



Part II. Black holes and the invisible universe

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(Técnico Lisboa)

Image: Jacques Henri Lartigue

Uniqueness: the Kerr solution

Theorem (Carter 1971; Robinson 1975; Chrusciel and Costa 2012):
A stationary, asymptotically flat, vacuum BH solution must be Kerr

$$ds^2 = \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} dt^2 + \frac{2a(r^2 + a^2 - \Delta) \sin^2 \theta}{\Sigma} dt d\phi$$
$$- \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2$$
$$\Sigma = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 + a^2 - 2Mr$$

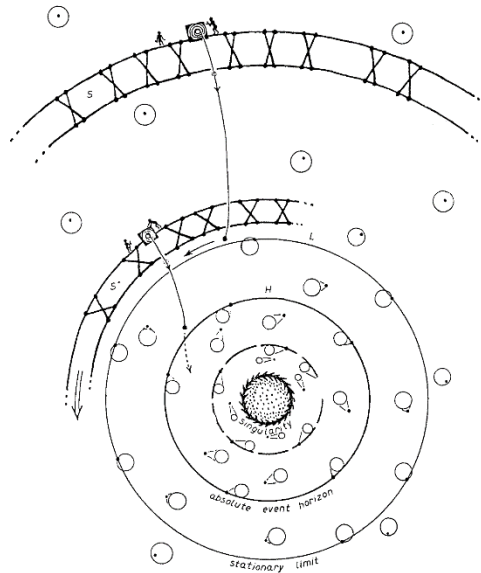
Describes a rotating BH with mass M and angular momentum $J=aM$, iff $a < M$

“In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein’s equations of general relativity provides the *absolutely exact representation* of untold numbers of black holes that populate the universe.”

S. Chandrasekhar, The Nora and Edward Ryerson lecture, Chicago April 22 1975

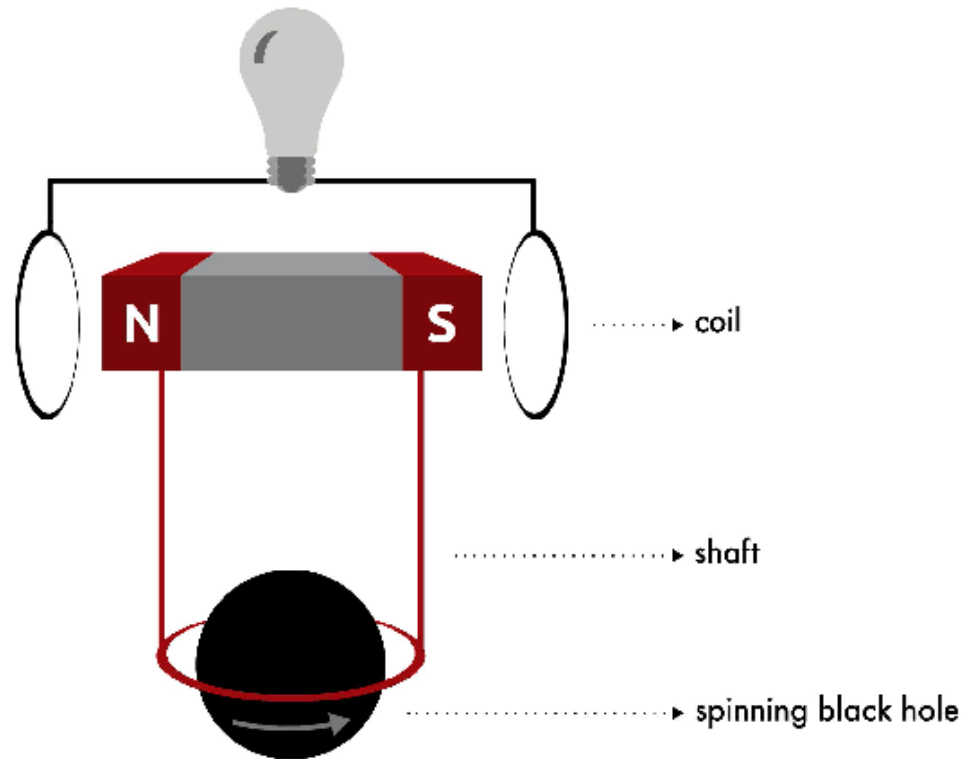
Gravitational Collapse: The Role of General Relativity

by Roger Penrose (1969)



I only wish to make a plea for “black holes” to be taken seriously and their consequences to be explored in full detail. For who is to say, without careful study, that they cannot play some important part in the shaping of observed phenomena?

Energy source?

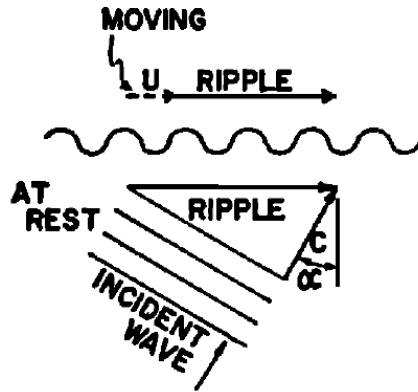


Penrose, *Gravitational Collapse: the role of General Relativity* (1969)

Brito, Cardoso & Pani, *Superradiance* (Springer-Verlag 2015)

