LECTURES ON COSMOLOGY & INFLATION

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Lesson 1: COSMOLOGY: REVIEW & MOTIVATIONS
Lesson 2: THE BOOTSTRAP APPROACH.
Lesson 3: UNITARITY & THE COSMOLOGICAL OPTICAL THEOREM

REVIEW AND MOTIVATIONS (LESSON 1)

* Expanding spacetime
* Inflation and primordial perturbations
* The wavefunction of the universe
* Observables
* Examples

On large scales ($\Delta x \gg 10$ Mpc) the universe is homogeneous & isotropic

cosmo time

$$ds^2_{FLRW} = -dt^2 + a(t)^2 \, d\bar{x}^2$$ ← comoving coordinates

$$= a^2(\eta)\left(-d\eta^2 + d\bar{x}^2\right)$$

scale factor → a ; ↑ conformal time

Dynamics is governed by G.R. + stuff.

$$S = \int d\eta \, d^3x \sqrt{-g}\left[\frac{M_p^2}{2} R + \mathcal{L}_{stuff}\right]$$

$$\hookrightarrow \quad R_{\mu\nu} \overset{?}{+} \frac{1}{2} g_{\mu\nu} R = \frac{1}{M_p^2} T_{\mu\nu}$$

↑ "reduced" Planck mass $M_{Pe}^2 = \frac{1}{8\pi G_N}$

We will solve this in perturbation theory.

$$g_{\mu\nu}(\bar{x},t) = \bar{g}^{FRLW}_{\mu\nu}(t) + \delta g_{\mu\nu}(\bar{x},t)$$

$$\phi(\bar{x},t) = \bar{\phi}(t) + \delta\phi(\bar{x},t)$$ → small perturbations

Perturbations are measured in cosmo observations

$$\delta(\bar{x},t) = \frac{\rho(\bar{x},t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

inhomogeneities of galaxies, Dark matter, neutrinos...

$$\Delta T(\hat{n}) = \frac{T(\hat{n}) - \bar{T}}{\bar{T}}$$

in the CMB ...

We can only predict (classical / quantum) averages

direction $\quad \hat{n}\cdot\hat{n} = 1$

$$\langle \Delta T(\hat{n}_1)\ \Delta T(\hat{n}_2) \ldots \rangle$$

or $\quad \langle \delta(\bar{x}_1)\ \delta(\bar{x}_2) \ldots \delta(\bar{x}_n)\rangle$

All are described by some probability distr.

In our universe the initial conditions of perturbations are observed to be:

- Statistically homog. & isotropic ($\mathbb{R}^3, SO(3)$)
- Adiabatic: all perturbations are proportional to each other.

$$\underset{\text{inflaton}}{\delta\phi} \sim \left(\frac{\delta\rho}{\bar{\rho}+\bar{p}}\right)_a = \left(\frac{\delta\rho}{\bar{\rho}+\bar{p}}\right)_b$$

