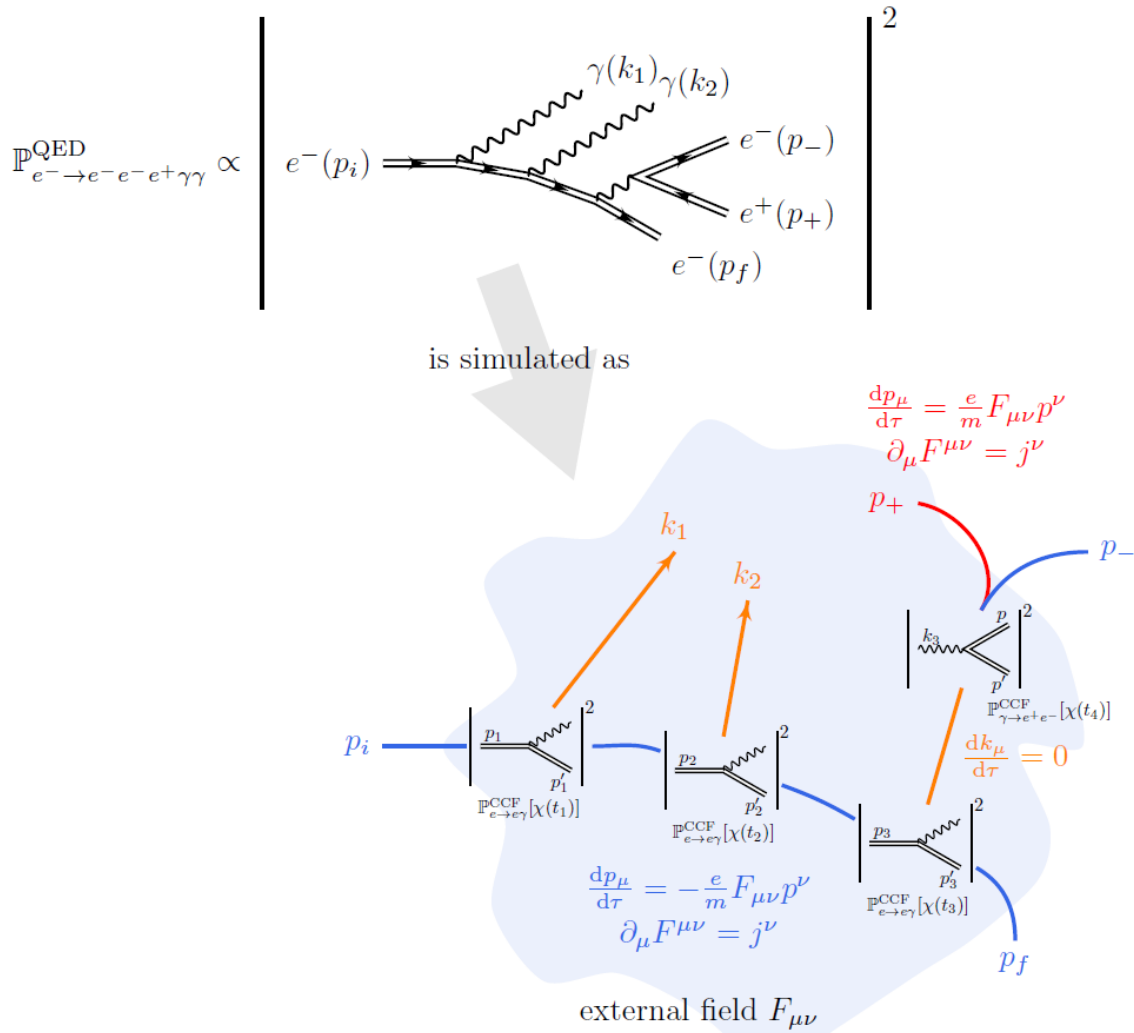
- Ratio of the critical frequency to the cyclotron frequency (harmonic index of the emitted photon):

$$\frac{\omega'}{\omega_c} \approx a_0^3$$

- Characteristic distance over which the photon is emitted:

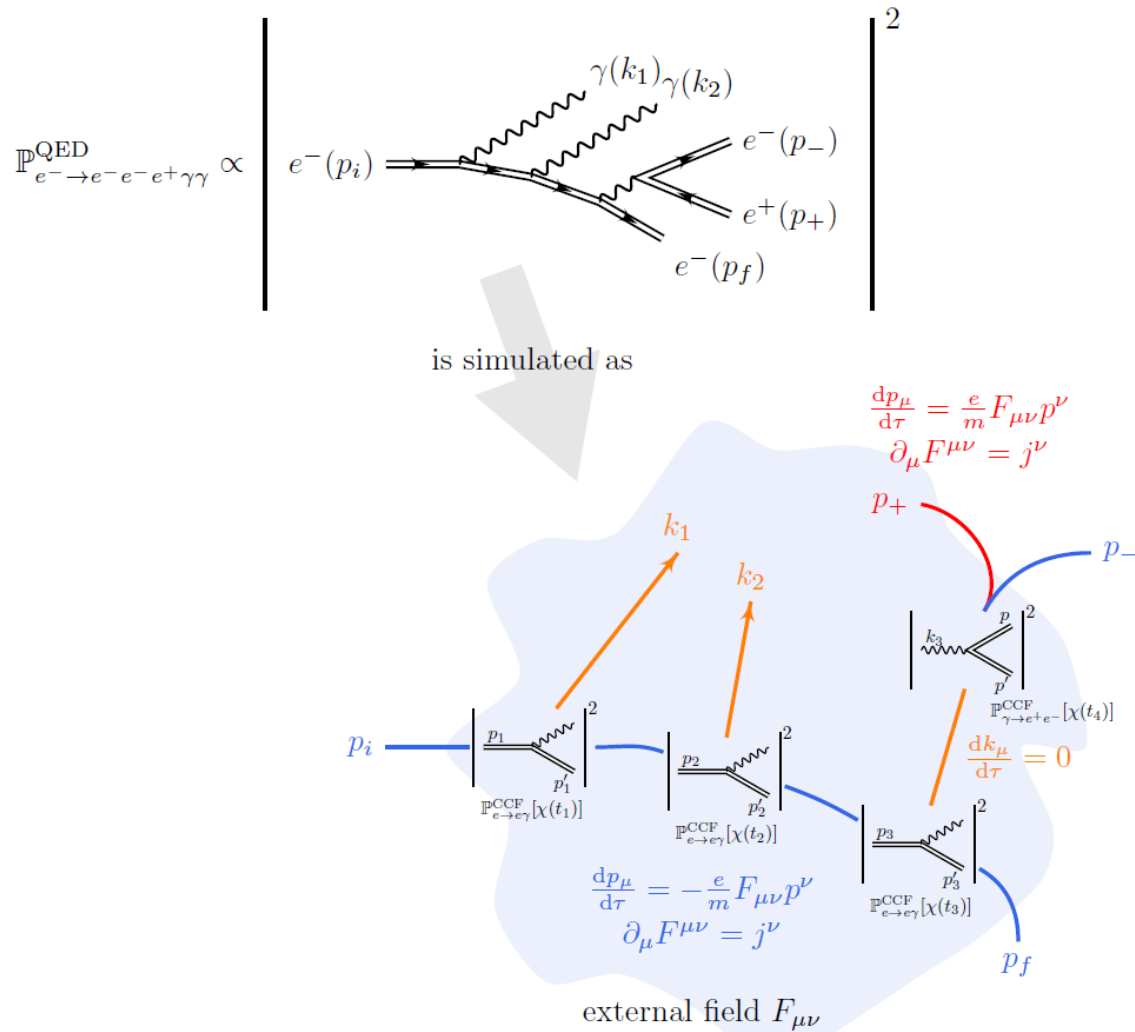
$$\frac{L_f}{C} = \frac{1}{2\pi a_0}$$

Theory and simulations LCFA



- Treat each QED event as occurring instantaneously, with the rates and spectra a function only of the local field invariants χ_e , χ_γ , f , g .
- In most cases, the two invariants $f = (E^2 - B^2)/E_{\text{crit}}^2$ and $g = \mathbf{E} \cdot \mathbf{B}/E_{\text{crit}}^2$ are much smaller than χ , so they can be neglected. The result is the **locally constant (crossed) field approximation**.

Theory and simulations LCFA



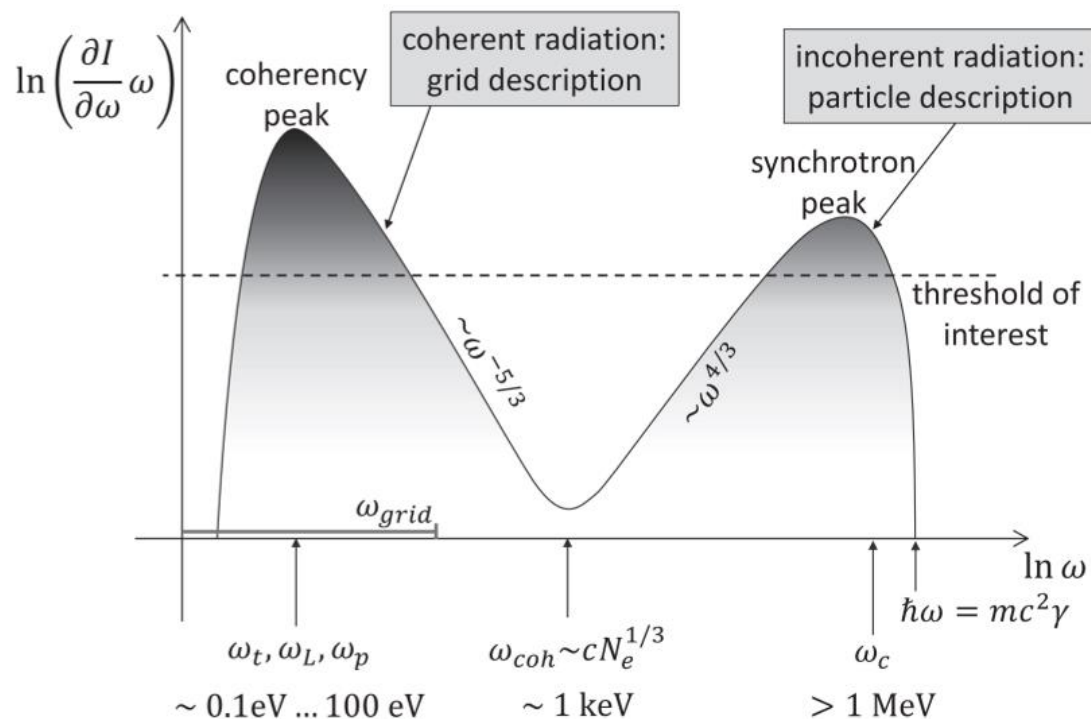
- QED rates in the locally constant (crossed) field approximation + point-like emission events linked by classical trajectories that are determined by Lorentz force.
- Higher-order processes are broken down into a chain of first-order processes.
- Requires $a_0 \gg 1$ (strictly, $a_0^3/\chi \gg 1$) and $\chi^2 \gg f, g$.

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Implementation

Two EM fields



- Separate the electromagnetic field into two components, one at lower frequency, dominated by coherent contributions, and one at higher frequency, dominated by the incoherent.
- Treat the first classically – specified/discretized on a grid (PIC) – and the second as ‘photons’ – uncharged, ballistically propagating particles.

Implementation

Kinetic equations

$$\begin{aligned} \frac{\partial \phi_e}{\partial t} + \frac{\mathbf{p}}{\gamma m} \cdot \frac{\partial \phi_e}{\partial \mathbf{r}} - e \left(\mathbf{E} + \frac{\mathbf{p} \times \mathbf{B}}{\gamma m} \right) \cdot \frac{\partial \phi_e}{\partial \mathbf{p}} = \\ - \phi_e \int w_{e \rightarrow e\gamma}(\mathbf{p}, \mathbf{q}) d^3 \mathbf{q} \\ + \int \phi'_e w_{e \rightarrow e\gamma}(\mathbf{p}', \mathbf{p}' - \mathbf{p}) d^3 \mathbf{p}', \\ \text{photon emission} \\ + \int_0^1 \left[\phi_{\pm} \frac{dP^c}{d\epsilon'} + \phi_{\gamma} \frac{dP^b}{d\epsilon'} \right] d\epsilon, \\ \text{pair creation} \end{aligned}$$

- Time-evolution of the distribution functions $\Phi_e(\mathbf{x}, \mathbf{p})$, $\Phi_{\gamma}(\mathbf{x}, \mathbf{p})$ etc
- Classical dynamics treated in the usual way.
- QED processes incorporated in the form of a collision operator that couples the different particle species. Physics encoded in the differential probability rates.