Superradiance

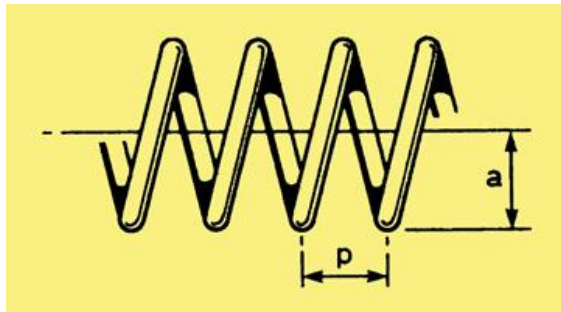
Zel'dovich JETP Lett. 14:180 (1971); Brito+ Lect. Notes Phys. 971 (2020)



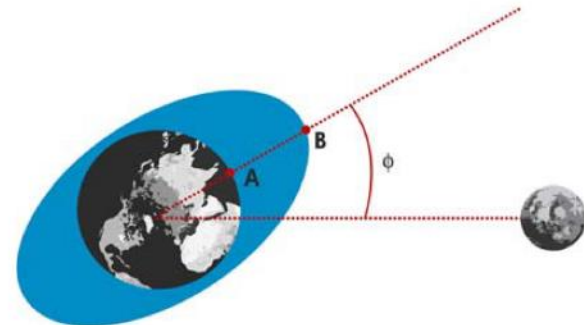
Ribner, J. Acous. Soc. Amer. 29 (1957)



Tamm & Frank, Doklady AN SSSR 14 (1937)

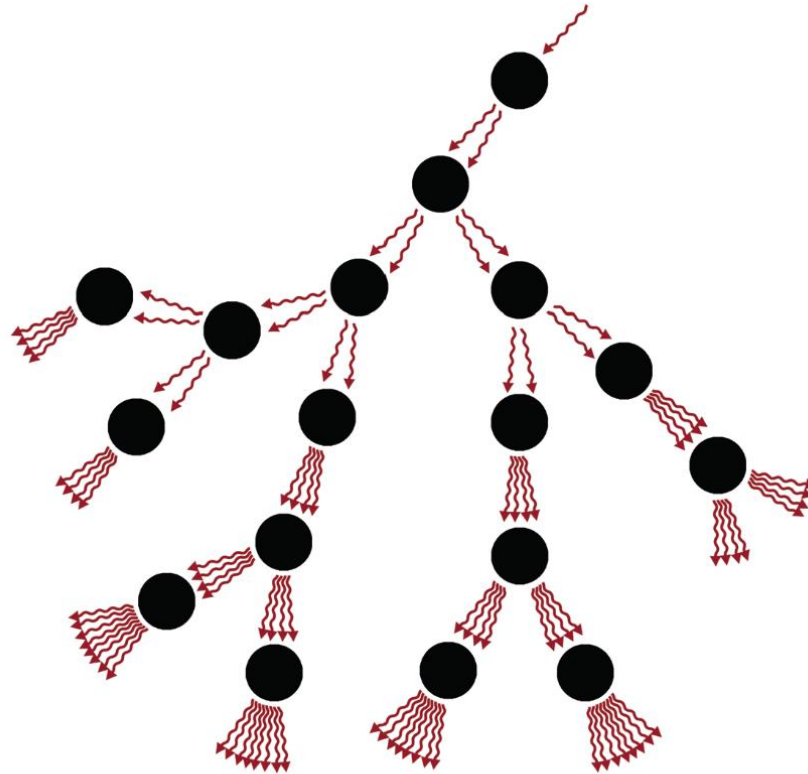


Pierce (& Kompfner), Bell Lab Series (1947)
Ginzburg, anomalous Doppler year



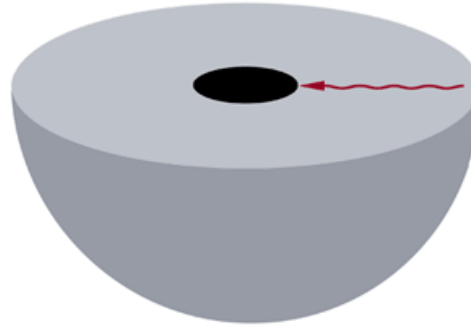
G. H. Darwin, Philos. Trans. R. Soc. London 171 (1880)

No fission



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Bombs and superradiant instabilities



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$$\nabla_\gamma \nabla^\gamma \Psi = \mu^2 \Psi, \quad \nabla_\gamma F^{\gamma\nu} = \mu^2 A^\nu, \quad \nabla_\gamma \nabla^\gamma h_{\mu\nu} = \mu^2 h_{\mu\nu}$$