a, b = Dark Matter, Baryons, photons, neutrinos, Dark En. ⋮

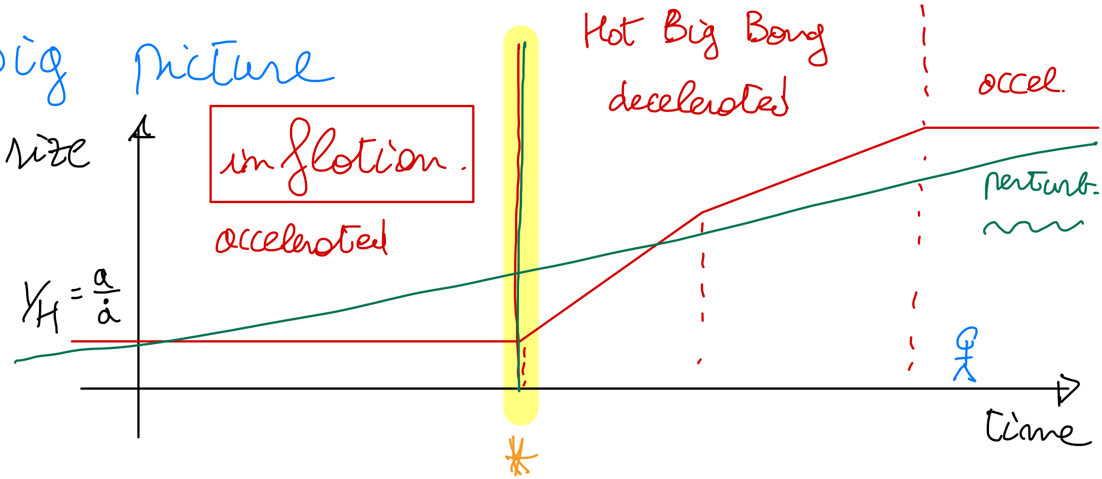
energy density $\quad$ pressure

- Close to Gaussian ($1/10^4$)
- Scale invariant

$$\langle \phi(\lambda\bar{x})\ \phi(\lambda\bar{y})\rangle \simeq \langle \phi(\bar{x})\ \phi(\bar{y})\rangle$$

- Correlated on large scales!

Big picture



Cosmo probes very high energy physics & perturbative regime of Quantum gravity.

The goal of primordial cosmology is to compute/predict the wavefunction of the Universe at t_*

normaliz.

$$\Psi[\phi(\bar{x}); t_*] = N \exp\Big[-\int_{\bar{q}_1\bar{q}_2} \frac{1}{2}\psi_2(\bar{q})\,\phi(\bar{q}_1)\,\phi(\bar{q}_2)$$

$$+\int_{\bar{q}_1\bar{q}_2\bar{q}_3}\psi_3(\bar{q})\,\frac{1}{3!}\,\phi(\bar{q}_1)\,\phi(\bar{q}_2)\,\phi(\bar{q}_3)+\dots\Big]$$

all fields in theory $\{g_{ij}, \phi, \rho_{DM}, h, A_\mu, \dots\}$

$$\psi_n = \psi_n(\bar{q}_1, \dots, \bar{q}_n) = (2\pi)^3\, \delta_D\Big(\sum_a \bar{q}_a\Big)\, \psi_n'$$

All observables can be computed from Ψ

$$\langle \mathcal{O} \rangle \equiv \int d\phi \; \Psi^* \mathcal{O} \Psi$$

Eg. can compute the 2 & 3-point correlators

power spectrum

$$\langle \phi(\bar{k}) \phi(\bar{k}') \rangle = (2\pi)^3 \delta_D(\bar{k}+\bar{k}') P(k)$$

$$P(k) = \frac{1}{2 \, \mathrm{Re}\, \psi_2(k)}$$

$$\langle \phi(\bar{k}_1) \phi(\bar{k}_2) \phi(\bar{k}_3) \rangle' = B_3 = \frac{\mathrm{Re}\, \psi_3}{\left(\prod_a^3 \mathrm{Re}\, \psi_2 \, k_a \right)}$$

bispectrum

To compute Ψ we usually need a model

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R + \mathcal{L}(X, \phi) \right]$$

inflaton

$$X \equiv \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi$$

e.g. $\mathcal{L} = X - V(\phi)$

The wavefunction can be computed w/ a path integral:

$$\Psi[\phi(\bar{x}); \eta_0] = \int^{\phi(\bar{x}, \eta_0)}_{\text{Bunch-Davies vacuum}} [D\phi] \; e^{iS[\phi]}$$

A Minkowski vacuum

E.g. at tree level

$$\Psi[\phi(\bar{x})] = e^{i S_{cl}[\phi_{cl}]}$$

with ϕ_{cl} a solution of classical eq of motion

$$(E.O.M)\, \phi_{cl} = 0$$

In perturbation theory

all non linearities

$$\phi_{cl} = \phi(\bar{k})\, K(\bar{k}, \eta) + \int G(\eta, \eta'; \bar{k}) \frac{\delta S_{int}}{\delta \phi(\bar{k})}$$