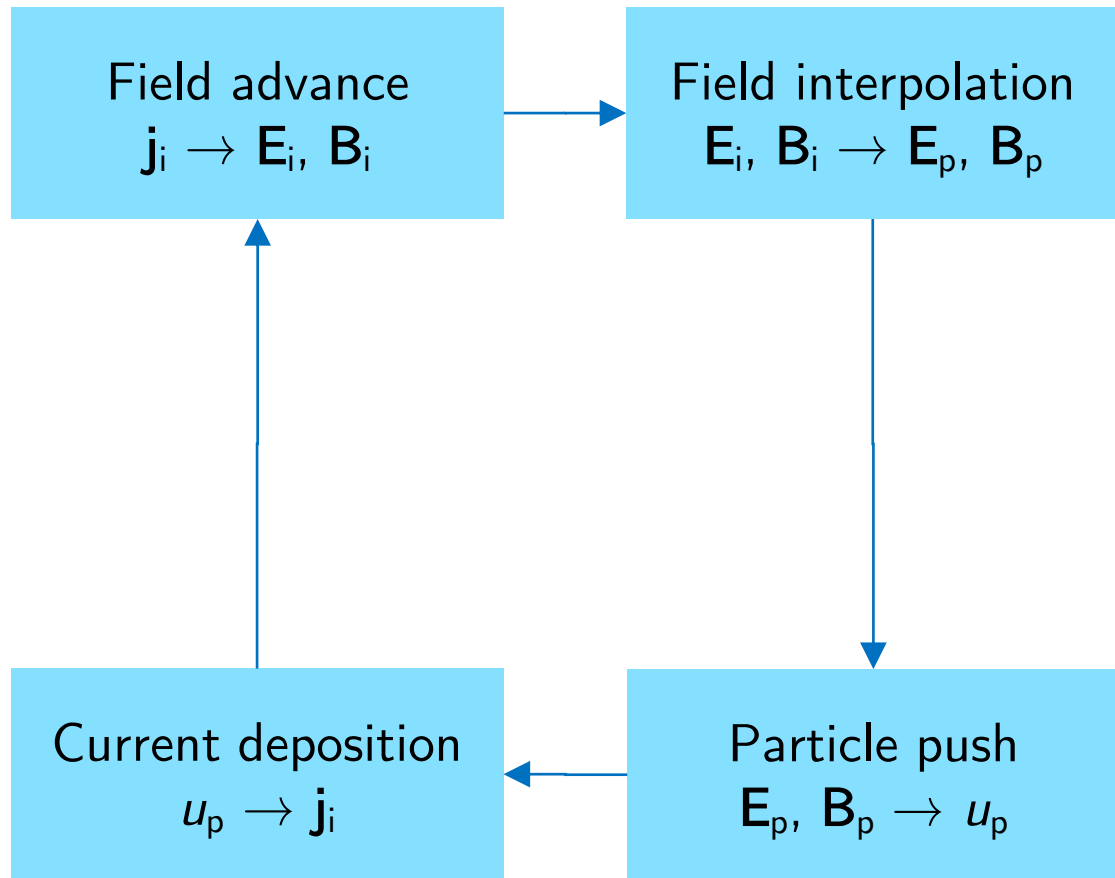
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Kinetic equations

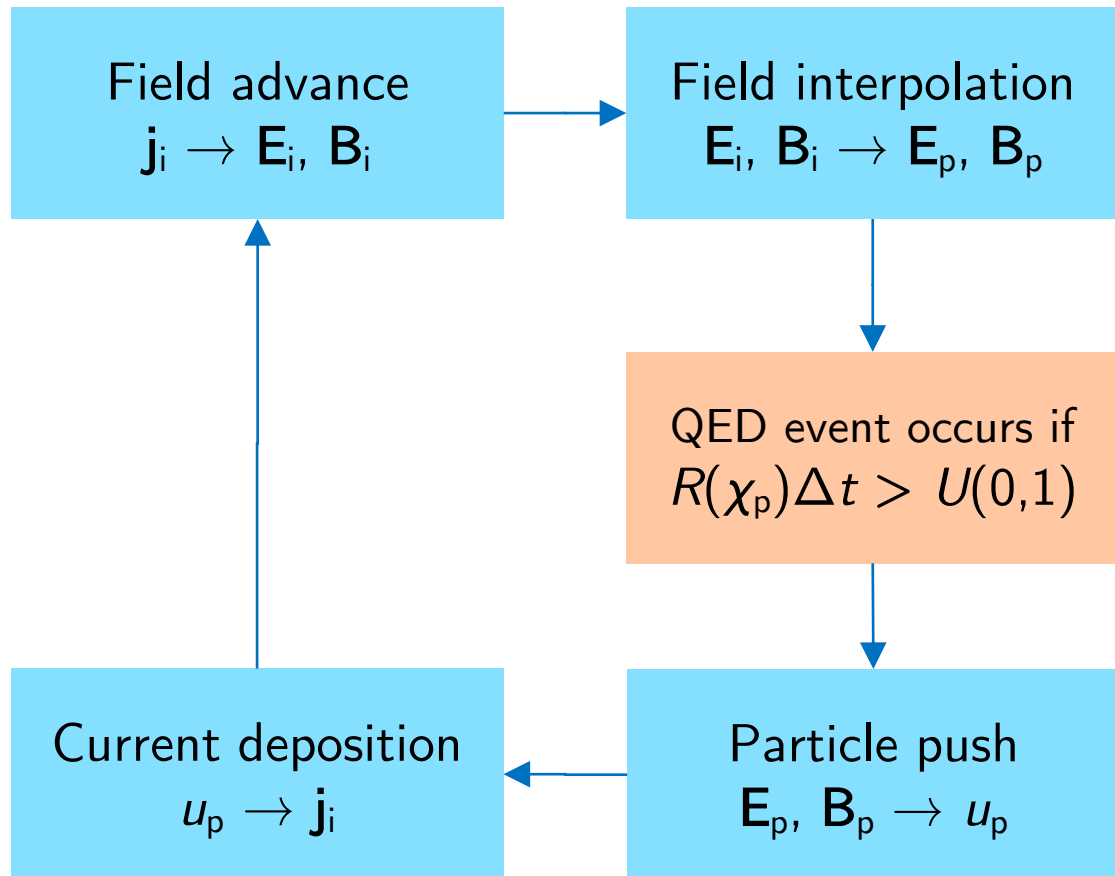
$$\begin{aligned} \frac{\partial \phi_e}{\partial t} + \frac{\mathbf{p}}{\gamma m} \cdot \frac{\partial \phi_e}{\partial \mathbf{r}} - e \left(\mathbf{E} + \frac{\mathbf{p} \times \mathbf{B}}{\gamma m} \right) \cdot \frac{\partial \phi_e}{\partial \mathbf{p}} = \\ - \phi_e \int w_{e \rightarrow e\gamma}(\mathbf{p}, \mathbf{q}) d^3 \mathbf{q} \\ + \int \phi'_e w_{e \rightarrow e\gamma}(\mathbf{p}', \mathbf{p}' - \mathbf{p}) d^3 \mathbf{p}', \\ \text{photon emission} \\ + \int_0^1 \left[\phi_{\pm} \frac{dP^c}{d\epsilon'} + \phi_{\gamma} \frac{dP^b}{d\epsilon'} \right] d\epsilon, \\ \text{pair creation} \end{aligned}$$

Sokolov et al, PRL 105, 195005 (2010)
Elkina et al, PRSTAB 14, 054401 (2011)
Bulanov et al, PRA 87, 062110 (2013)

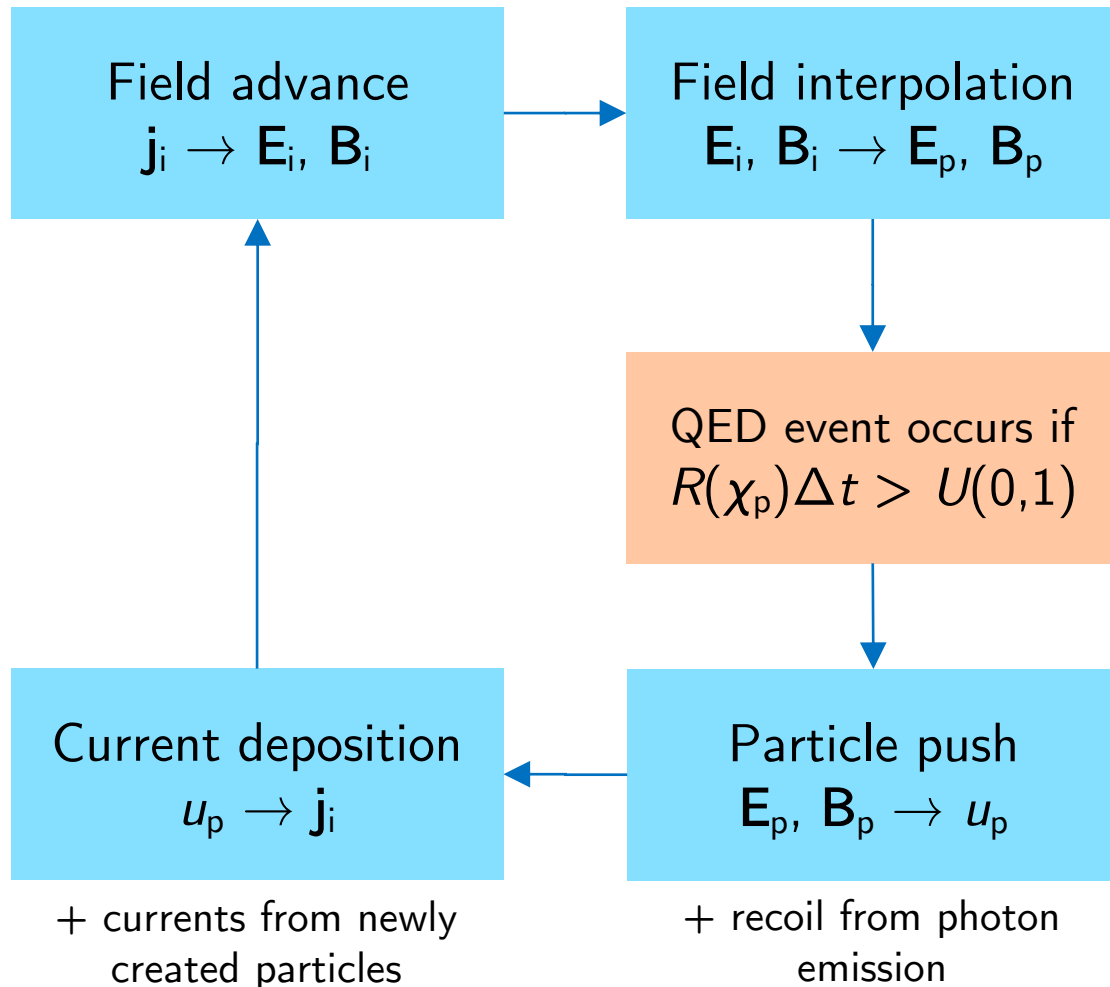
- System can be reduced to a Fokker-Planck equation for quantum RR [Neitz and Di Piazza, PRL 111, 054802 (2013); Vranic et al, NJP 18, 073035 (2016)]
- Stochastic differential equation for electron dynamics [Niel et al, PRE 97, 043209 (2018)]
- In classical limit, Landau-Lifshitz equation (& quantum-corrected power)



- Particle-in-cell codes solve for the classical evolution of the electron (etc) distribution functions, as sampled by 'macroparticles'.

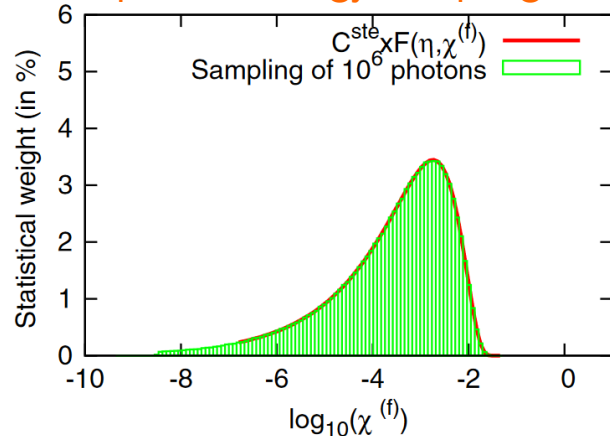


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- Probability rates for all QED processes integrated along macroparticle trajectory.



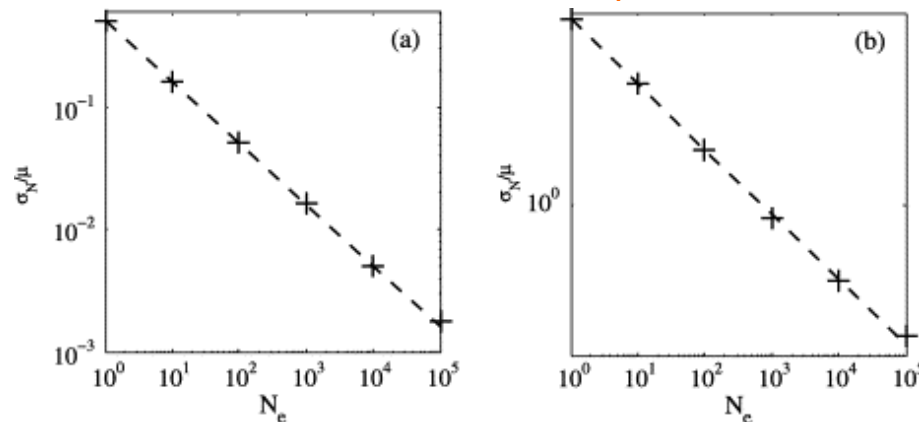
- Particle-in-cell codes solve for the classical evolution of the electron (etc) distribution functions, as sampled by ‘macroparticles’.
- Probability rates for all QED processes integrated along macroparticle trajectory.
- Electrons recoil on photon emission, new electrons and positrons added on photon decay.

photon energy sampling



Duclous et al, PPCF
53, 015009 (2011)

error reduction with macroparticle number



Ridgers et al,
JCP 260, 273
(2014)

- Pseudorandomly determine if event occurs in a single timestep (comparing a dice roll to $W\Delta t$ or integrated optical depth)
- Momenta (spin etc) of newly created particles selected by sampling the differential rates
- Introduces noise due to finite number of sampling points, can be overcome with biasing.