$$\Psi \sim e^{-i\omega t} Y_{lm}$$

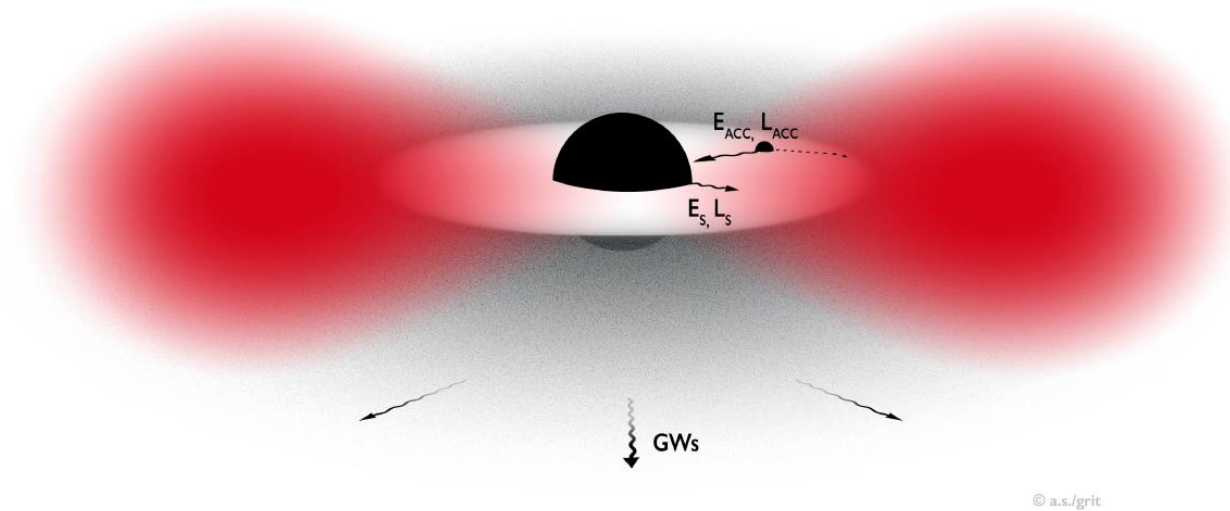
$$\omega \sim \mu + i(m\Omega_H - \mu)(M\mu)^{4l+5+S}$$

$$S = -s, -s + 1, \dots, s - 1, s$$

Massive bosonic “states” around Kerr are linearly unstable

See review Brito+ Lect. Notes Phys. 971 (2020)

Fundamental fields: bounding the boson mass



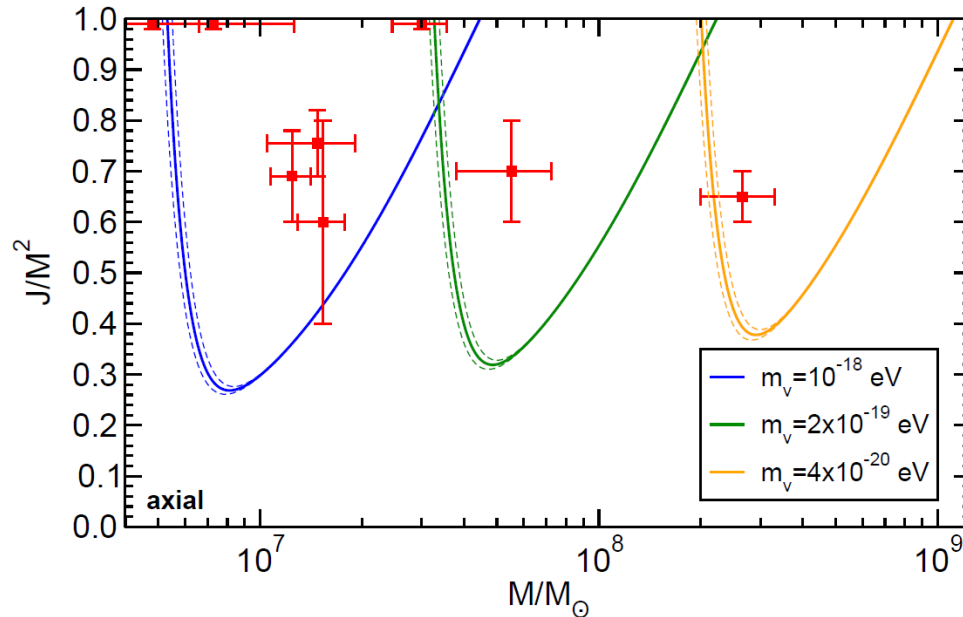
$$\tau \sim 100 \left(\frac{10^6 M_\odot}{M} \right)^8 \left(\frac{10^{-16} \text{eV}}{\mu} \right)^9 \text{ seconds}$$

Wonderful sources of GWs

Brito+ Lecture Notes Physics 971 (2020)

Bounding the boson mass with EM observations

Pani et al PRL109, 131102 (2012)

