KATARZYNA KRAJEWSKA
Institute of Theoretical Physics, Faculty of Physics UW

Threshold effects in electron-positron pair creation from the vacuum

---

ExHILP, Jena, Germany

13 September 2021

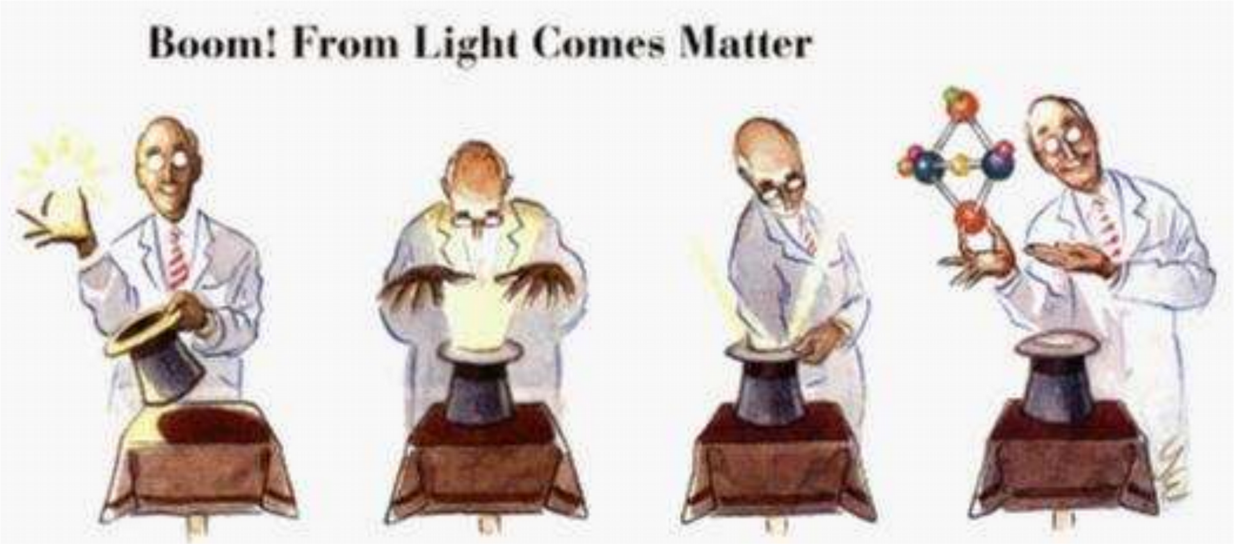
UNIVERSITY
OF WARSAW

Content

Introduction

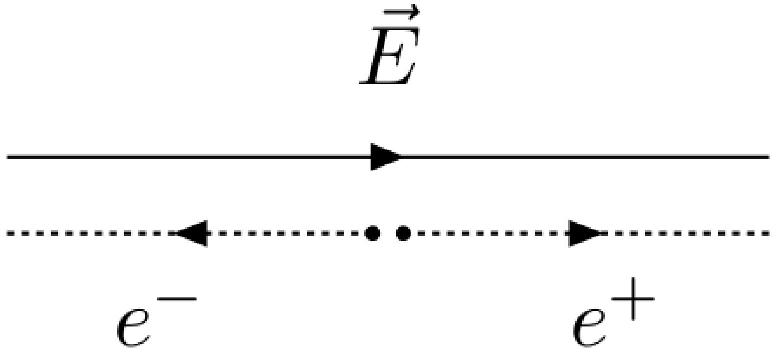
Theoretical formulation

Threshold effects



Introduction

Schwinger process



Schwinger critical field

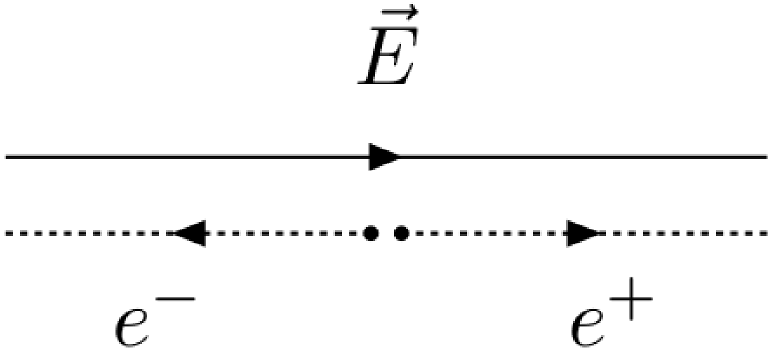
$$2e\mathcal{E}_S \frac{\hbar}{m_\mathrm{e} c} \approx 2 m_\mathrm{e} c^2 \qquad \mathcal{E}_S \approx \frac{m_\mathrm{e}^2 c^3}{|e|\hbar} \approx 10^{16}\ \mathrm{V/cm} \qquad I_S \approx 10^{29}\ \mathrm{W/cm^2}$$

Sauter 1931, Euler and Heisenberg 1936, Schwinger 1951

UNIVERSITY
OF WARSAW

Schwinger process

Brezin, Itzykson, *Phys. Rev. D* **2**, 1191 (1970)