EPOCH

CALDER

Smilei)

P I C  D O R

PICon  

WarpX

Osiris
4.0

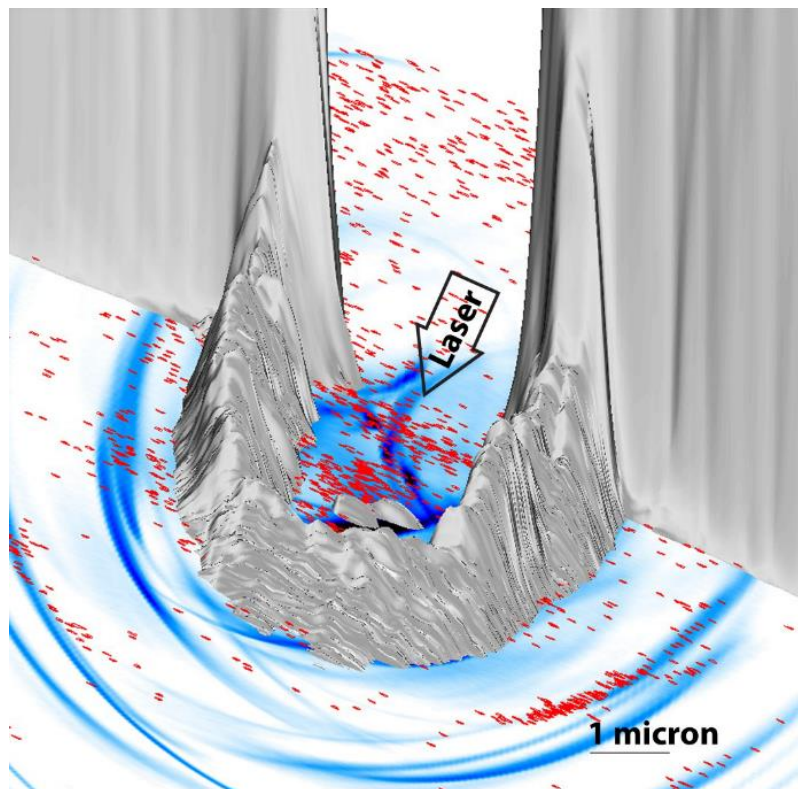
VLPL

QUILL

If your code isn't here and should be, let me know!

Implementation

Laser-driven plasmas



C P Ridgers et al, PRL 108, 165006 (2012)

- Next-generation laser facilities producing intensities 10^{23} W/cm² (ELI, Apollon etc).
- Critical density pair plasmas formed in laser-foil, laser-laser, laser-gas interactions.
- Coupling between classical plasma dynamics and nonlinear QED in fields with complex structure.



Implementation

Laser-driven plasmas

Classical plasma
dynamics

Quantum processes

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Implementation

Laser-driven plasmas

Classical plasma
dynamics

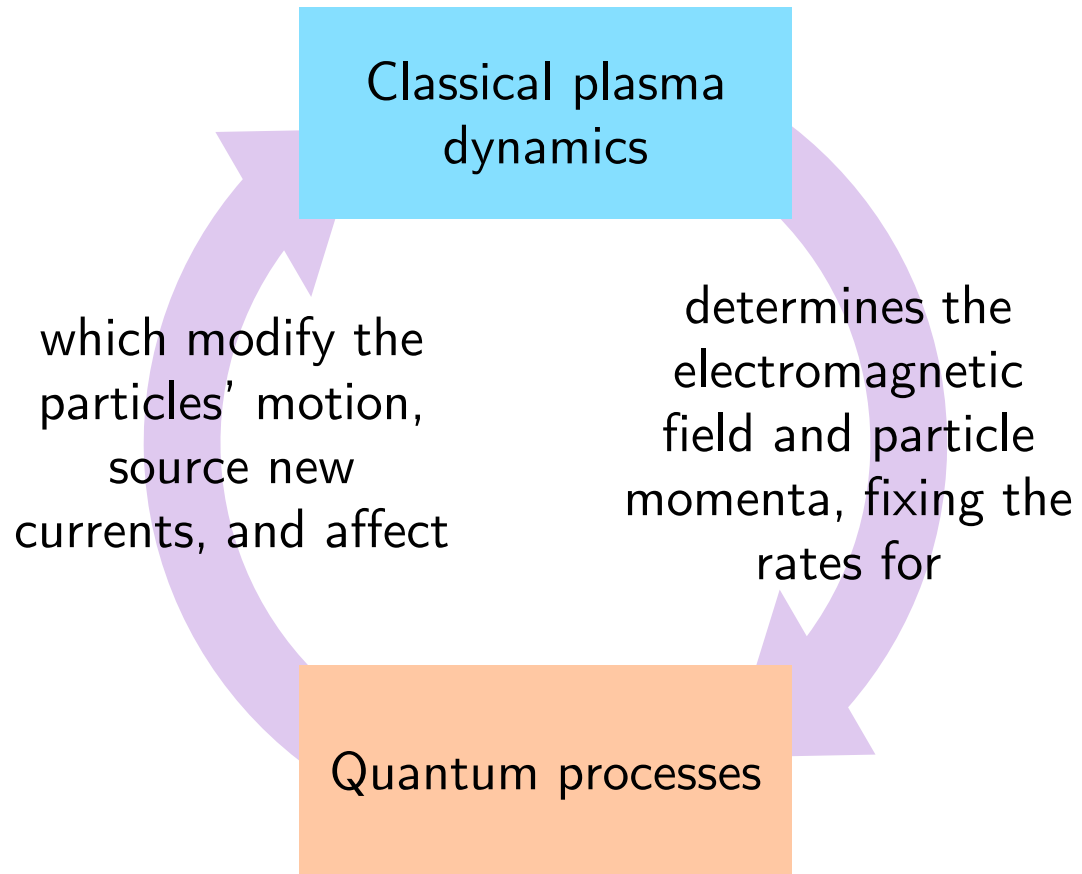
determines the
electromagnetic
field and particle
momenta, fixing the
rates for

Quantum processes

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Implementation

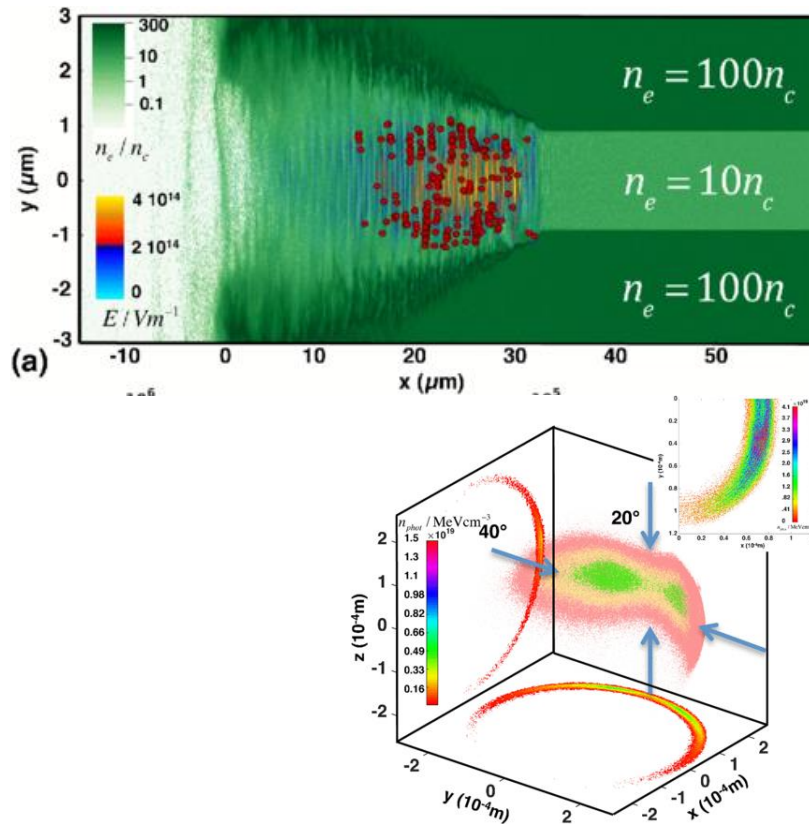
Laser-driven plasmas



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- Coupling between classical plasma dynamics and nonlinear QED in fields with complex structure.

Implementation

Single laser-driven plasmas

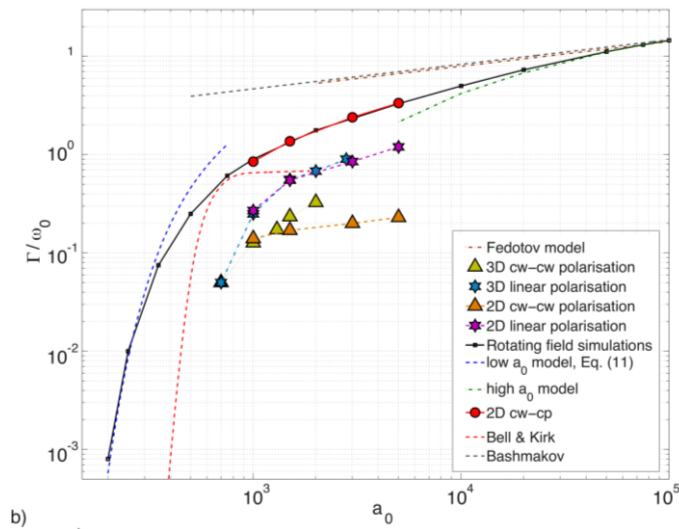
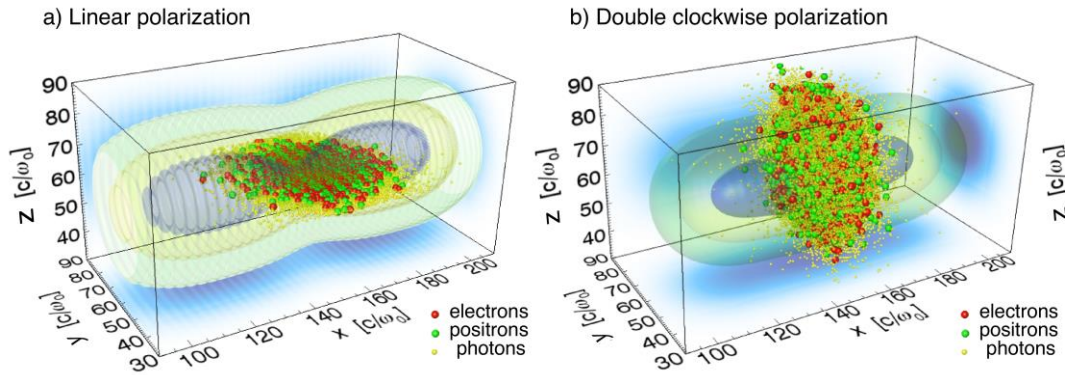


D J Stark et al, PRL 116, 185003 (2016)

- e.g. laser irradiation of a compound target creates a quasistatic magnetic field that guides electron acceleration.
- Combination of the laser and plasma fields gives a quantum parameter $\chi = 0.1$, leading to high-energy photon emission.
- 10s TW emitted in >10 MeV photons, collimated within tens of degrees, at $a_0 = 200$.

Implementation

Dual laser-driven cascades



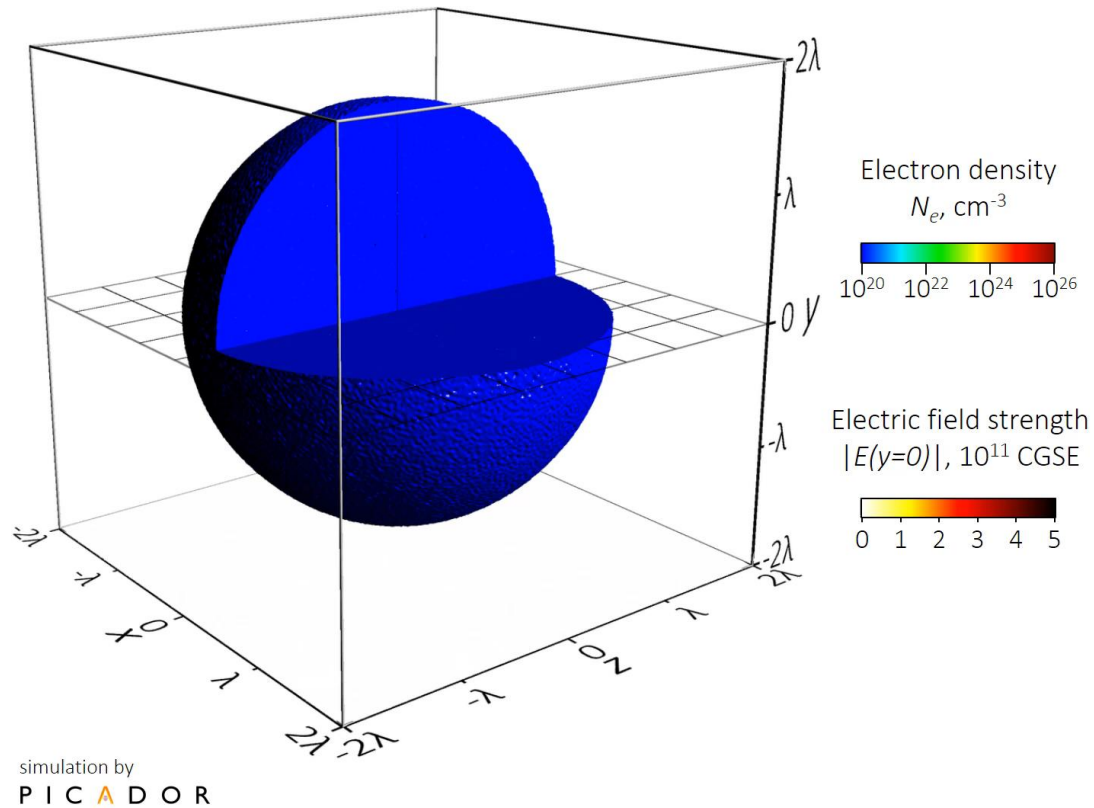
T Grismayer et al, PRE 95, 023210 (2017)

- e.g. exponential growth of the positron density when two counterpropagating lasers accelerate seed electrons to high energy.
- Formation of critical-density electron-positron plasmas that absorb and convert laser energy to γ rays.
- Threshold intensity for cascade initiation (linear polarization) predicted to be $7 \times 10^{23} \text{ W/cm}^2$.