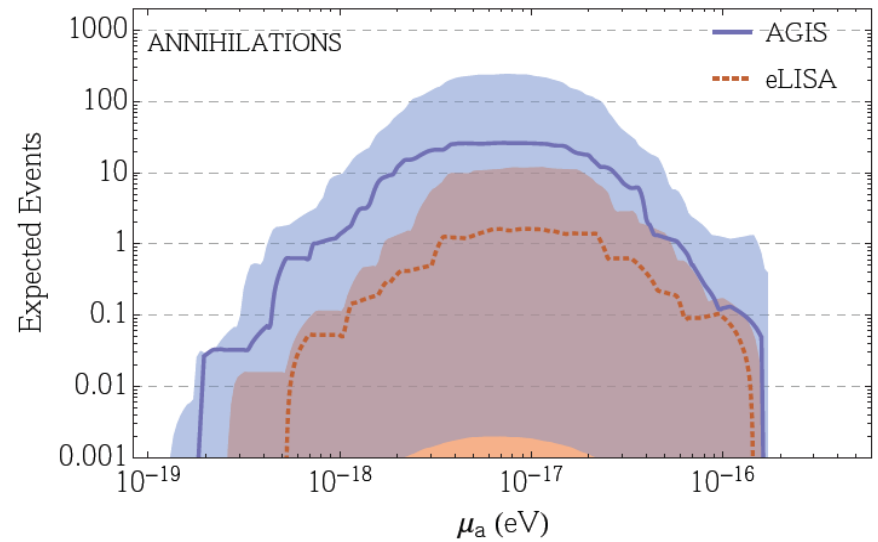
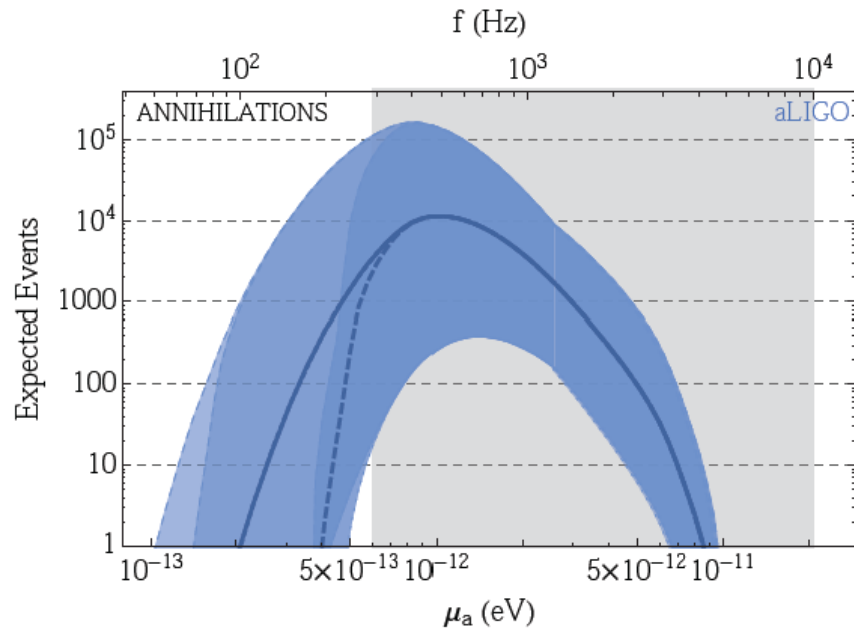


Bound on photon mass is model-dependent: details of accretion disks or intergalactic matter are important... but gravitons interact very weakly!

$$m_g < 5 \times 10^{-23} \text{ eV}$$

Brito + PRD88:023514 (2013); Review of Particle Physics 2014

Wonderful sources for different GW-detectors!



Arvanitaki+ PRD91:084011 (2015); Brito+CQG32:134001 (2015);

Brito+ Lect.Notes Physics 971 (2020)

Wonderful sources for different GW-detectors!

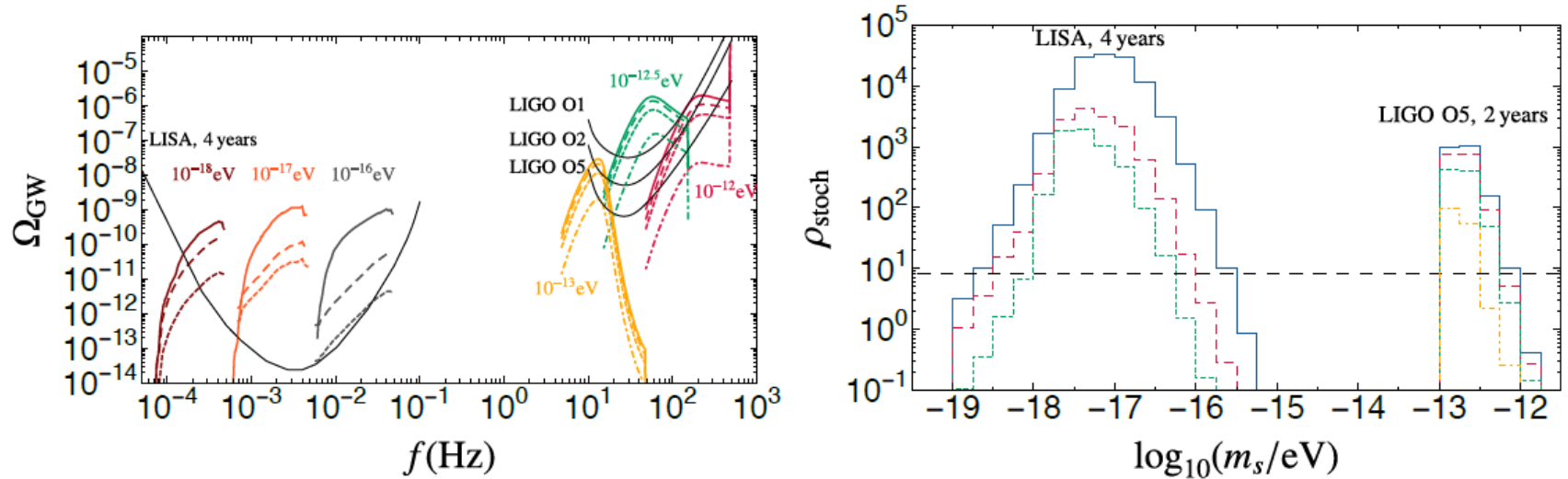
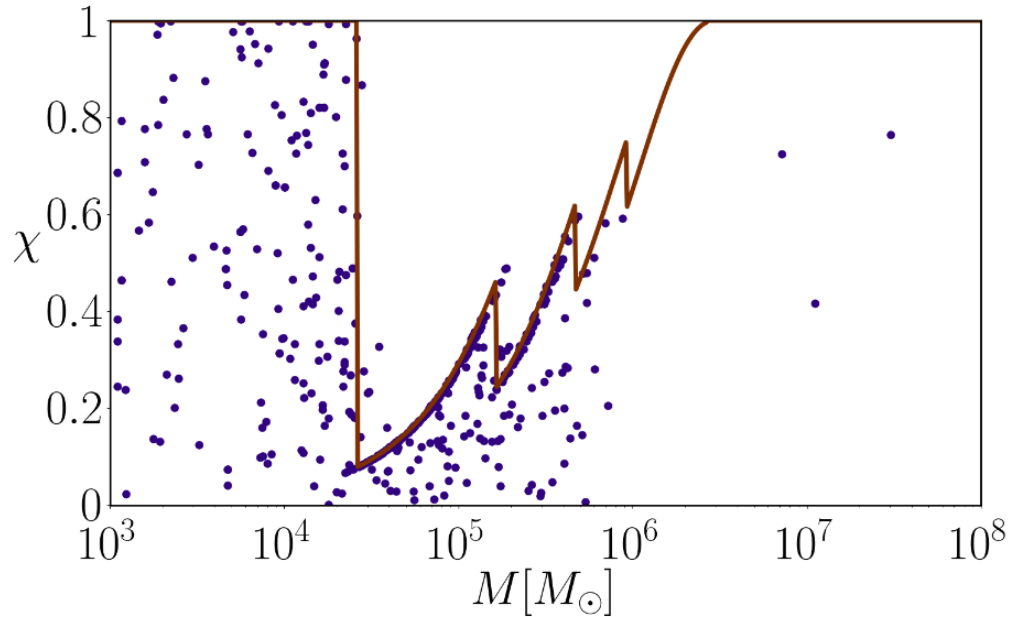