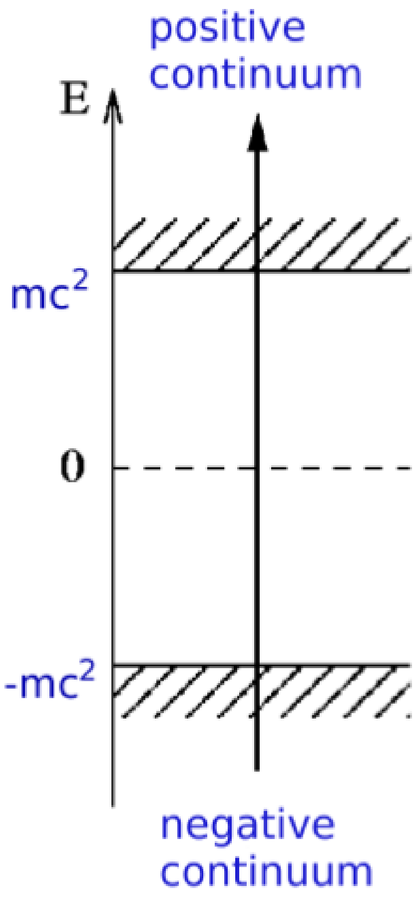


Keldysh parameter $$\gamma = \frac{\hbar\omega/m_e c^2}{\mathcal{E}_0/\mathcal{E}_S} = \frac{m_e c\omega}{|e|\mathcal{E}_0} = \frac{1}{\mu}$$ inverse normalized vector potential

$\mu \gg 1$ $\quad w \sim \exp(-\pi\mathcal{E}_S/\mathcal{E}_0)$ tunneling regime

$\mu \ll 1$ $\quad w \sim \mathcal{E}_0^{2n_0}$ multiphoton regime

Schwinger process



Schwinger, *Phys. Rev.* **82**, 664 (1951)

◆ coherent enhancement

Akkermans, Dunne, PRL **108**, 030401 (2012)
Kamiński, Twardy, Krajewska, PRD **98**, 056009 (2018)
Krajewska, Kamiński, PRA **100**, 062116 (2019)

◆ dynamically-assisted Schwinger effect

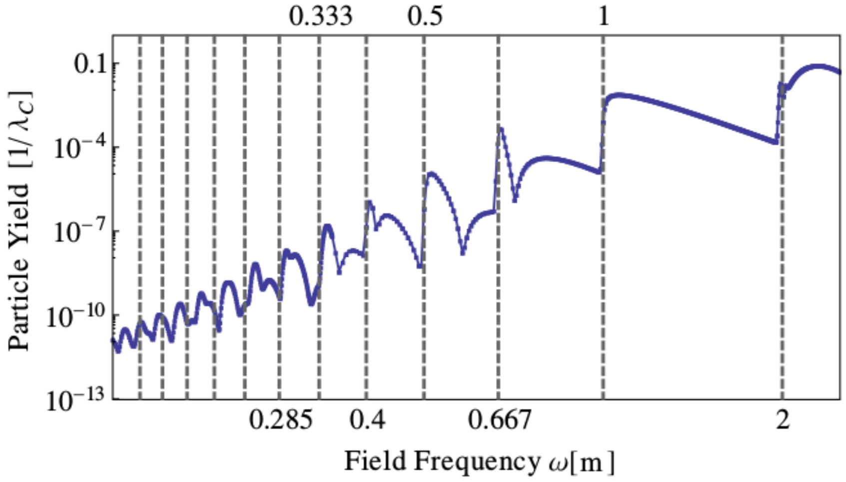
Schützhold, Gies, Dunne, PRL **101**, 130404 (2008)
Aleksandrov, Plunien, Shabaev, PRD **97**, 116001 (2018)
Torgrimsson, Schneider, Schützhold, PRD **97**, 096004 (2018)

◆ effective mass concept

Kohlfürst, Gies, Alkofer, PRL **112**, 050402 (2014)
Li, Lu, Xie, PRD **92**, 085001 (2015)
Oertel, Schützhold, PRD **99**, 125014 (2019)
Krajewska, Kamiński, PRA **100**, 012104 (2019)

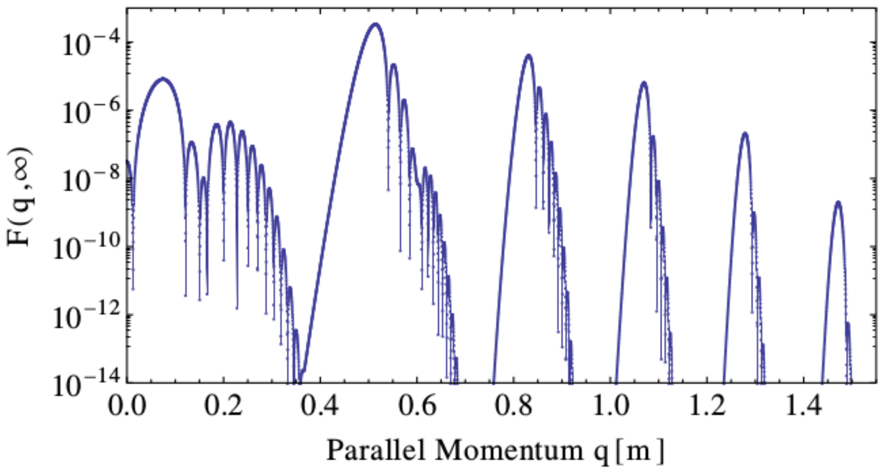
Threshold effects and effective mass

Kohlfürst, Gies, and Alkofer, *Phys. Rev. Lett.* **112**, 050402 (2014)