Implementation

Multiple laser-driven plasmas

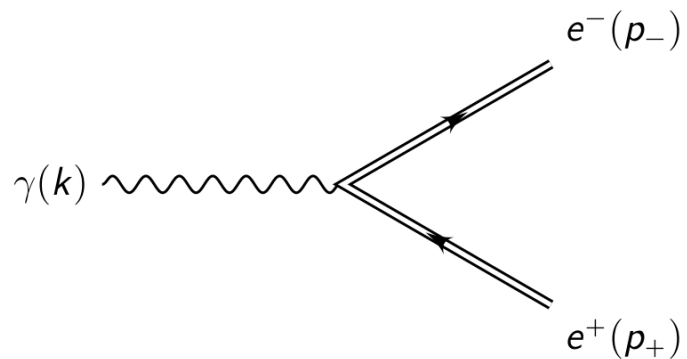
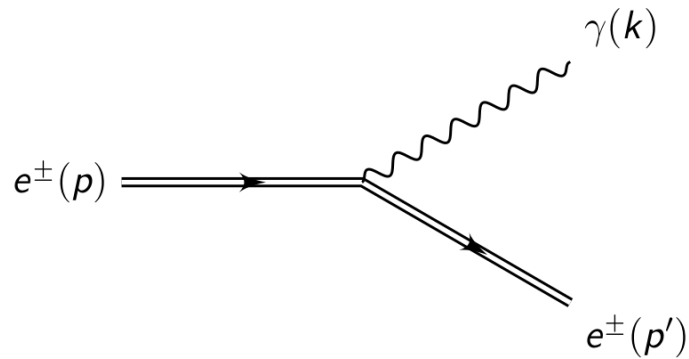


A Gonoskov et al, PRX 7, 041003 (2017)

- Multi-beam configurations mean higher intensity is reached for lower input power.
- 4π irradiation of a plasma target with 40 PW of laser power (divided among 12 pulses) traps electrons on special trajectories.
- These electrons oscillate back and forth along the field axis, leading to collimated emission of GeV photons.

Implementation

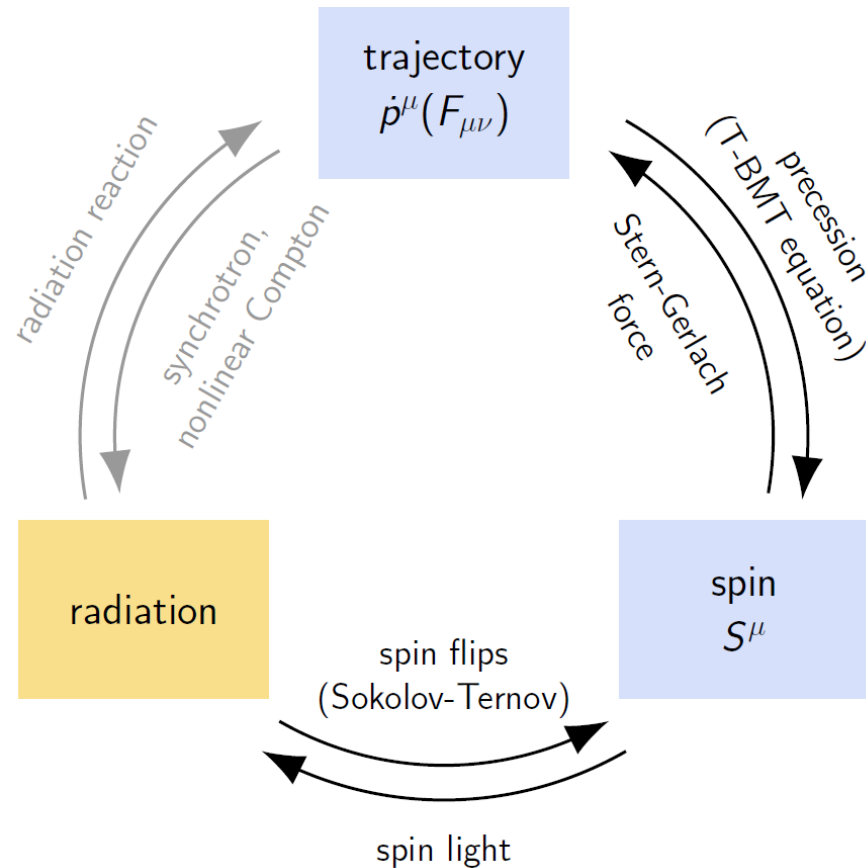
More physics to come



- “Standard” implementation include both first-order processes using spin and polarization averaged rates.
- Can these processes be modelled more accurately?
- What other processes are there?

Implementation

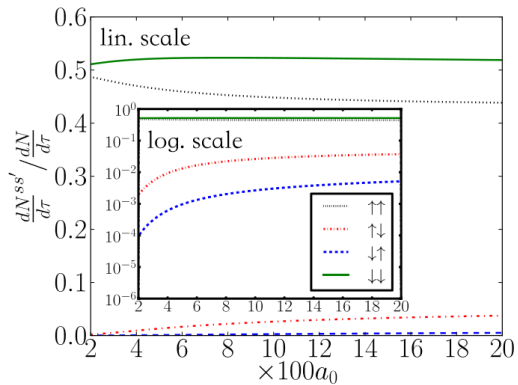
Spin and polarisation



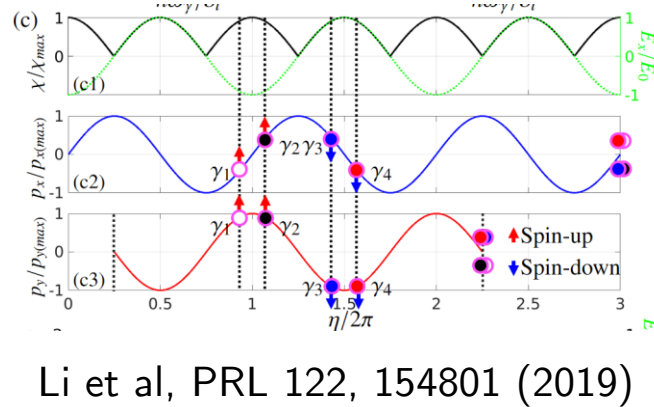
- Electron/positron spin and photon polarisation affect the probability rates and spectra for QED processes.
- QED processes and EM fields affect the spin and polarisation in turn. (A plane EM wave does not change the asymptotic value of the spin in the absence of radiation.)

Implementation

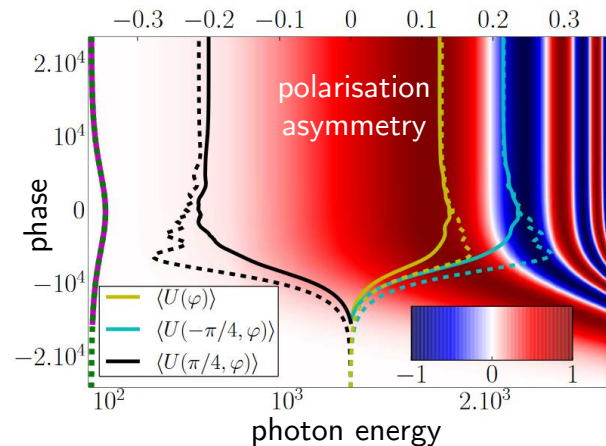
Spin and polarisation



Del Sorbo et al, PRA 96,
043407 (2017)



Li et al, PRL 122, 154801 (2019)

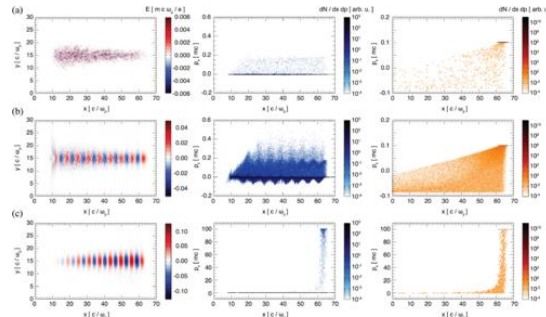
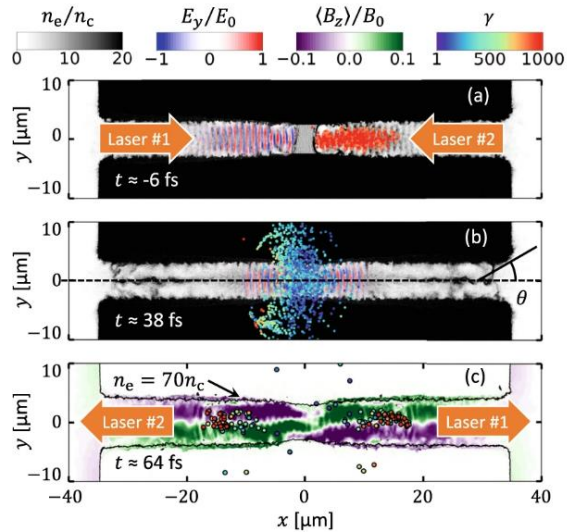


King and Elkina, PRA 94, 062102 (2016)

- Photon emission: need spin-resolved emission rates, defined w.r.t. a suitable basis, and a way to transport the electron spin between events (BMT equation)
- Pair creation: need polarisation-resolved pair creation rates, polarisation change (of, e.g. LP photons in a CP background) modelled by associating a refractive index with the EM field

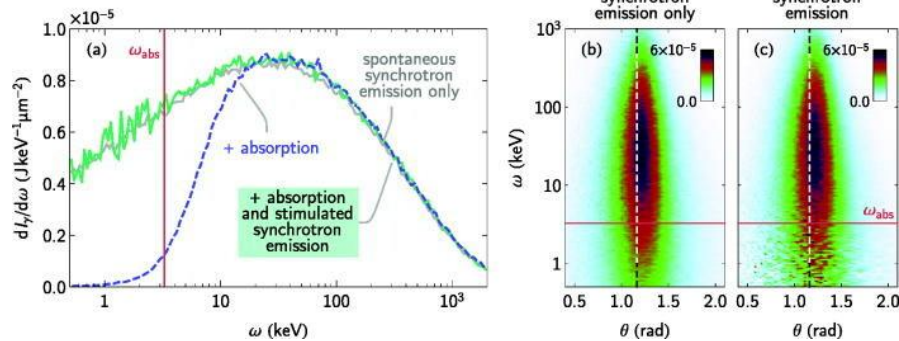
Implementation

Particle-particle interactions



Del Gaudio et al, PRL 125, 265001 (2020)

He et al, Comm Phys 4, 139 (2021)

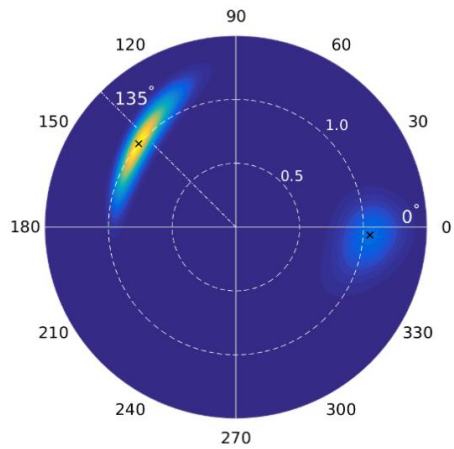


Blackburn et al, PoP 28, 053001 (2021)

- Collisional processes in strong-field environments
- Linear Breit-Wheeler ($\gamma\gamma \rightarrow e^+e^-$): postprocessing of simulated photon distributions
- Linear Compton scattering ($e\gamma \rightarrow e\gamma$): direct evaluation of cross sections
- Photon absorption/stimulated emission ($e\gamma \rightarrow e, e\gamma \rightarrow e\gamma\gamma$)

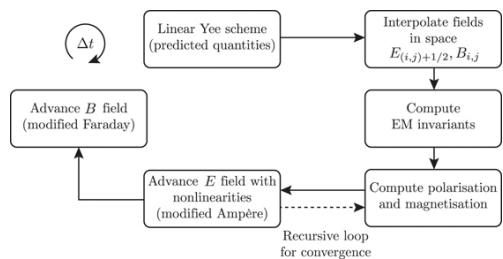
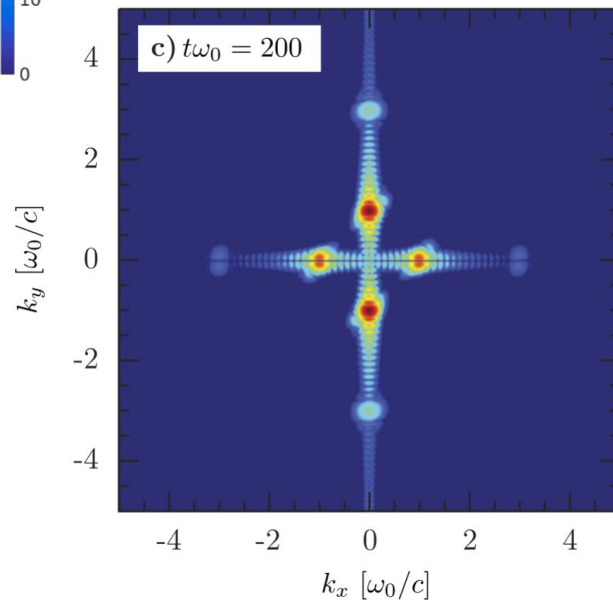
Implementation

Nonlinear vacuum



distribution of
signal from
photon-photon
scattering

vacuum HHG



- Vacuum emission picture: weak-field limit, use Euler-Heisenberg effective Lagrangian to obtain polarisation current for driving lasers, solved using Fourier methods [e.g. [Blinne et al, PRD 99, 016006 \(2019\)](#)]
- Adapted Yee scheme (FDTD) to solve nonlinear Maxwell equations in Osiris. Field solver iterated to convergence with additional P and M [[Grismayer et al, NJP 23, 095005 \(2021\)](#)]