FIG. 2. Left panel: stochastic background in the LIGO and LISA bands. For LISA, the three different signals correspond to the “optimistic” (top), “less optimistic” (middle) and “pessimistic” (bottom) astrophysical models. For LIGO, the different spectra for each scalar field mass correspond to a uniform spin distribution with (from top to bottom) $\chi_i \in [0.8, 1]$, $[0.5, 1]$, $[0, 1]$ and $[0, 0.5]$. The black lines are the power-law integrated curves of Ref. [61], computed using noise PSDs for LISA [9], LIGO’s first two observing runs (O1 and O2), and LIGO at design sensitivity (O5) [62]. By definition, $\rho_{\text{stoch}} \geq 1$ when a power-law spectrum intersects one of the power-law integrated curves. Right panel: ρ_{stoch} for the backgrounds shown in the left panel. We assumed $T_{\text{obs}} = 2$ yr for LIGO and $T_{\text{obs}} = 4$ yr for LISA.

Brito + PRL119: 131101 (2017); arXiv: 1706:05097

PRD96:064050 (2017); arXiv: 1706.06311

Signatures in Regge plane



Two-year simulation for LISA and a boson with 10^{-16} eV. Saw-tooth due to different m harmonics. Final estimate from LISA: $(0.88 - 1.35) \times 10^{-16}$ eV

Brito + PRL119: 131101 (2017); arXiv: 1706:05097

PRD96:064050 (2017); arXiv: 1706.06311

Constraints on fundamental fields via superradiance

Review in Brito+ Lect. Notes Phys.971 (2020)

Resolvable events from single sources

Arvanitaki+ PRD91 (2015) 084011; Brito CQG32 (2015)134001; Brito+PRD96:064050; D'Antonio+PRD98 (2018)103017; Isi+ PRD99 (2019)084042; Palomba+ PRL123 (2019) 171101

Stochastic background

Brito+PRL119: 131101 (2017); PRD96:064050 (2017); Tsukada+PRD99 (2019) 103015; LIGO/Virgo PRD100 (2019) 061101

Accurate measurements of BH spin (via EM or GW measurements)

Pani+ PRL 109 (2012) 131102; Brito+PRD88 (2013) 023514

Spin distribution

Arvanitaki+ PRD83 (2011) 044026; Brito CQG32 (2015)134001; Brito+PRL119: 131101 (2017); PRD96:064050 (2017);

Polarization of light if field is axionlike

Plascencia+JCAP1804 (2018) 059; Chen+ arXiv:1905.02213

Motion of stars close to supermassive BHs

Ferreira+PRD96 (2017)083017; Boskovic+PRD98 (2018) 024037; Davoudiasl+PRL123 (2019)021102; Bar+ JCAP1907 (2019)045; GRAVITY MNRAS 489 (2019) 4606

Constraints on fundamental fields via superradiance

Review in Brito+ Lect. Notes Phys.971 (2020)