$e\varepsilon/m^2 = 0.1$

$\omega = 0.3m \quad e\varepsilon = 0.2m^2$



Effective mass model

$$n\omega = 2m_* c^2$$

$$m_* = m\sqrt{1+\mu^2}$$

Threshold effects

UNIVERSITY
OF WARSAW

Theoretical formulation

Grib, Mostepanenko, and Frolov, *Teor. Mat. Fiz.* **13**, 377 (1972)

$$\boxed{i\frac{d}{dt}\begin{bmatrix} c_{\boldsymbol{p}}^{(1)}(t)\\ c_{\boldsymbol{p}}^{(2)}(t)\end{bmatrix} = \begin{pmatrix}\omega_{\boldsymbol{p}}(t) & i\Omega_{\boldsymbol{p}}(t)\\ -i\Omega_{\boldsymbol{p}}(t) & -\omega_{\boldsymbol{p}}(t)\end{pmatrix}\begin{bmatrix} c_{\boldsymbol{p}}^{(1)}(t)\\ c_{\boldsymbol{p}}^{(2)}(t)\end{bmatrix}}$$

where $$\omega_{\boldsymbol{p}}(t) = \sqrt{[m_{\mathrm{e}}(\boldsymbol{p}_\perp)c^2]^2 + c^2[p_\parallel - eA(t)]^2}$$

$$m_{\mathrm{e}}(\boldsymbol{p}_\perp) = \frac{1}{c}\sqrt{(m_{\mathrm{e}}c)^2 + \boldsymbol{p}_\perp^2} \qquad \Omega_{\boldsymbol{p}}(t) = -ce\mathcal{E}(t)\frac{m_{\mathrm{e}}(\boldsymbol{p}_\perp)c^2}{2\omega_{\boldsymbol{p}}^2(t)}$$

initial conditions: $$\lim_{t\to-\infty} c_{\boldsymbol{p}}^{(1)}(t) = 1, \qquad \lim_{t\to-\infty} c_{\boldsymbol{p}}^{(2)}(t) = 0$$

◆ Expectation value of the number of pairs produced from vacuum into a given eigenmode

$$\boxed{f(\boldsymbol{p}) = \lim_{t\to+\infty}\left|c_{\boldsymbol{p}}^{(2)}(t)\right|^2}$$

Pulsed electric field

- The QED vacuum interacts with a linearly polarized electric field $\boldsymbol{\mathcal{E}}(t) = (0, 0, \mathcal{E}(t))$

where $$\int_{-\infty}^{+\infty} \mathrm{d}t\, \mathcal{E}(t) = 0$$

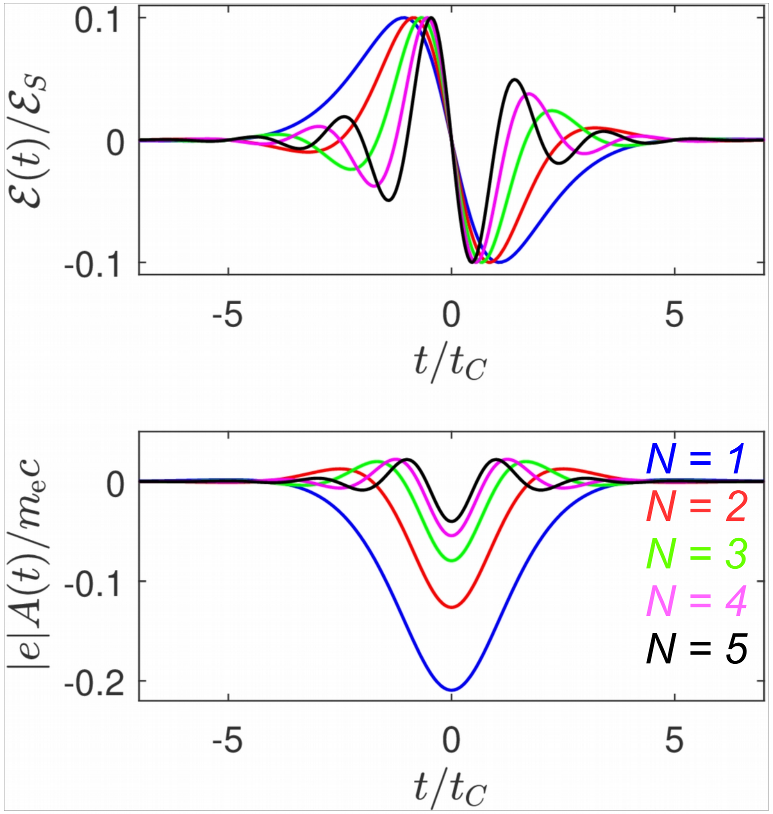
Hence, $$\boldsymbol{A}(t) = (0, 0, A(t))$$