Implementation

Numerical challenges

emission time (1/rate), compared to CFL timestep (n cells per wavelength):

$$\frac{\Delta t_{QED}}{\Delta t_{CFL}} \sim \frac{10n}{a}$$

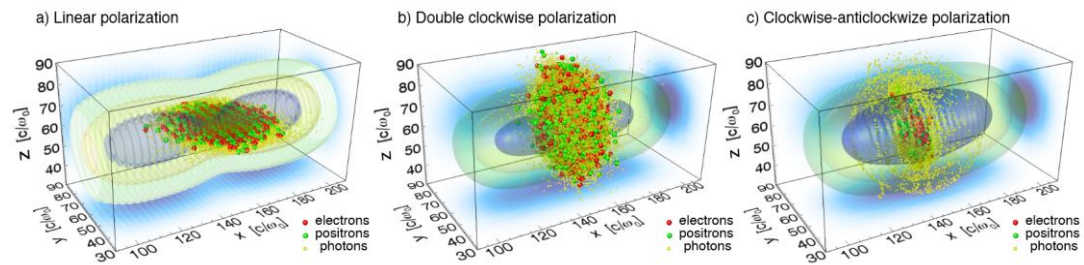
emission time (1/rate), compared to Debye length ($/c$):

$$\frac{\Delta t_{QED}}{\Delta t_D} \sim \frac{100}{a} \left(\frac{n_e}{n_c} \right)^{1/2} \left(\frac{m_e c^2}{k_b T_e} \right)^{1/2}$$

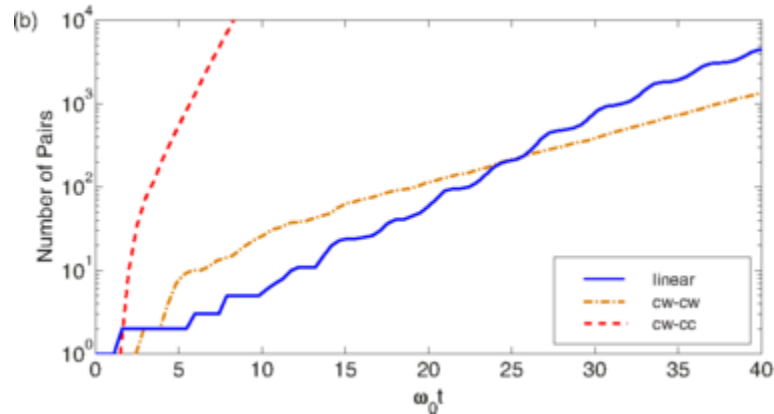
- New constraints on the timestep: the probability of a single QED event should be much smaller than one.
- **Sub-cycling**: if the field does not change very much over the timestep (which it shouldn't), do several particle pushes + tests for emission/pair creation to occur.

Implementation

Numerical challenges

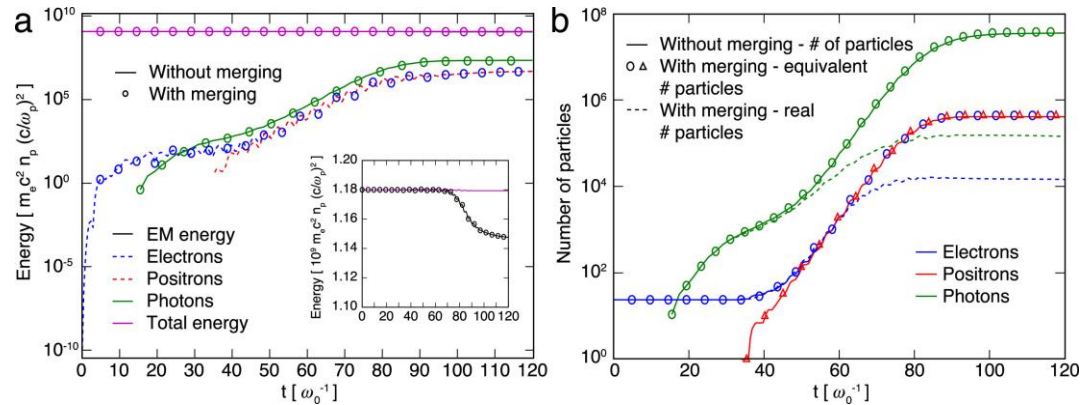


- Growth in macroparticle number: several photons per initial electron in a shower cascade, many pairs per electron in an avalanche cascade

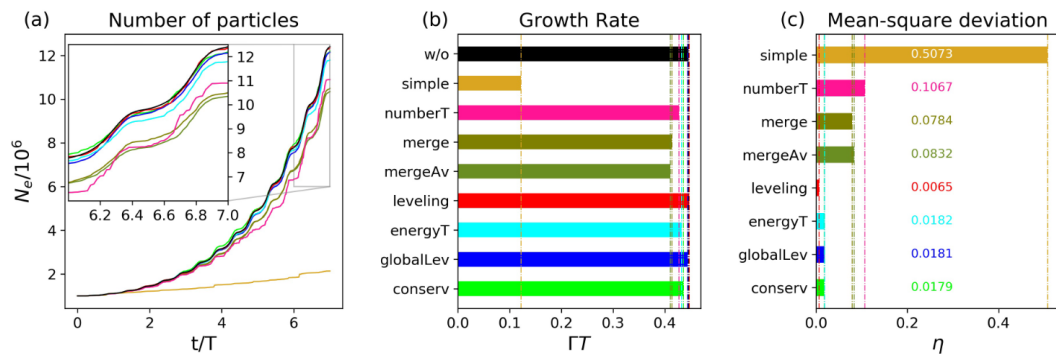


Implementation

Numerical challenges



Vranic et al, CPC 191, 65 (2015)

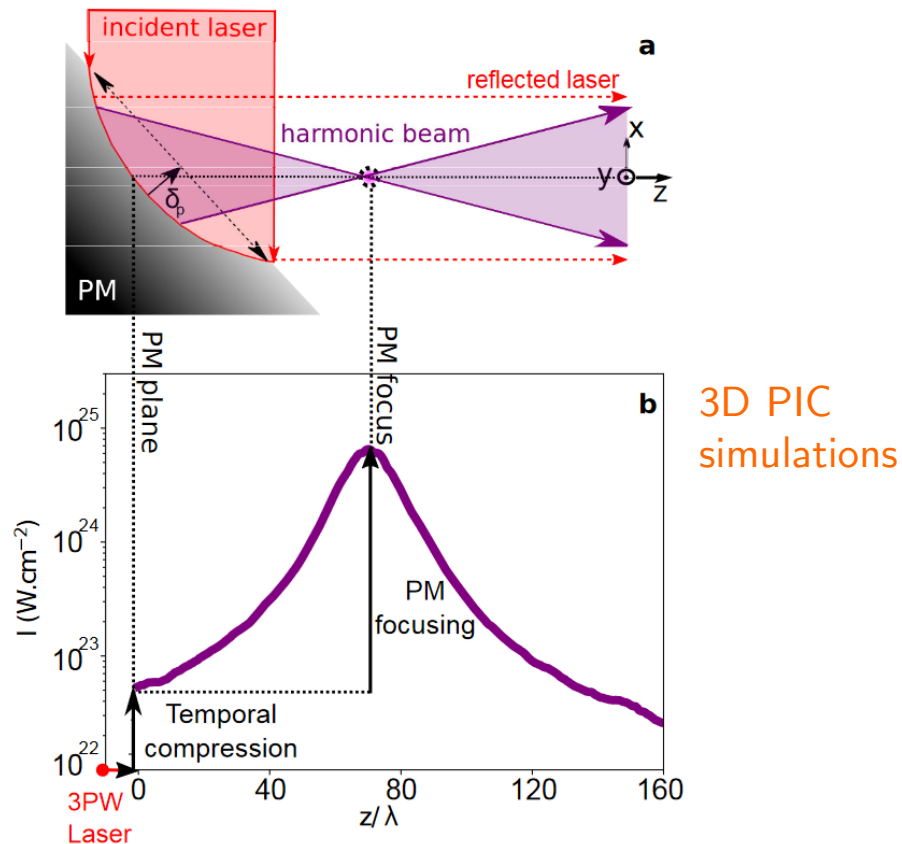


Muraviev et al, CPC 262, 107826 (2021)

- Growth in macroparticle number: several photons per initial electron in a shower cascade, many pairs per electron in an avalanche cascade
- **Merging**: take particles close in phase space and combine into two new particles (conserves energy/momentum and weight)
- **Thinning**: removal of macroparticles and increase in weight of others

Implementation

Numerical challenges

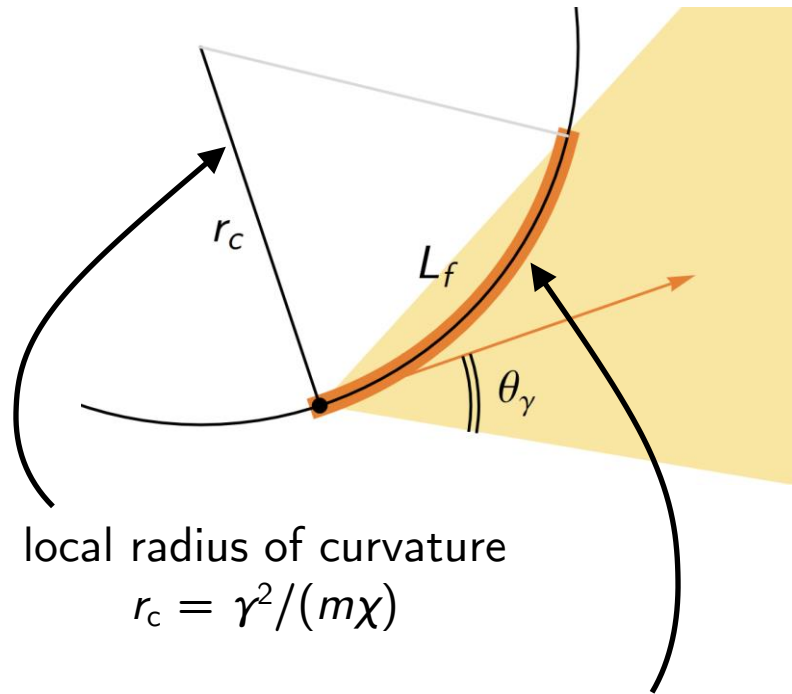


- Propagation of tightly focused laser pulses affected by non-ideal dispersion of FDTD field solvers
- Parallelised pseudo-spectral methods available [e.g. [Vay et al, JCP 243, 260 \(2013\)](#)]
- High spatial resolution needed, especially for focusing of high harmonics from a plasma surface

Overview

- Motivation. What are we interested in? And why simulate at all?
- What does the theory tell us? From strong-field QED to probability rates
- How is this implemented? The Monte Carlo approach, particle-in-cell codes and more, physics processes, numerical challenges
- How are we doing? Benchmarking, new approaches, experiments
- What's next?

How small is the formation length?



local radius of curvature
 $r_c = \gamma^2 / (m\chi)$

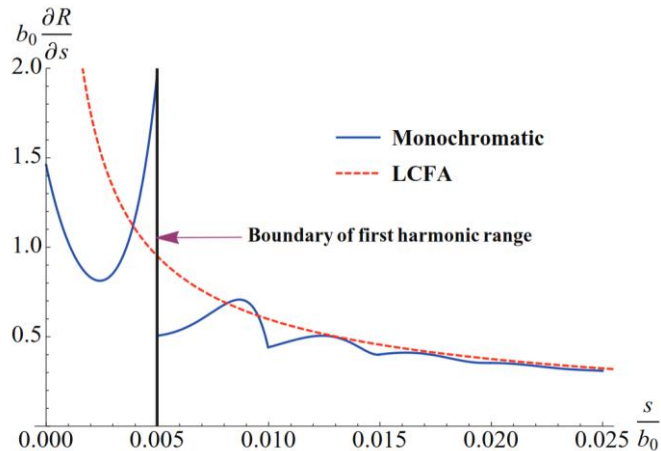
emission angle fixes **overlap between
 electron and photon trajectories**

- QED processes included via a local approximation, i.e. formation length assumed to be vanishingly small.
- If θ is RMS angle of entire spectrum:

$$L_f \simeq \frac{\lambda}{2\pi a_0}$$

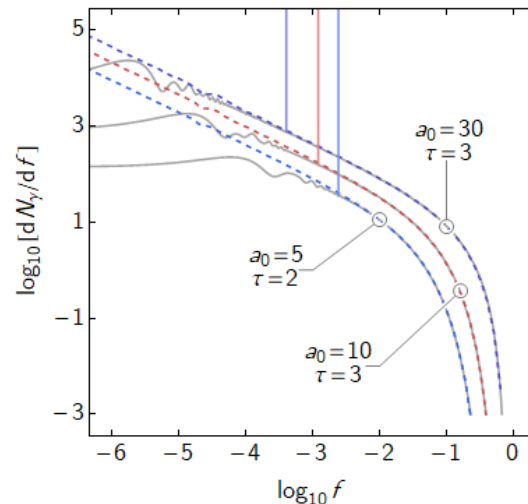
- Or angle at energy $f = \omega / (\gamma m)$:

$$L_f \simeq \frac{\chi_e^{1/3} \lambda}{2\pi a_0} \left(\frac{1-f}{f} \right)^{1/3}$$



flat-top PW:
Harvey et al, PRA 91,
013822 (2015)

