To any p order

$$N_{\text{of } \varphi\text{'s}} = N_{\text{Amplitudes}} + 1$$

field redef.

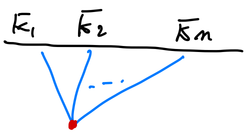
Comment:

* This extends to any number of $m=0$ fields including gravitons
* We did not mention time.

UNITARITY (LESSON 3)

(all based on recent papers: 2009.02898, 2010.12818, 2103.08649, 2103.09832)

Last lecture we bootstrapped



To bootstrap exchange & loop diagrams,

we need unitarity.

In Quantum Mechanics unitarity is:

★ States have a non-negative norm

$$\langle \phi | \phi \rangle \geq 0$$

★ Time evolution is unitary

$$|\phi(t)\rangle = U |\phi(t_0)\rangle \qquad U U^{\dagger} = \mathbb{1}$$

So that probabilities make sense.

In particle physics (QFT in Minkowski) we have the Optical Theorem for Amplitudes

$$S \equiv (\mathbb{1} + iT) \qquad \underset{\text{unitary}}{S^{\dagger} S = \mathbb{1}}$$

$$(1 - iT^{\dagger})(1 + iT) = \mathbb{1}$$

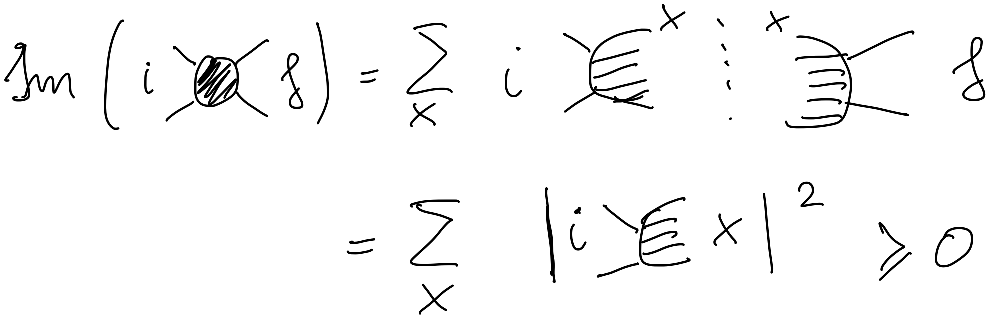
$$i(T^{\dagger} - T) = T^{\dagger} T$$

Evaluate this inside $\langle f | \ \ldots \ | i \rangle$ and we insert the identity on the R.H.S.

$$i \langle f | T^{\dagger} - T | i \rangle = \sum_X \langle f | T^{\dagger} | X \rangle \langle X | T | i \rangle$$

$$A_{i \to f} - A^{*}_{f \to i} = \sum_X \int d\Pi_X \, A_{i \to X} \, A^{*}_{f \to X}$$

Graphically this becomes (for $|i\rangle = |f\rangle$)



Very powerful:

* One can numerically bootstrap the non-perturbative amplitude.
* Constrain Effective Field Theories that do not admit a unitary UV-completion (a.k.a. positivity bounds).

Order by order in perturbation theory the Optical Th. reduces to Cutkosky Cutting Rules ('60)

COSMOLOGICAL OPTICAL THEOREM AND CUTTING RULES

Let's do the same for the wavefunction Ψ.

We need two simple properties

① The propagators are Hermitian analytic

$$\bar{K}^*(-k^*, \eta) = \bar{K}(k, \eta) \qquad k \in \mathbb{C}$$

Bunch-Davies

$$G^*(-k^*; \eta, \eta') = G(k; \eta, \eta')$$

This is trivial to check in examples

$$\bar{K}(-k^*) = \left[(1 - i(-k^*)\eta)\, e^{i(-k^*)\eta} \right]^*$$

massless scalar in dS

$$= (1 - ik\eta)\, e^{ik\eta} = \bar{K}(k)$$

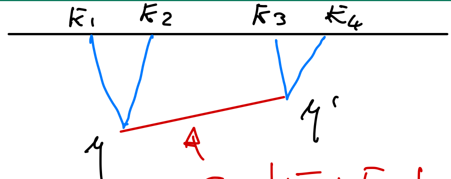
This can be proven on any FLRW for fields of any mass & spin with B.D. vacuum.