$$\lim_{t\to-\infty} A(t) = \lim_{t\to+\infty} A(t)$$

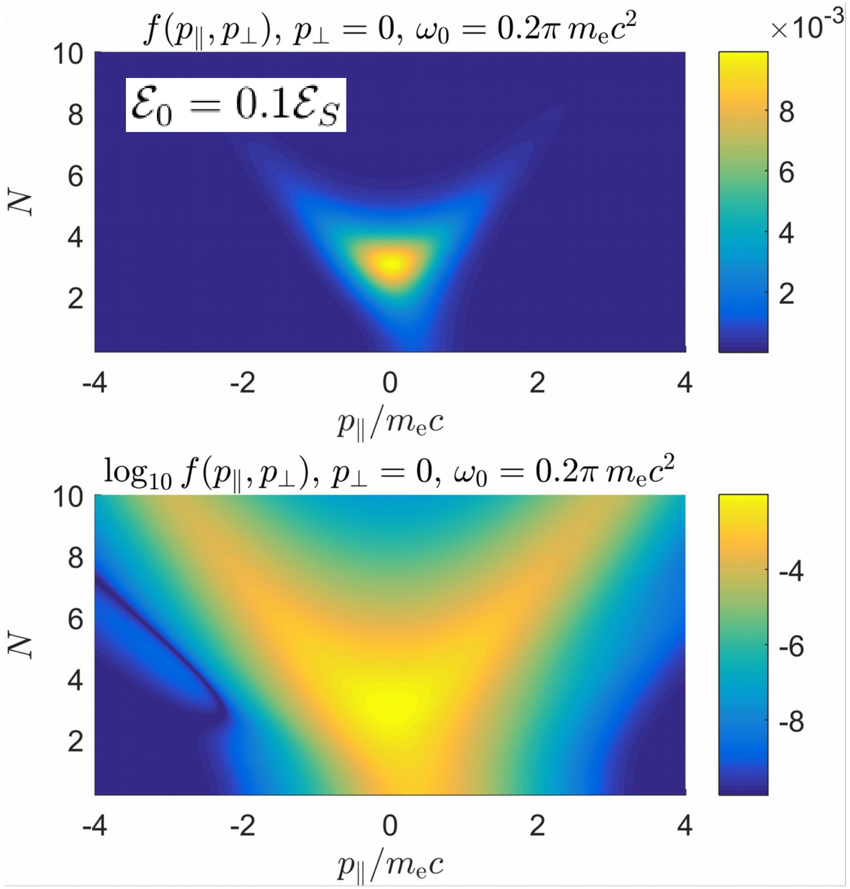
$$\mathcal{E}(t) = \mathcal{E}_0 \frac{\mathcal{N}_0}{\cosh(\beta t)} \sin(\omega t)$$

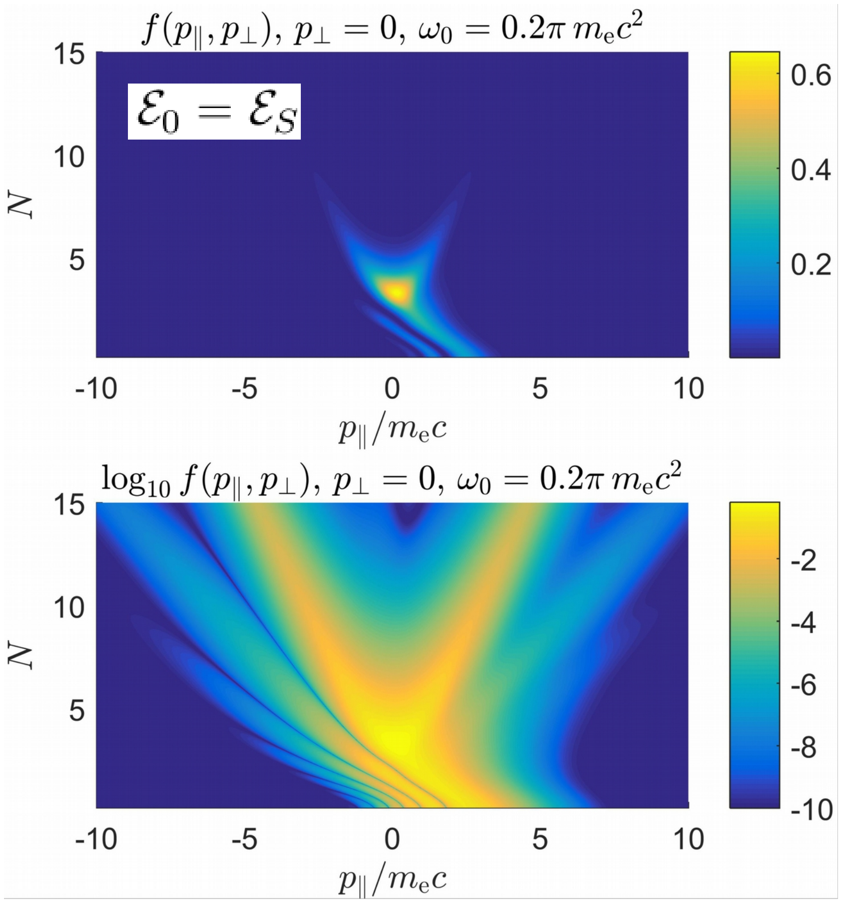
$$\omega = N\omega_0 \qquad \omega_0 = 0.2\pi\, m_{\mathrm{e}}c^2$$

$$\omega = 2m_{\mathrm{e}}c^2 \to N_{\mathrm{th}} = 2m_{\mathrm{e}}c^2/\omega_0 = 3.18$$