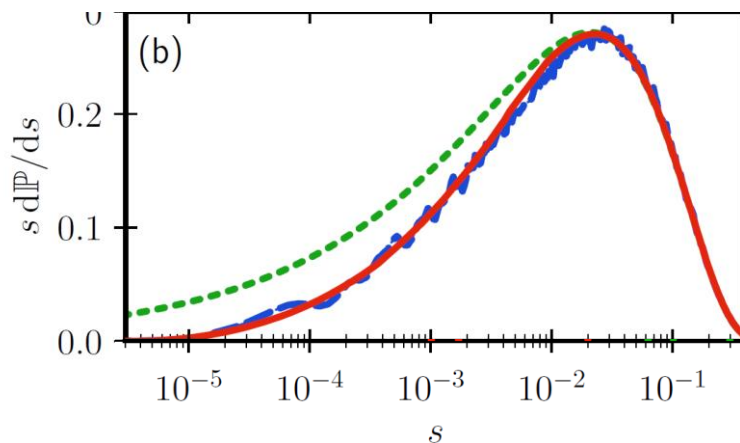
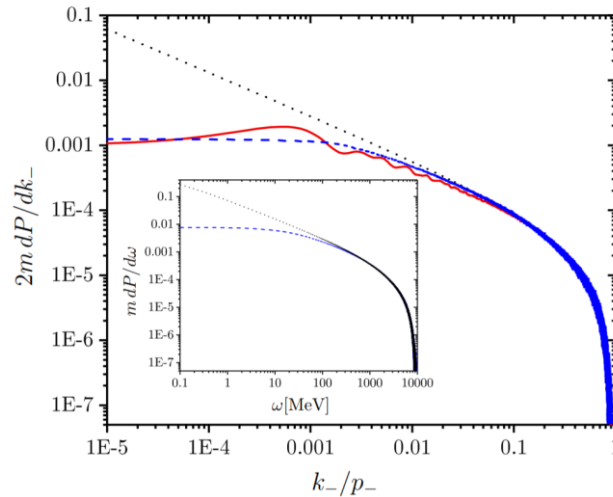
\cos^2 pulsed PW:
Blackburn et al, PoP 25,
083108 (2018)



- Expect the error to be large for a_0 not large and small photon energies (LCFA rate diverges as $\omega^{-1/3}$).
- Formation length becomes comparable to laser wavelength right at the first nonlinear Compton edge:

$$f_C \simeq \frac{2\chi_e}{a_0^3}$$
- Include neglected interference contributions?

QED
LCFA
improved

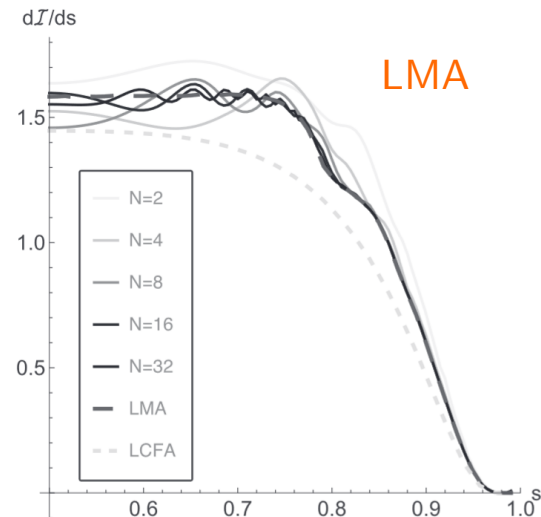
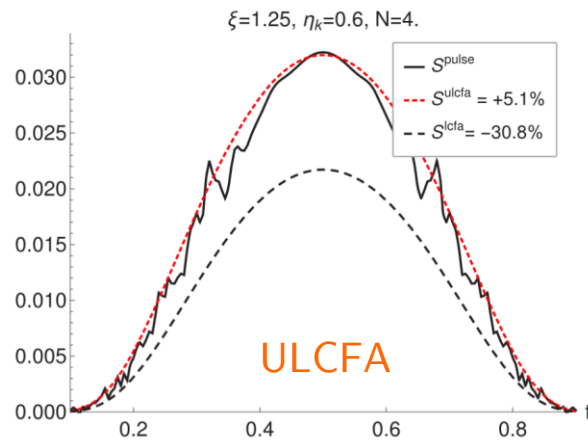


QED
LCFA
improved

- When about to emit a photon, verify its formation length is small enough, otherwise switch to a linear Compton rate – low f spectrum tends to a constant, finite value [Di Piazza et al, PRA 99, 022125 (2019)].
- Include lowest order (in $1/a_0$) correction to the LCFA, i.e. the effect of a field gradient [Ilderton et al, PRA 99, 042121 (2019)].

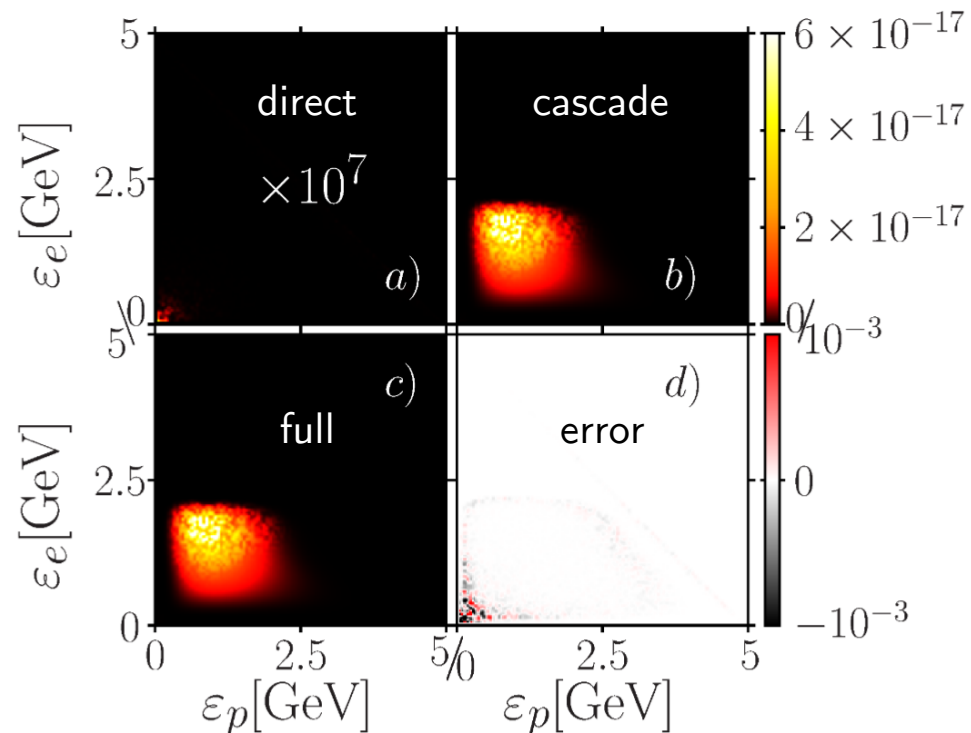
Benchmarking

Interference corrections: pair creation



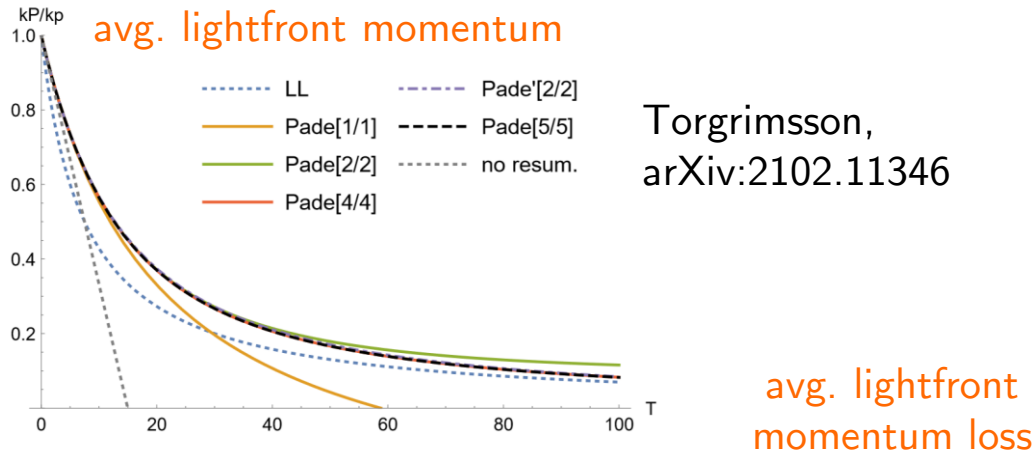
- Uniform LCFA: include field gradient corrections “inside the Airy function” [King, PRA 101, 042508 (2020)].
- For sufficiently “plane-wave-like” fields, use rates for a monochromatic PW rather than a CCF [see Ben King’s talk on Monday, also Heinzl et al, PRA 102, 063110 (2019)]. Already compared against experimental data in E144 [see description in Bamber et al, PRD 60, 092004 (1999)].

Benchmarking Higher order processes

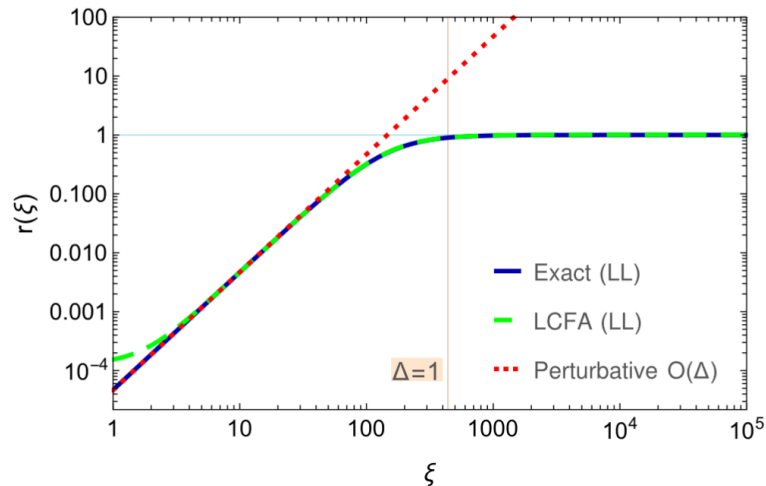


- How well does “factorisation” work? (Splitting a high-order process into a sequence of first-order processes.)
- No direct benchmarking with simulations exist, but there are theoretical calculations.
- “Cascade” contribution for trident (photon emission followed by pair creation) accurate to 0.1% at $a_0 = 50$ and electron energy = 5 GeV.

Benchmarking Higher order processes



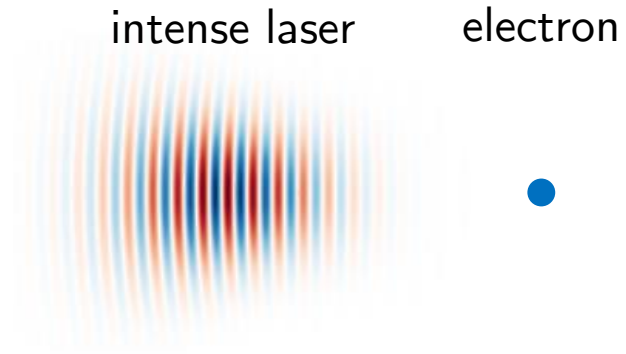
Heinzl et al,
PRL 127,
061601 (2021)



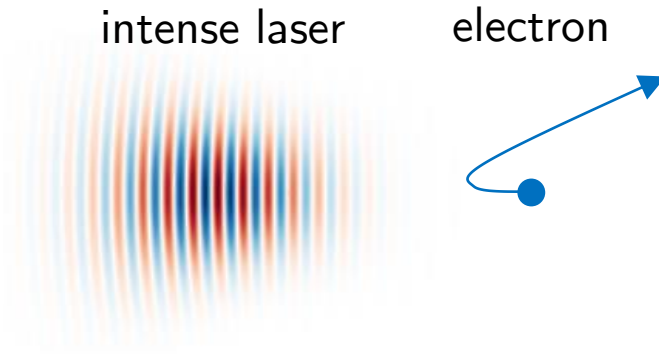
- “Resummed” theoretical calculations available for classical and quantum RR
- Emission of many photons and loop contributions (latter neglected in simulations)
- Are there spectral signatures of these contributions?

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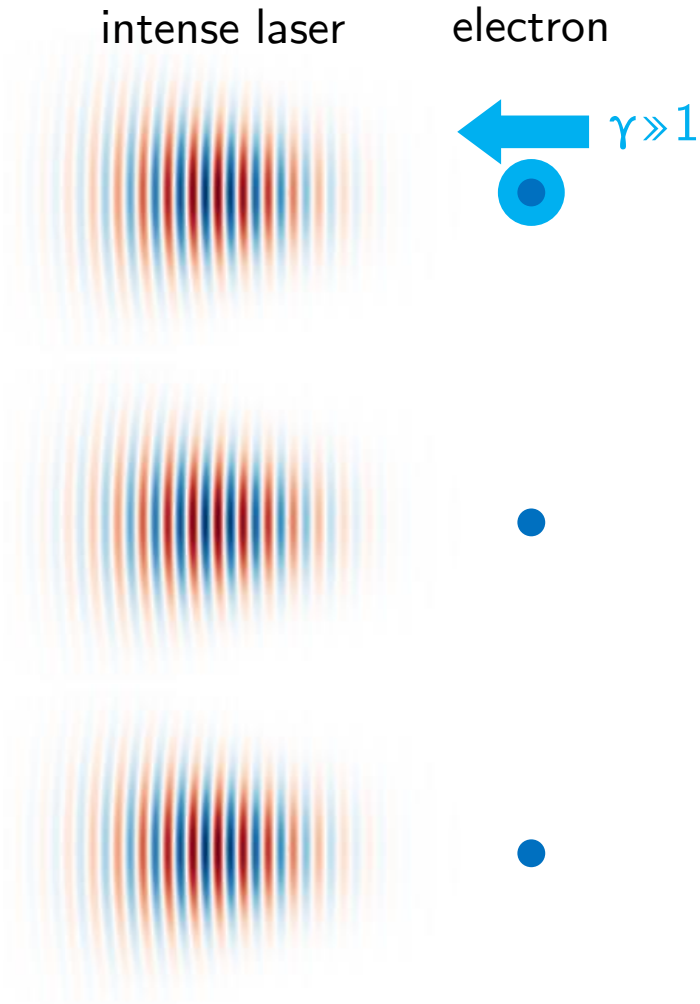
- RR effects (classical or quantum), pair creation etc, require energetic particles to be embedded in strong EM fields.