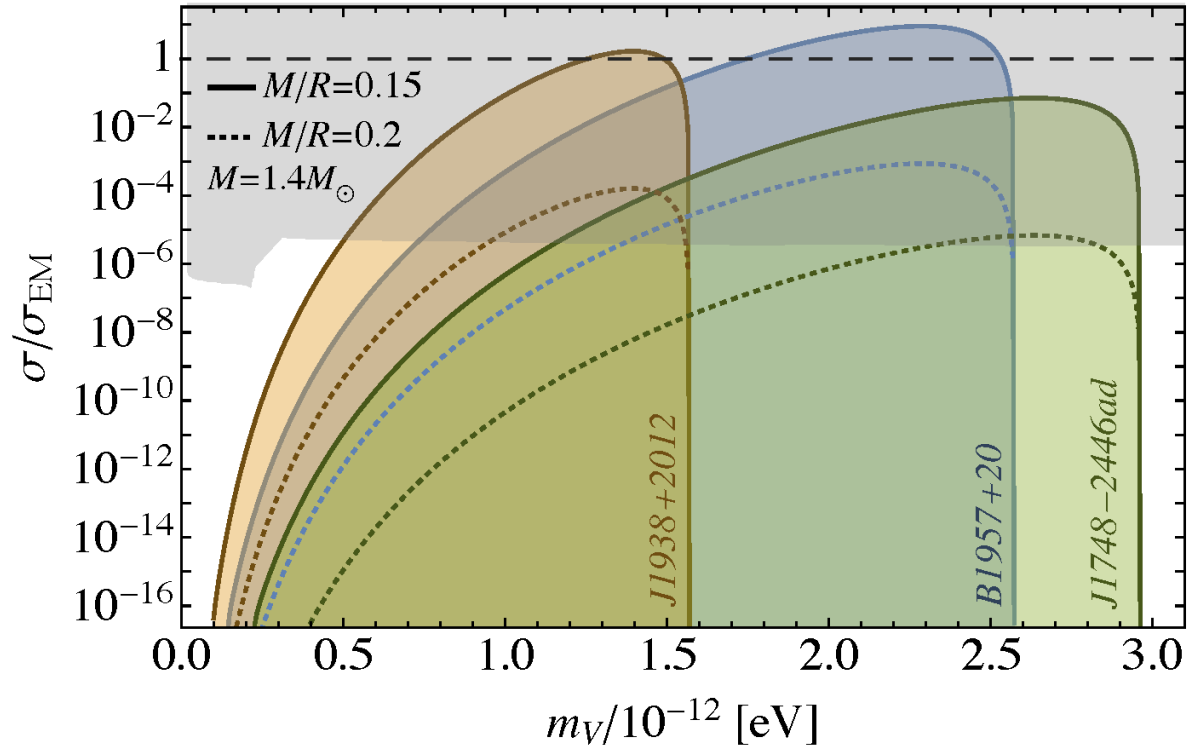
	excluded region (in eV)	source
*	$5.2 \times 10^{-13} < m_S < 6.5 \times 10^{-12}$	Direct bounds from absence of spin down in Cyg X-1.
*	$1.1 \times 10^{-13} < m_V < 8.2 \times 10^{-12}$	
*	$2.9 \times 10^{-13} < m_T < 9.8 \times 10^{-12}$	
	$6 \times 10^{-13} < m_S < 2 \times 10^{-11}$	Indirect bounds from BH mass-spin measurements.
	$7 \times 10^{-20} < m_S < 1 \times 10^{-16}$	
*	$2 \times 10^{-14} < m_V < 1 \times 10^{-11}$	
*	$1 \times 10^{-20} < m_V < 9 \times 10^{-17}$	
*	$6 \times 10^{-14} < m_T < 1 \times 10^{-11}$	
*	$3 \times 10^{-20} < m_T < 9 \times 10^{-17}$	
	$1.2 \times 10^{-13} < m_S < 1.8 \times 10^{-13}$	Null results from blind all-sky searches for continuous GW signals.
	$2.0 \times 10^{-13} < m_S < 2.5 \times 10^{-12}$	
	$m_V: \text{NA}$ $m_T: \text{NA}$	
	$6.4 \times 10^{-13} < m_S < 8.0 \times 10^{-13}$	Null results from searches for continuous GW signals from Cygnus X-1.
	$m_V: \text{NA}$ $m_T: \text{NA}$	
	$2.0 \times 10^{-13} < m_S < 3.8 \times 10^{-13}$	
	$m_V: \text{NA}$ $m_T: \text{NA}$	Negative searches for a GW background.
	$5 \times 10^{-13} < m_S < 3 \times 10^{-12}$	Bounds from pulsar timing.
	$m_V \sim 10^{-12}$	
	$m_T: \text{NA}$	
	$2.9 \times 10^{-21} < m_S < 4.6 \times 10^{-21}$	Bounds from mass and spin measurement of M87 with EHT.
	$8.5 \times 10^{-22} < m_V < 4.6 \times 10^{-21}$	
*	$1.0 \times 10^{-21} < m_T < 8.2 \times 10^{-21}$	

Constraints on fundamental fields via superradiance

M. Stott arXiv:2009.07206

Boson Spin	95% Confidence Limit Mass Bounds
Spin-0	$4.3 \times 10^{-14} \text{ eV} \leq \mu_0 \leq 2.7 \times 10^{-11} \text{ eV}$
	$1.7 \times 10^{-19} \text{ eV} \leq \mu_0 \leq 5.9 \times 10^{-17} \text{ eV}$
	$2.7 \times 10^{-21} \text{ eV} \leq \mu_0 \leq 4.5 \times 10^{-21} \text{ eV}$
Spin-1	$6.5 \times 10^{-15} \text{ eV} \leq \mu_1 \leq 2.9 \times 10^{-11} \text{ eV}$
	$2.9 \times 10^{-22} \text{ eV} \leq \mu_1 \leq 1.2 \times 10^{-16} \text{ eV}$
Spin-2	$2.5 \times 10^{-14} \text{ eV} \leq \mu_2 \leq 2.2 \times 10^{-11} \text{ eV}$
	$3.1 \times 10^{-20} \text{ eV} \leq \mu_2 \leq 9.1 \times 10^{-17} \text{ eV}$
	$6.4 \times 10^{-22} \text{ eV} \leq \mu_2 \leq 7.7 \times 10^{-21} \text{ eV}$

Stars?