② Factorization of the bulk-to-bulk prop:

power spectrum

$$\operatorname{Im} G(\eta, \eta') = 2P \operatorname{Im} \bar{K}(\eta) \operatorname{Im} \bar{K}(\eta')$$

Example:



$s = |\vec{k}_1 + \vec{k}_2|$

Let

k_1, k_2, k_3, k_4 &

$$\operatorname*{Disc}_s (i\psi_4) \equiv i\left[\psi_4(\{k\}; s) + \psi_4^*(-\{k^*\}; s) \right]$$

Then

nested integral

$$\underset{s}{\mathrm{Disc}}\, i\psi_4 = \underset{s}{\mathrm{Disc}} \int d\eta\, d\eta'\, K_1(\eta)\, K_2(\eta)\, G_s(\eta,\eta') \times K_3(\eta')\, K_4(\eta')$$

$$= \int d\eta\, d\eta'\, \bar{K}_1(\eta)\, K_2(\eta)\, \bar{K}_3(\eta')\, \bar{K}_4(\eta') \times \mathrm{Im}\, G_s(\eta,\eta')$$

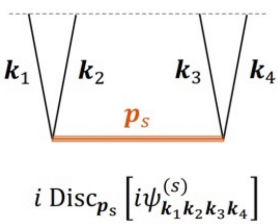
Un-nested!

$$= \left[\int d\eta\, K_1(\eta)\, K_2(\eta)\, (\mathrm{Im}\, K_s)\right] \times 2 P_s \left[\int d\eta'\, \bar{K}_3(\eta')\, \bar{K}_4(\eta')\, (\mathrm{Im}\, K_s)\right]$$

$$= P_s\, \underset{s}{\mathrm{Disc}}\left[i\psi_3(k_1,k_2,s)\right] \times \underset{s}{\mathrm{Disc}}\left[i\psi_3(k_3,k_4,s)\right]$$

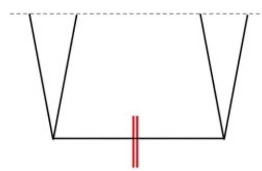
The Disc of ψ_4 is fixed by ψ_3!

Graphically

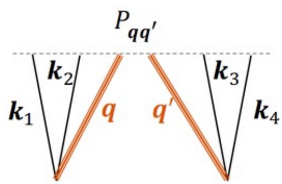


$i\,\mathrm{Disc}_{p_s}\left[i\psi^{(s)}_{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3\boldsymbol{k}_4}\right]$