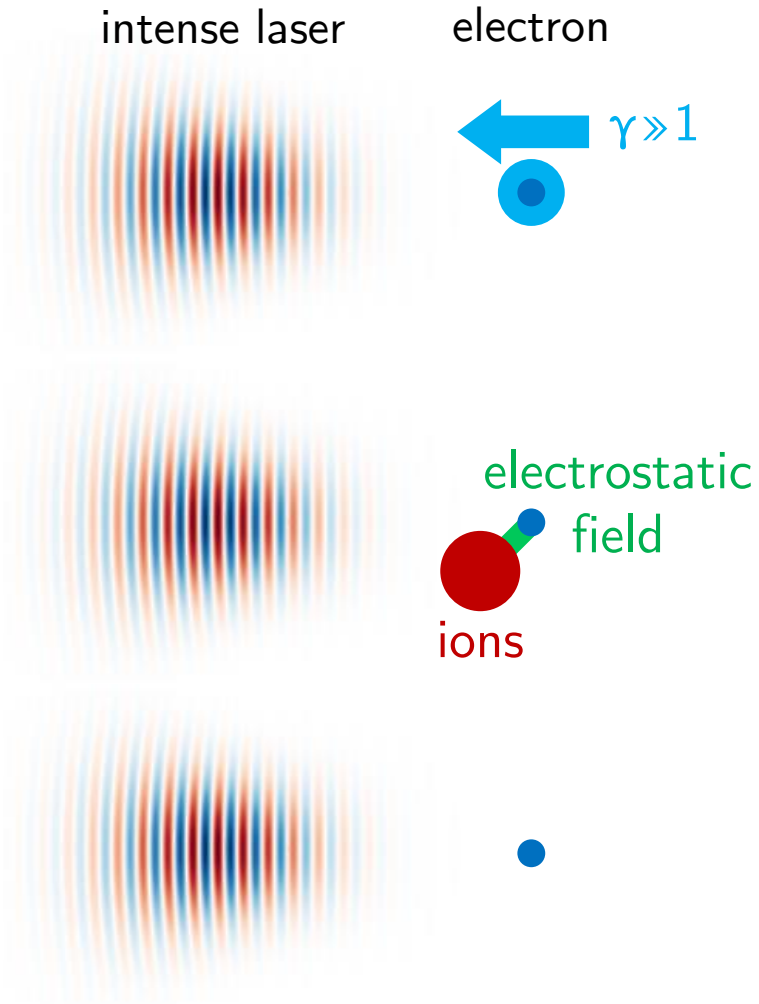
- RR effects (classical or quantum), pair creation etc, require energetic particles to be embedded in strong EM fields.
- But strong spatiotemporal gradients in a short, focussed pulse can expel electrons from the region of high intensity.



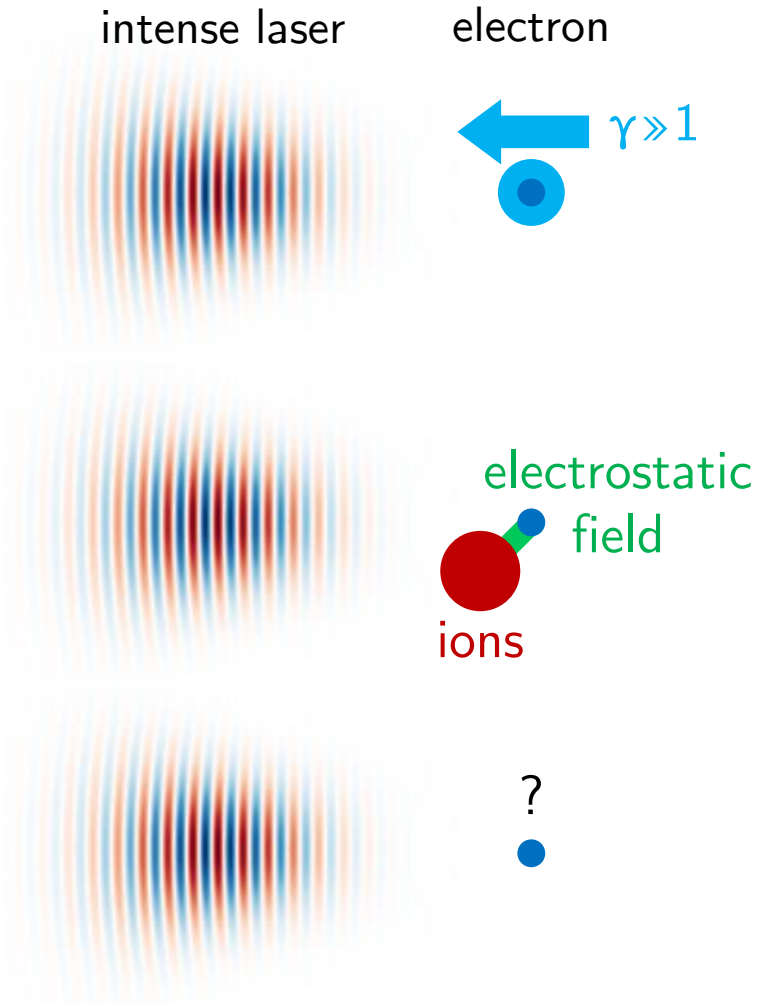
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- Three typical interaction scenarios.



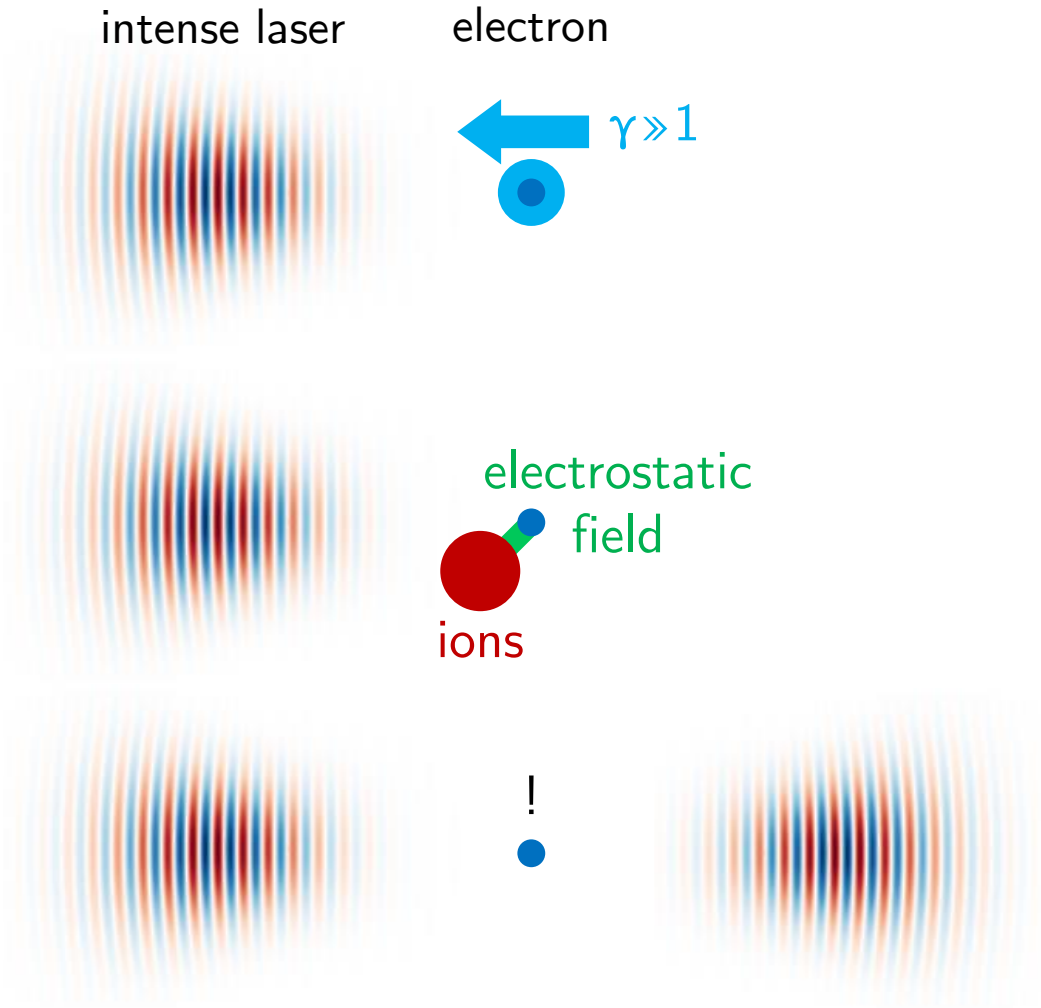
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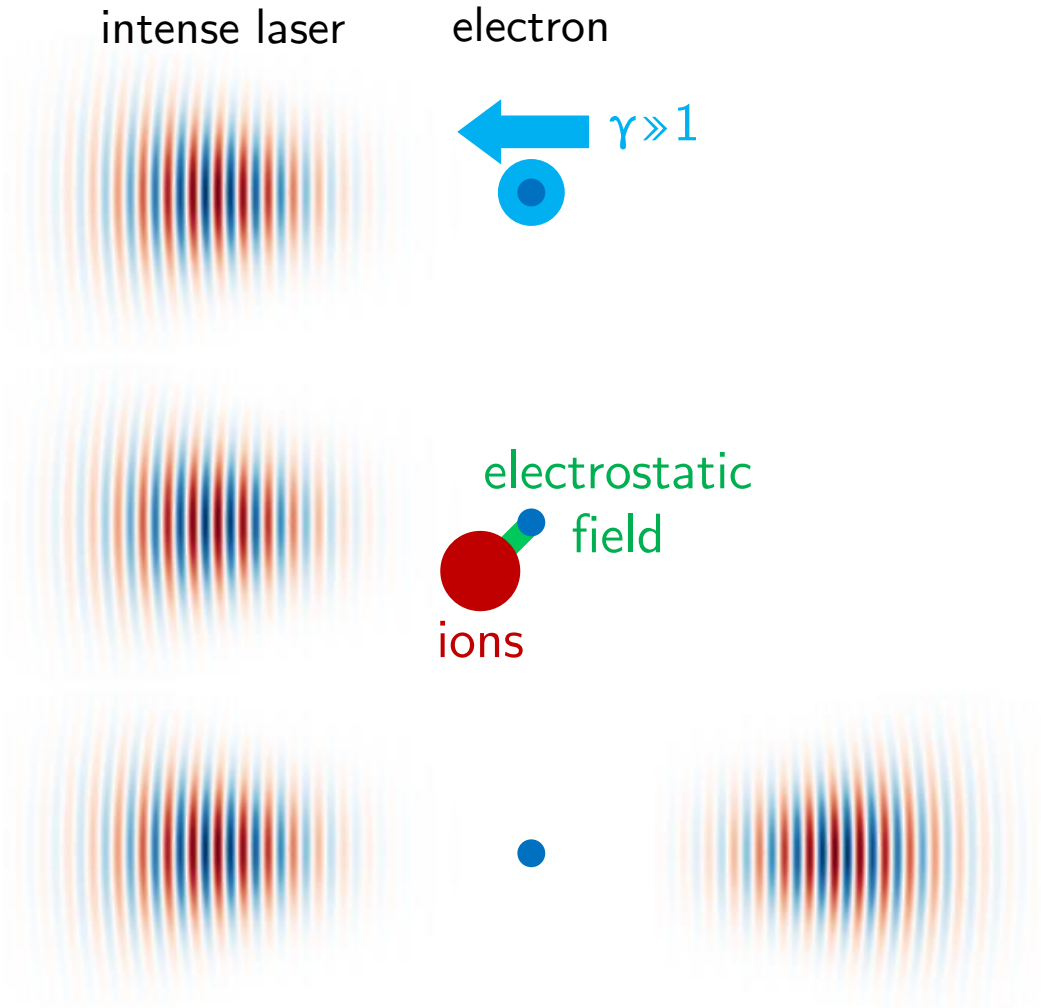
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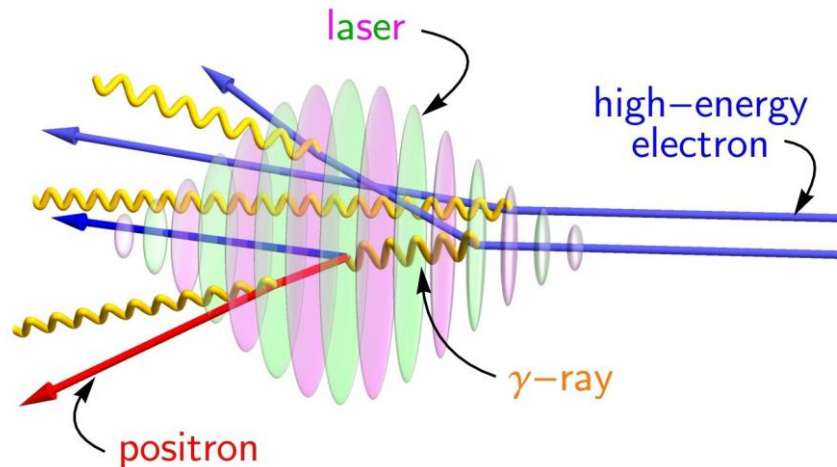
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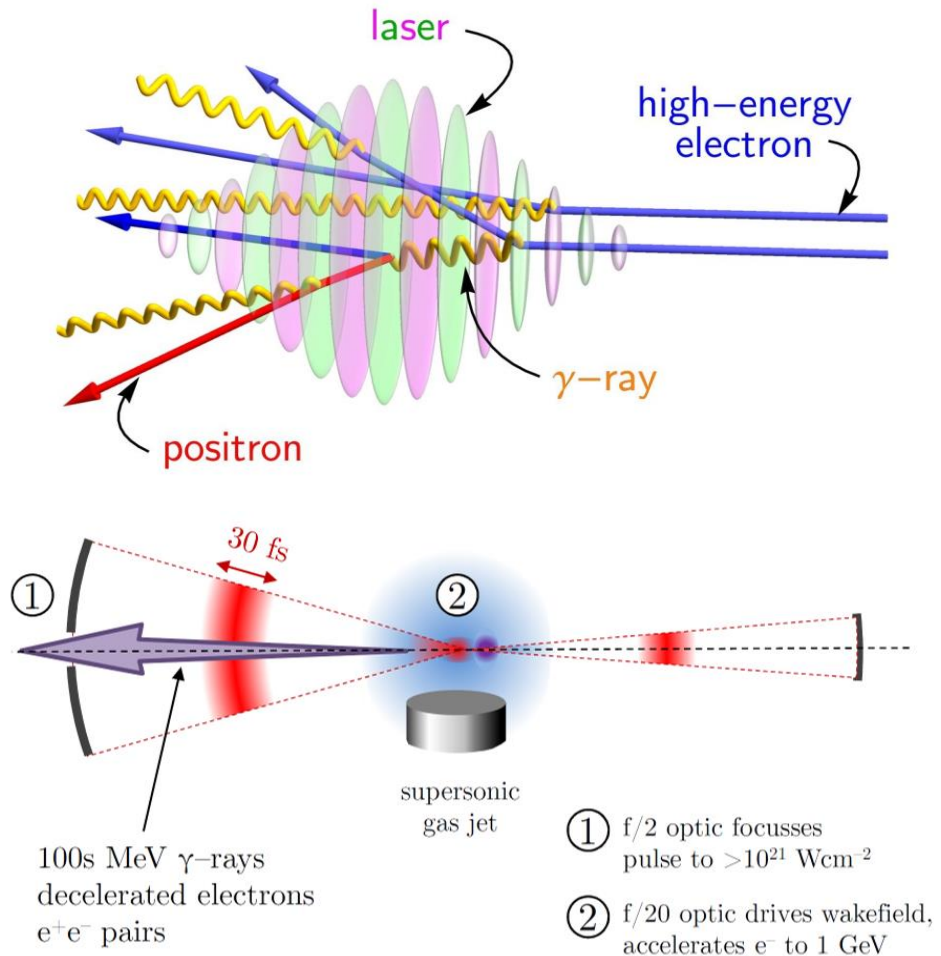
- RR effects (classical or quantum), pair creation etc, require energetic particles to be embedded in strong EM fields.
- But strong spatiotemporal gradients in a short, focussed pulse can expel electrons from the region of high intensity.
- Three typical interaction scenarios: **laser-particle beam**, **laser-plasma** and **laser-laser**.