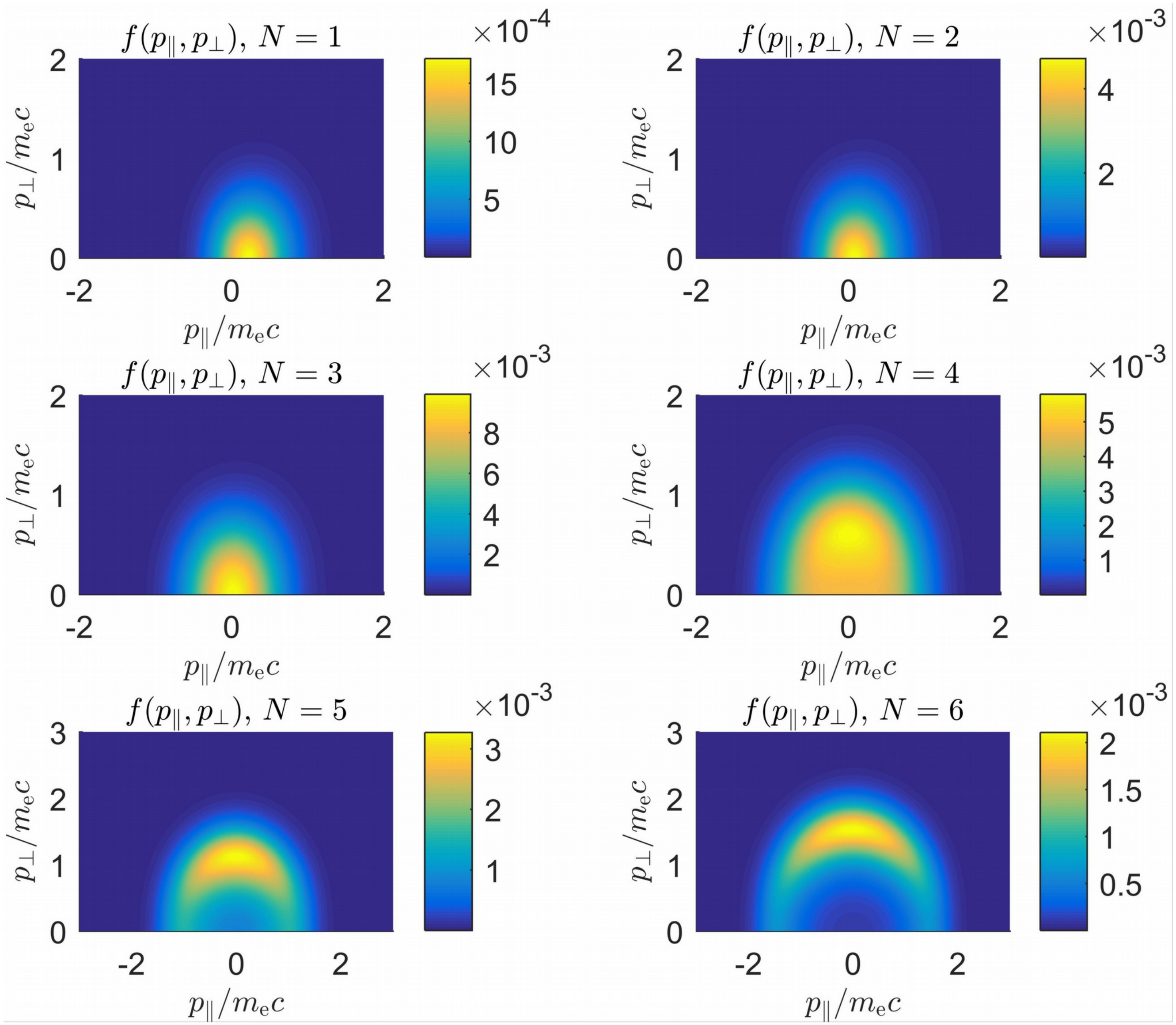


Momentum distributions of one-photon pair creation



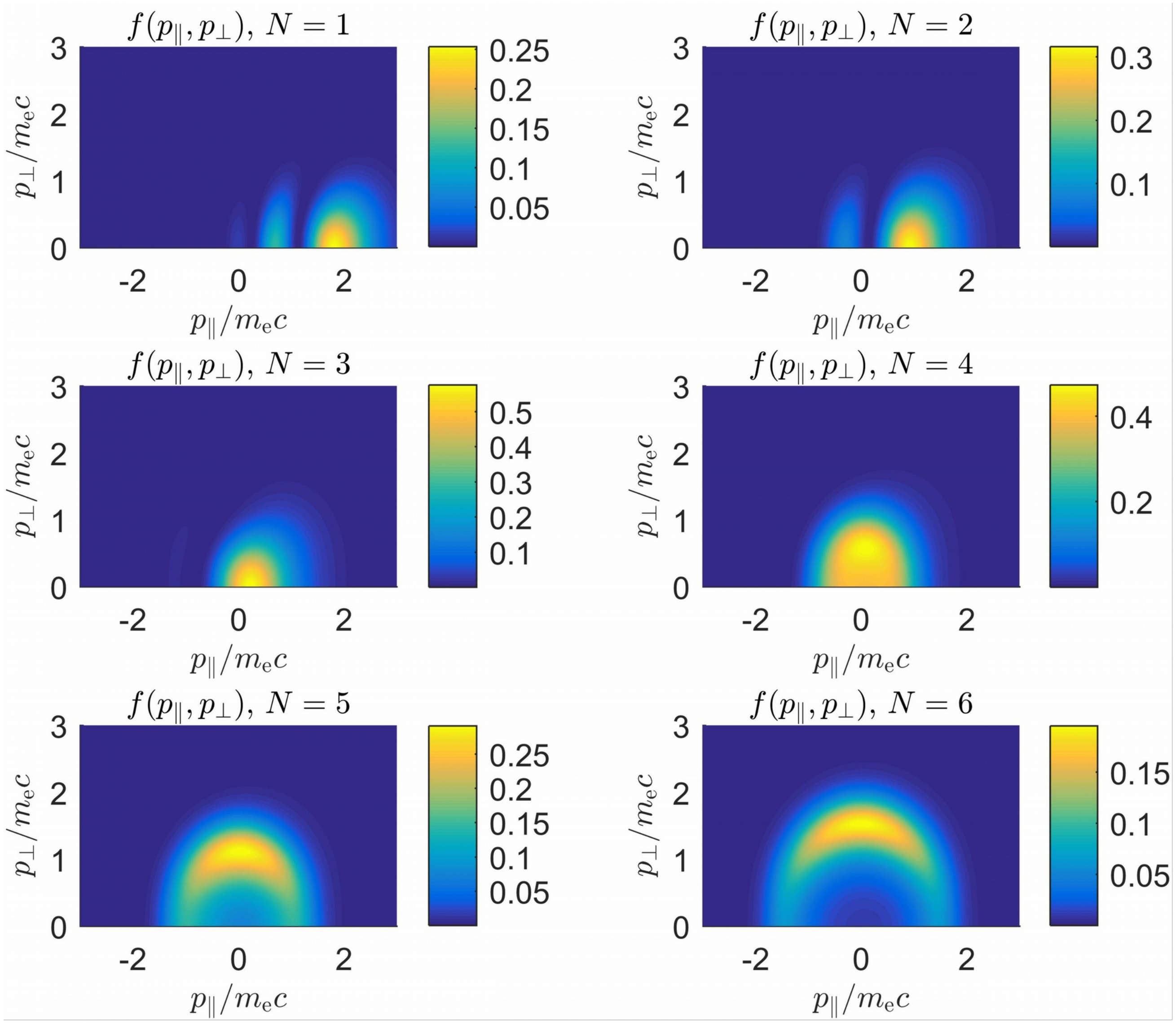