$=$

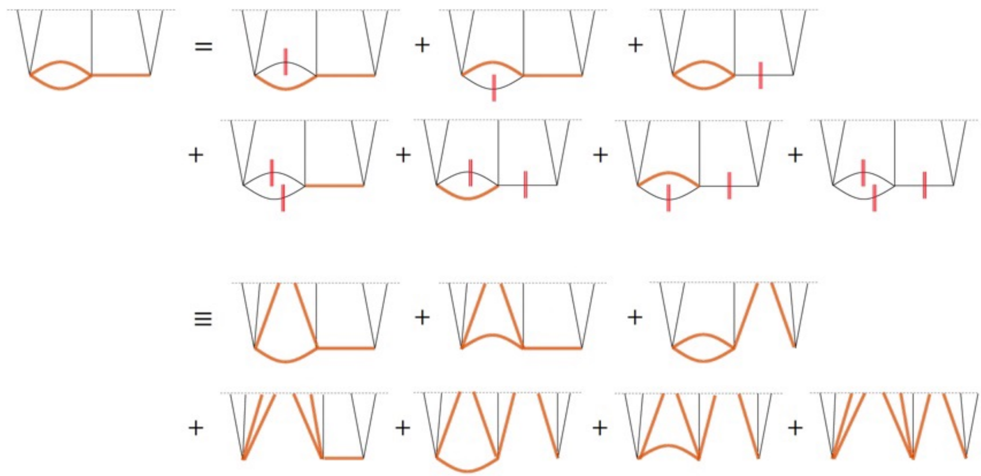


$\equiv$



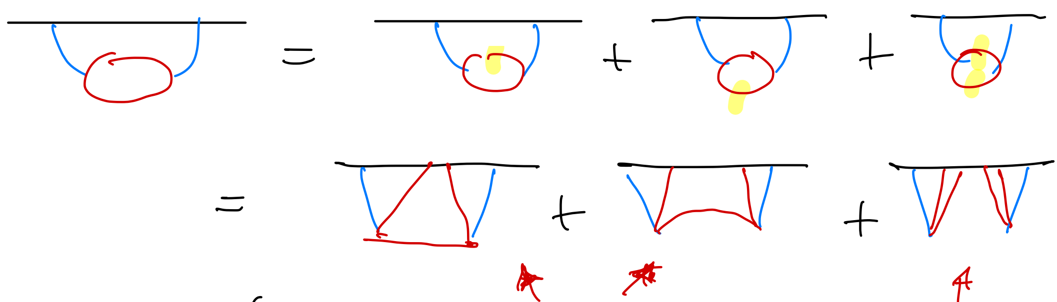
$i\,\mathrm{Disc}_{\boldsymbol{q}}\left[i\psi_{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{q}}\right]\ P_{qq'}\ i\,\mathrm{Disc}_{\boldsymbol{q}'}\left[i\psi_{\boldsymbol{q}'\boldsymbol{k}_3\boldsymbol{k}_4}\right]$

This generalizes to all orders including all loops. E.g.



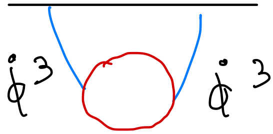
LOOP CORRECTIONS

We can compute loops!



$$\mathrm{Disc}\, i\psi_2 = \int_{\bar{q}} P_{\bar{q}} \; \mathrm{Disc}\left(i\psi_4^{\text{tree}}\right) + \int_{qq'} \mathrm{Disc}\left(i\psi_3\right)^2$$

We reproduced known calculations of



$$\mathrm{Disc}\, i\psi_2 = \frac{H^2}{16\pi} k^3 \frac{2}{15} \lambda^2$$

We can use to get perturbative unitarity bounds.

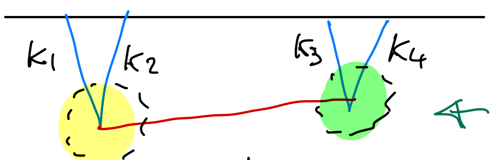
RECURSION RELATIONS

Wavefunction coefficients have only very specific singularities:

★ Total energy singularity as $k_T = \sum^{n} k_a \to 0$

The residue is the flat space Amplitude

★ Partial energy singularities



$$E_R = k_3 + k_4 + s$$

$$s = |\vec{k}_1 + \vec{k}_2|$$

$$E_L = k_1 + k_2 + s$$

This has poles at $E_L = 0$ or $E_R = 0$.

$$\psi_4 \sim \ell_4^2 \left[\frac{6}{k_T^5}\left(\frac{1}{E_L}+\frac{1}{E_R}\right) + \frac{3}{k_T^4}\left(\frac{1}{E_L^2}+\frac{1}{E_R^2}\right) + \frac{1}{k_T^3}\left(\frac{1}{E_R^3}+\frac{1}{E_L^3}\right) + \frac{1}{E_L^3 E_R^3} \right]$$

The residues of all partial energy poles are fixed by the Cosmo Optical Theorem!

More formally, change variables

$$\{k_1, k_2, k_3, k_4, s\} \to \{E_L, E_R, k_1 k_2, k_3 k_4, s\}$$

We introduce partial energy shifts:

$$E_L \to E_L + z \qquad z \in \mathbb{C}$$

$$E_R \to E_R - z$$

$$\tilde{\psi}_4(z) = \psi_4(E_L + z, E_R - z, s)$$

$$\psi_4 = \tilde{\psi}_4(z=0) = \oint \frac{dz}{2\pi i} \frac{\tilde{\psi}_4(z)}{z}$$

$= \sum$ residues

So we find the general result

$$\psi_4 = \psi_4^{Res} + B$$

$\leftarrow$ Boundary

$\uparrow$ residues

$$\psi_4^{Res} = \sum_{0<n<m} \frac{A_L}{E_L^m} + \frac{A_L}{E_R^m}$$

$$A_n = \text{Res}\,(\text{Right-hand side of C.O.T})$$

Note: time did not appear anywhere

SUMMARY & OUTLOOK

- Cosmo observations measure quantum correlators / the wavefunction Ψ of pert. quantum gravity in quasi de Sitter.
- We can compute Ψ in perturbation theory model by model.
- We now know the constraints of manifest locality & unitarity on Ψ

Future:

- Find a non-perturbative Cosmological Optical Theorem
- How does this constraint the hypothetical Conformal Field Theory at the future boundary of de Sitter??
- Can we bootstrap non-perturbative correlators?

- What EFT's admit a UV-completion? (Cosmological positivity bounds).