- Accelerating the electron beam, before it interacts with the laser, allows us to reach the regime $\chi > 1$ at lower intensity.
- SLAC experiment E144: 50 GeV electrons + ps laser with $a_0 = 0.4$ gives $\chi = 0.3$, nonlinear effects in photon emission and pair creation observed [Bula et al, PRL 76, 3116 (1996); Burke et al, PRL 79, 1626 (1997)].

Current experiments

Laser-driven particle beams

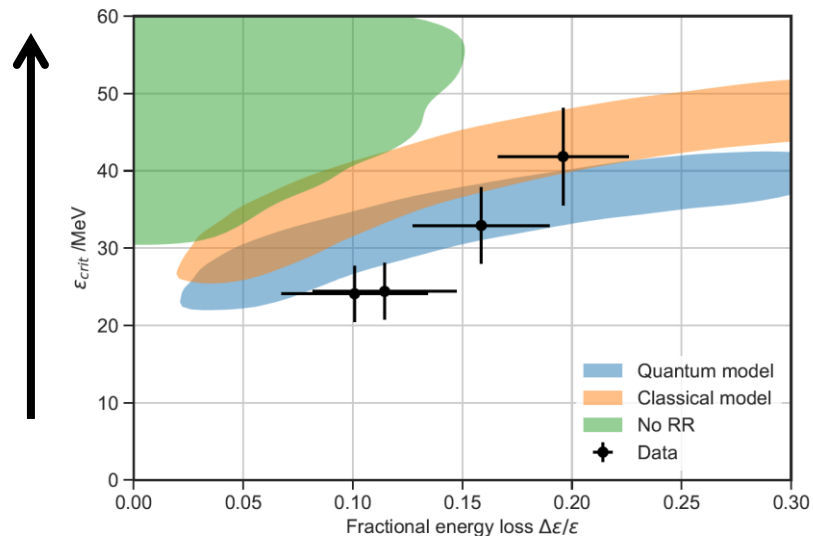


- With higher laser intensities available, we could reach a similar χ with much lower electron energies, as well as the radiation-reaction regime $R_c > 1$.
- In the “all-optical” configuration, one laser is used to drive a wakefield, accelerating the electron beam; the other acts as the target. This exploits the small size of the electron beam and inherent synchronization of the lasers.

Current experiments

Laser-driven particle beams

harder gamma spectrum



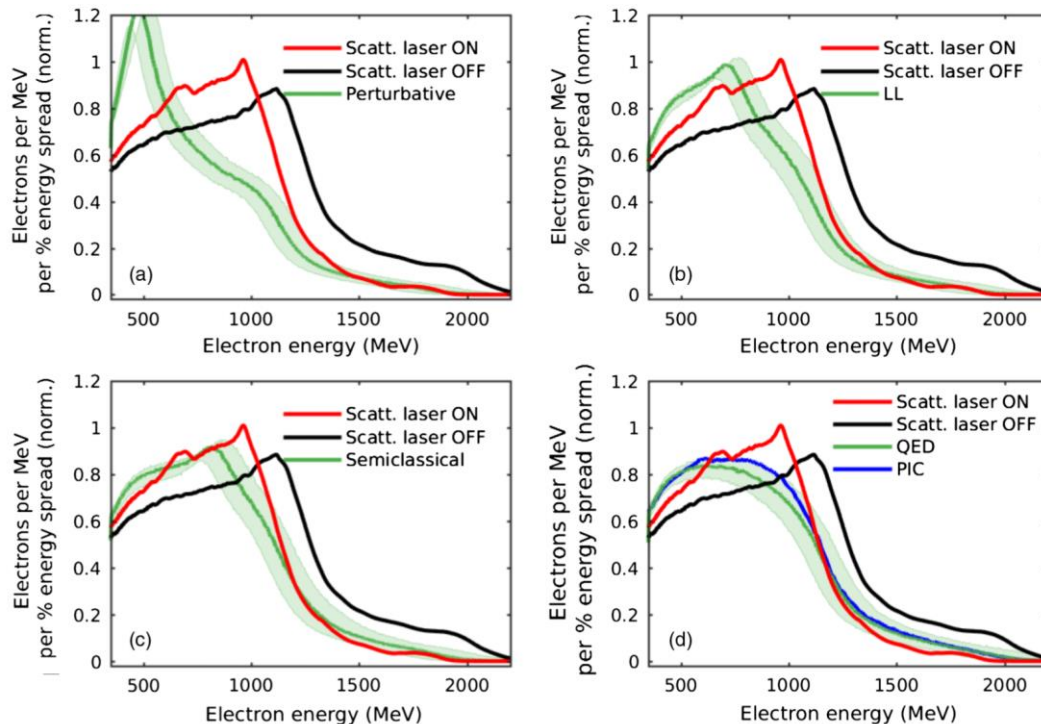
J M Cole et al, PRX 8, 011020 (2018)

- 4 shots with CsI signal significantly above background.
- Hardest gamma rays associated with lowest energy in electron beam: indicative of a radiation reaction process.
- Data inconsistent with **neglect of RR**. **Classical RR** overpredicts critical energies. **Quantum RR** slightly better, but not distinguishable beyond 1 sigma.



Current experiments

Laser-driven particle beams

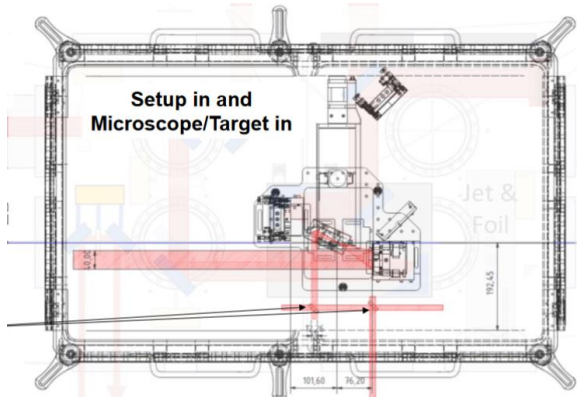


K Poder et al, PRX 8, 031004 (2018)

- Comparison of predicted and measured electron energy spectra for various models.
- $R^2 = 87\%$ for classical (Landau-Lifshitz) vs 92% (96%) for stochastic (deterministic) quantum RR.
- Quantum corrections present, but agreement lacking. Failure of the approximations in simulations?
Characterization of initial conditions?

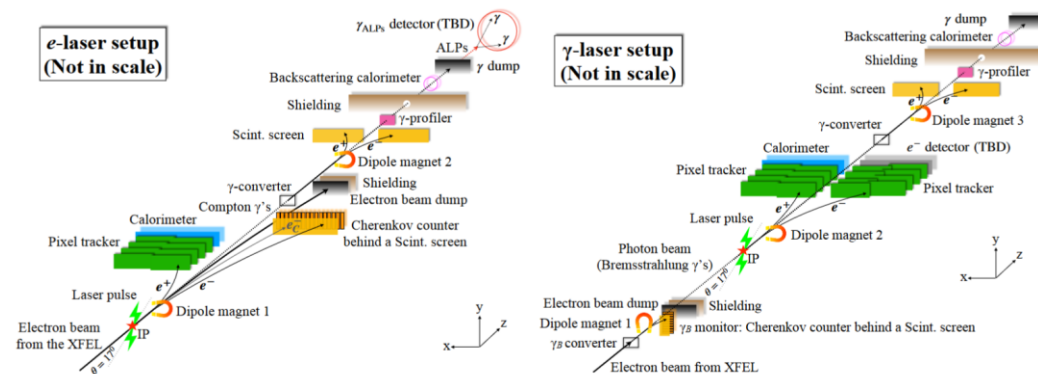
Current experiments

What's next?



Meuren, E320 collaboration

- Increased electron beam energy and laser intensity → stronger quantum effects in RR, observation of nonlinear pair creation.

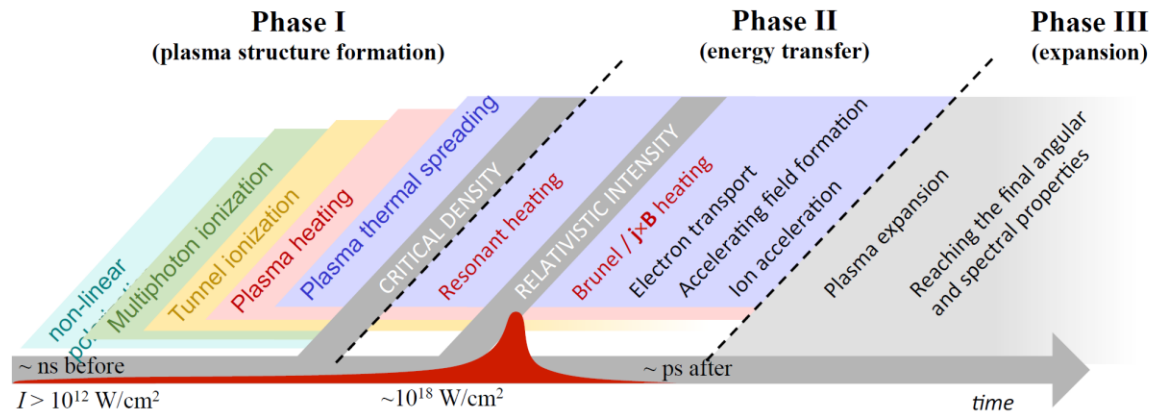


- Combination of a high-intensity laser and a conventional accelerators. Precision tests of strong-field QED.
- How good are our predictions?

LUXE: Abramowicz et al, arXiv:2102.02032

Current experiments

Error budgeting



Atoms: TDSE, ionization
theoretical/empirical rates

Plasma: Vlasov equation,
MHD, other kinetic
approaches

Plasma: Particle-In-Cell
- Maxwell's eq. (FFT, FDTD+)
- Particles motion (relativistic solvers)

Particles:
- Maxwell's eq. (FFT, FDTD+)
- Particles (ballistic propagation)

- Laser-plasma interactions contain rich dynamics over a wide range of timescales.
- Simulate to match or interpret experimental results? (Why not both?)
- Precision experiments with electron beams and/or high rep-rate lasers.
- What is the error we make in simulations and what contributes to it?



“It’s dangerous to make predictions, especially about the future.”

-- Baseballer and philosopher Niels Bohr/Danish physicist Yogi Berra [subs: please check]



“It’s dangerous to make predictions, especially about the future.”

-- Baseballer and philosopher Niels Bohr/Danish physicist Yogi Berra [subs: please check]

... but we’re going to do it anyway.

Thank you!