Momentum distributions across one-photon threshold



$$\mathcal{E}_0 = 0.1\mathcal{E}_S$$

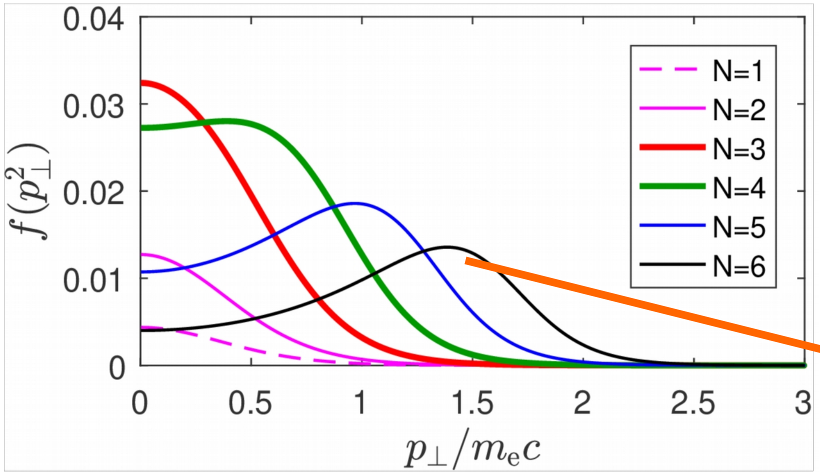
Momentum distributions across one-photon threshold