Where: $(E.O.M)^{lin}_{\eta} K \doteq 0$ Bulk-to-boundary propagator

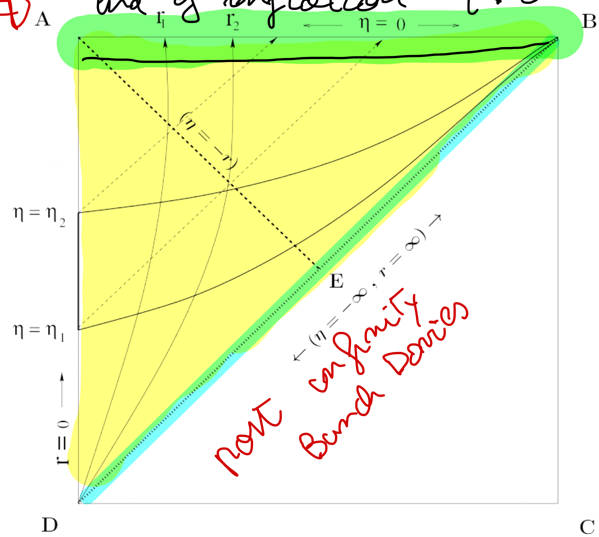
$$(E.O.M)^{lin}_{\eta}\, G(\eta, \eta') = -\delta(\eta - \eta')$$ Bulk to bulk propagator

Boundary conditions at past & future infinity of (quasi) de Sitter

$$ds^2 = \frac{-d\eta^2 + d\bar{x}^2}{H^2 \eta^2}$$

constant Hubble $\rightarrow$ H^2

Future boundary ⟹ end of inflation $\eta \to 0$



Penrose diagram of de Sitter

past infinity Bunch Davies

Bunch-Davies vacuum

$$\lim_{\eta \to -\infty(1-i\varepsilon)} K(\eta)\,,\ G(\eta,\eta') \doteq 0$$

Future boundary

$$K(\eta \to 0) = 1 \quad \& \quad G(\eta \to 0, \eta') = 0$$

For a massless scalar & graviton the solutions are:

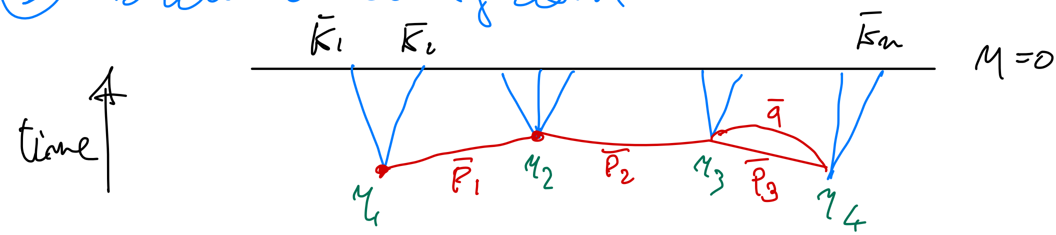
$$f^{\pm}(\bar{k},\eta) = \frac{1}{\sqrt{2k}}\,\frac{H}{k}\,(1 \mp ik\eta)\, e^{\pm ik\eta}$$

$$K = \frac{f^{+}(\bar{k},\eta)}{f^{+}(\bar{k},\eta_0)}$$

$$G = i\,\Theta(\eta-\eta')\Big[\overbrace{f^+(\eta')\,f^-(\eta)}^{\text{Feynman prop.}} + \underbrace{- \frac{f^-(\eta_0)}{f^+(\eta_0)}\, f^+(\eta)\, f^+(\eta')}_{\text{Boundary.}}\Big] + \eta \leftrightarrow \eta'$$

The Feynman–Witten rules:

① Draw a diagram



② Assign $\bar{k}_a$ to external legs & $\bar{p}_1 \ldots \bar{p}_I$ to internal legs

③ External line $\rightarrow$ $K(\bar{k}, \eta_A)$

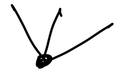
Internal line $\rightarrow$ $G(\bar{p}; \eta, \eta')$

④ Integrate over all vertices.

$$\prod_a^V \int d\eta_a$$

⑤ Add the appropriate vertices. E.g.

$\lambda \phi^3/3! \Rightarrow$



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EXAMPLES