$$\mathcal{E}_0 = \mathcal{E}_S$$

Marginal momentum distributions

$\mathcal{E}_0 = 0.1\mathcal{E}_S$

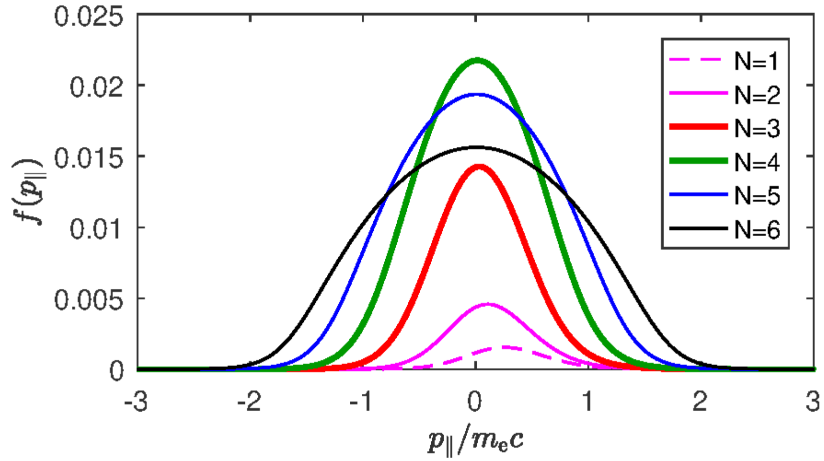


Transverse momentum distribution

$$f(p_\perp^2) = \pi \int_{-\infty}^{+\infty} dp_\parallel\, f(p_\parallel, p_\perp)$$

$$2m_\mathrm{e}(\boldsymbol{p}_\perp)c^2 = \omega = N\omega_0$$

$$p_\perp = \sqrt{(N\omega_0/2c)^2 - (m_\mathrm{e}c)^2}$$



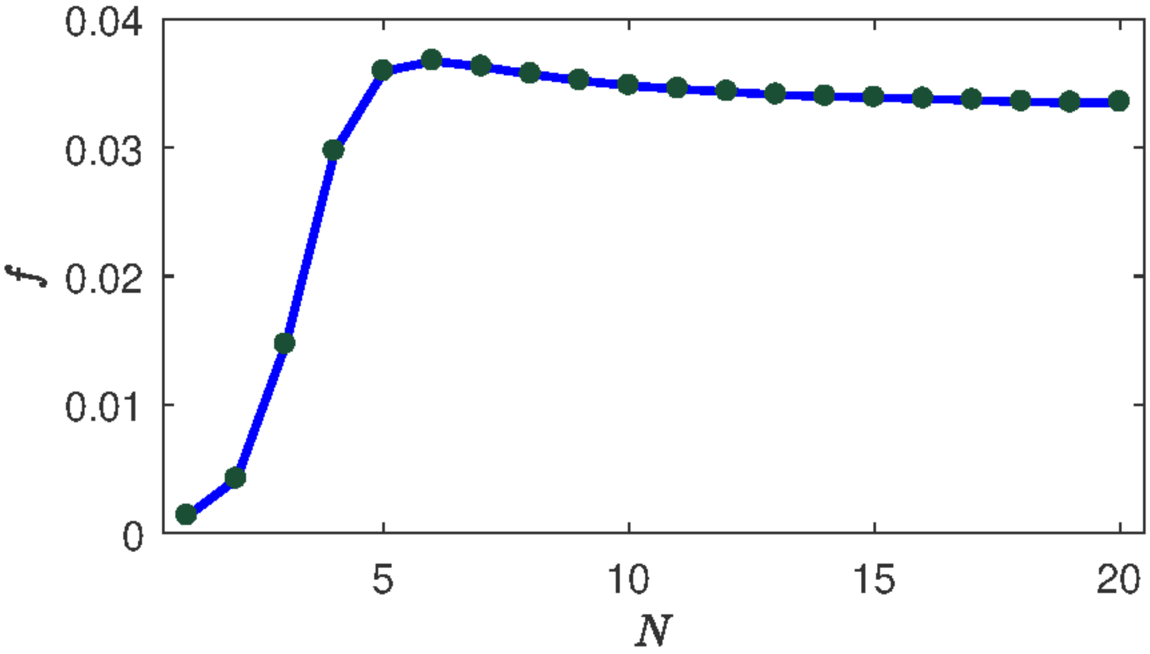
Longitudinal momentum distribution

$$f(p_\parallel) = 2\pi \int_0^{+\infty} dp_\perp\, p_\perp f(p_\parallel, p_\perp)$$

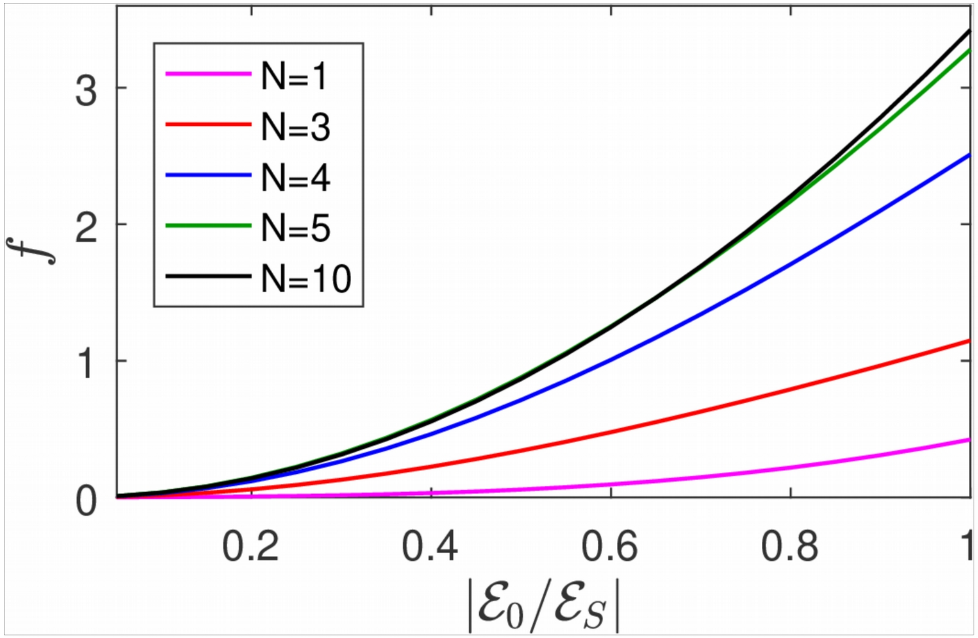
Saturation

Total number of created pairs

$$f = \int d^3p f(p_\parallel, p_\perp) = 2\pi \int_{-\infty}^{+\infty} dp_\parallel \int_0^{+\infty} p_\perp dp_\perp f(p_\parallel, p_\perp)$$



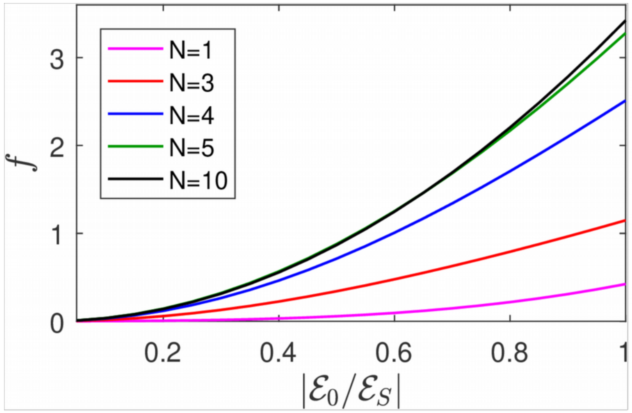
Total number of pairs



Behavior typical for the perturbative regime

for $N > N_{\rm th}$ we have $f = 3.5(\mathcal{E}_0/\mathcal{E}_S)^2$

Perturbative treatment



$$i\frac{d}{dt}\begin{bmatrix} c_{\boldsymbol{p}}^{(1)}(t)\\ c_{\boldsymbol{p}}^{(2)}(t)\end{bmatrix} = \begin{pmatrix}\omega_{\boldsymbol{p}}(t) & i\Omega_{\boldsymbol{p}}(t)\\ -i\Omega_{\boldsymbol{p}}(t) & -\omega_{\boldsymbol{p}}(t)\end{pmatrix}\begin{bmatrix} c_{\boldsymbol{p}}^{(1)}(t)\\ c_{\boldsymbol{p}}^{(2)}(t)\end{bmatrix}$$

$$\max_t |\Omega_{\boldsymbol{p}}(t)| \ll \min_t |\omega_{\boldsymbol{p}}(t)|$$

$$i\dot{c}_{\boldsymbol{p}}^{(1)}(t) = \omega_{\boldsymbol{p}}(t)c_{\boldsymbol{p}}^{(1)}(t)$$

$$i\dot{c}_{\boldsymbol{p}}^{(2)}(t) = -i\Omega_{\boldsymbol{p}}(t)c_{\boldsymbol{p}}^{(1)}(t) - \omega_{\boldsymbol{p}}(t)c_{\boldsymbol{p}}^{(2)}(t)$$

$$\boxed{\frac{\mathcal{E}_0}{2\mathcal{E}_S} \ll \left[\frac{m_{\rm e}(\boldsymbol{p}_\perp)}{m_{\rm e}}\right]^2}$$

$$\boxed{f(\boldsymbol{p}) \approx \left|\int_{-\infty}^{\infty} dt\, \Omega_{\boldsymbol{p}}(t) e^{-2i\int_{-\infty}^{t} d\tau\, \omega_{\boldsymbol{p}}(\tau)}\right|^2}$$

$$\mathcal{E}_0 \ll \mathcal{E}_S$$

$$m_{\rm e}(\boldsymbol{p}_\perp) \gg m_{\rm e}$$

Summary

(a) We have shown **dramatic changes of the energy sharing between the longitudinal and transverse motion of created particles.** What is **essential** in this respect **is how quickly the electric field changes in time.**

(b) **With increasing the frequency of the electric field above the one-photon threshold**, we observe the **increasing effective mass of the electron and saturation of the number of pairs being produced.**

Such electric field can be generated in heavy ion collisions.

(c) We have presented **that the perturbative scaling of the total number of produced pairs with the electric-field strength can happen for arbitrary strong electric field**.

We acknowledge the financial support from **the National Science Centre** (Poland) under **Grant Nos. 2014/15/B/ST2/02203 and 2018/31/B/ST2/01251**.

NATIONAL SCIENCE CENTRE
POLAND

UNIVERSITY
OF WARSAW