Exclusion plots for known pulsars, based on measured spin-down rates. M is assumed to be $M=1.4 M_{\text{sun}}$. Grey is excluded region from CMB distortion (photon \rightarrow X depletion)

Cardoso+ PRD95: 124056 (2017); also Kaplan+arXiv:1908.10440

III. Transitions, disruption and floating by tides

$$V = -\theta (t - t_0) \frac{M_c \mu}{R} \sum_{|m| \leq 2} \frac{4\pi}{5} \left(\frac{r}{R}\right)^2 Y_{lm}^* (\theta_c, \phi_c) Y_{lm} (\theta, \phi)$$

$$\mathcal{H}\psi_n = E_n\psi_n$$

$$\mathcal{H} = \mathcal{H}_0 + V$$

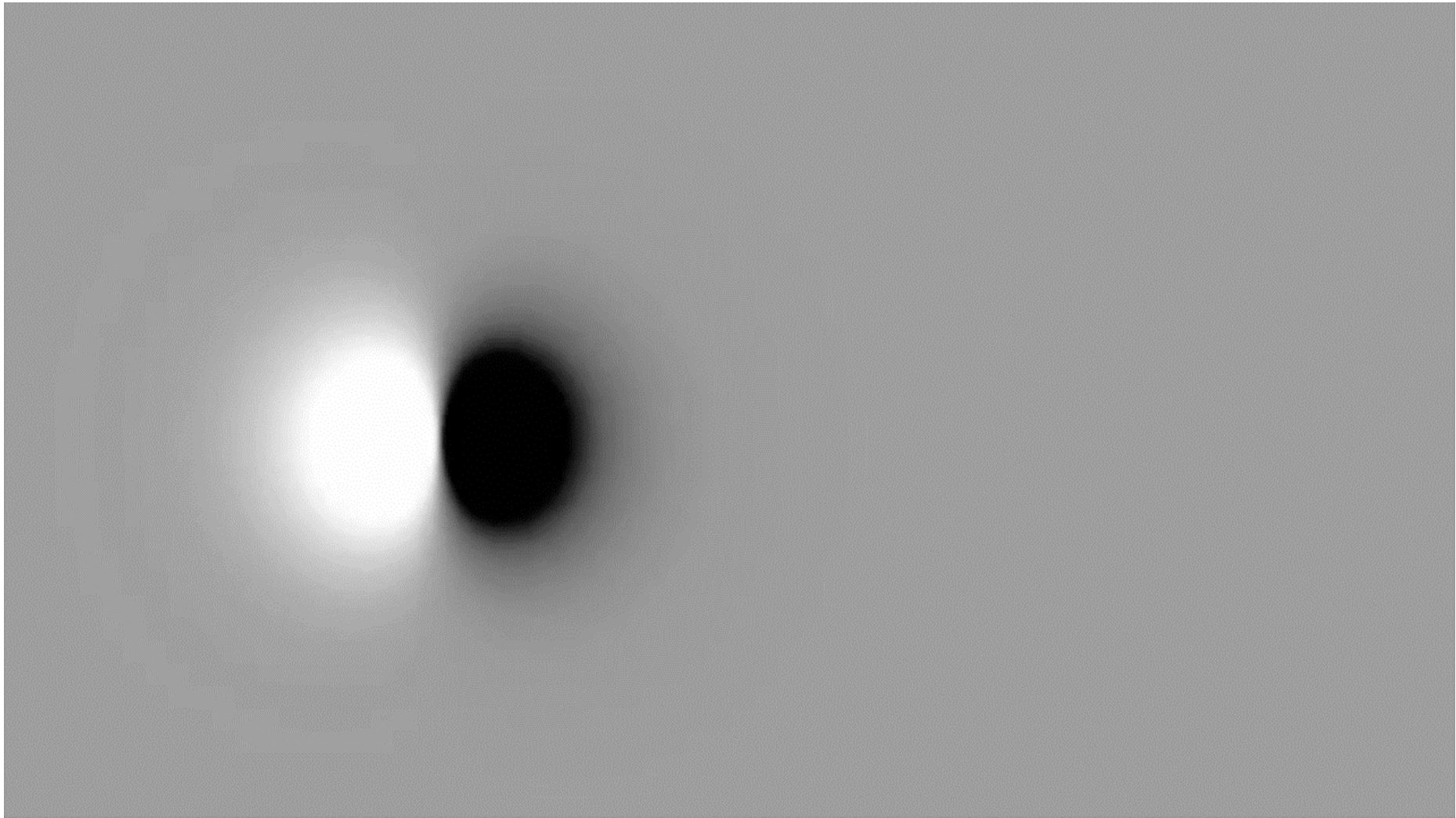
Use standard perturbation theory:

Find level transitions which lead to puffing up of cloud

Levels which change multipolar structure

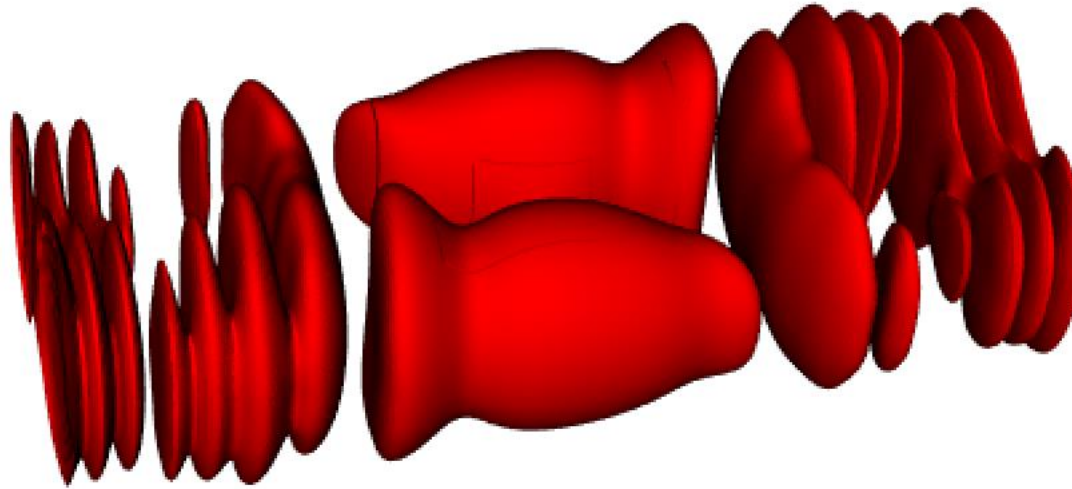
Resonances and cloud destruction

III. Transitions, disruption and floating by tides



Zhang+ PRD99 (2019) 064018; Baumann+ PRD99 (2019) 044001; Cardoso+ PRD101 (2020) 064054

Tidal disruption of clouds



Couplings to Standard Model: plasmas

Plasma gives photon an effective mass

$$\omega_p = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}} = 1.8 \times 10^3 \left(\frac{n_e}{10^{-3} \text{ cm}^{-3}} \right)^{1/2} \text{ rad s}^{-1}$$

...but it is position-dependent, plasma distribution-dependent... and nonlinearities make it transparent:

$$\omega_p \left(1 + \frac{e^2 E^2}{m_e^2 \omega^2} \right)^{-1/2} < \omega < \omega_p$$

Couplings to Standard Model

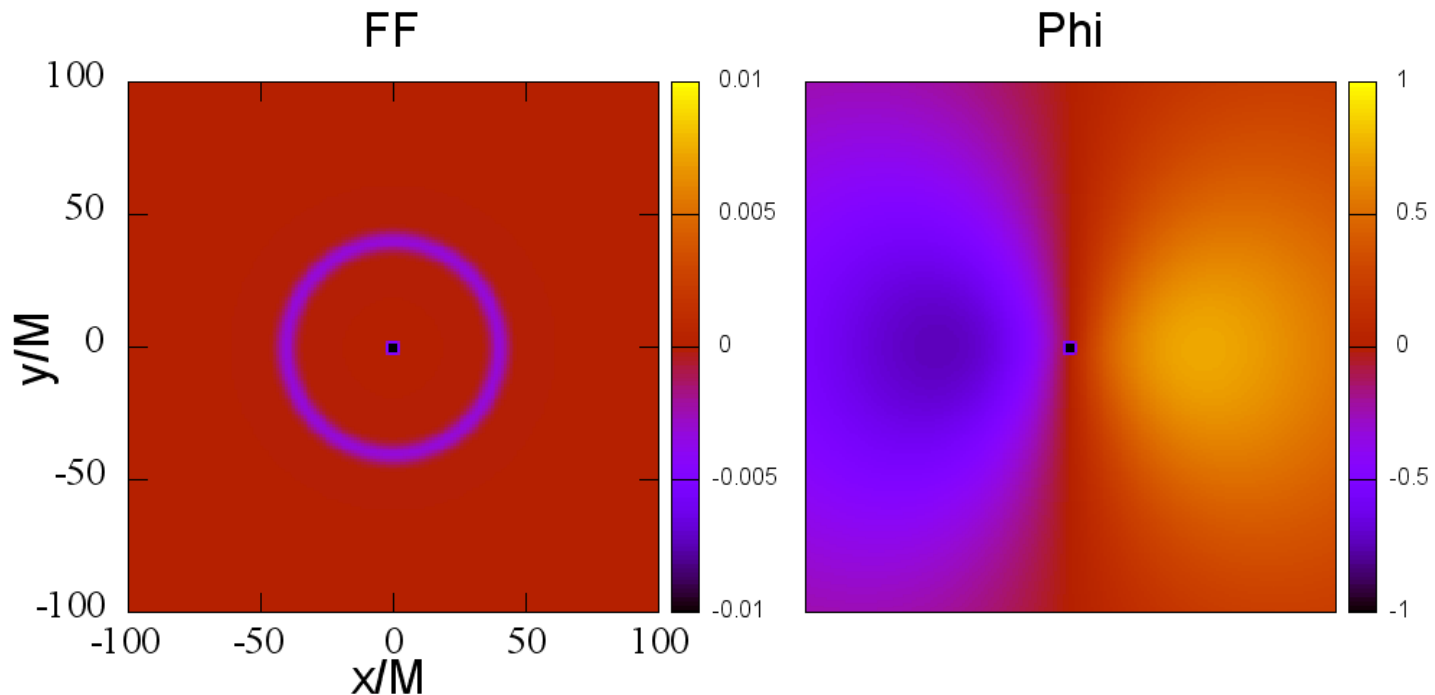
$$\mathcal{L} = \frac{R}{k} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Psi \partial_\nu \Psi - \frac{\mu_S^2}{2} \Psi \Psi - \frac{k_{\text{axion}}}{2} \Psi * F^{\mu\nu} F_{\mu\nu}$$

Boskovic+ PRD99:035006 (2019); Ikeda+ PRL122:081101 (2019)

Couplings to Standard Model

$$\mathcal{L} = \frac{R}{k} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}g^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi - \frac{\mu_S^2}{2}\Psi\Psi - \frac{k_{\text{axion}}}{2}\Psi * F^{\mu\nu}F_{\mu\nu}$$

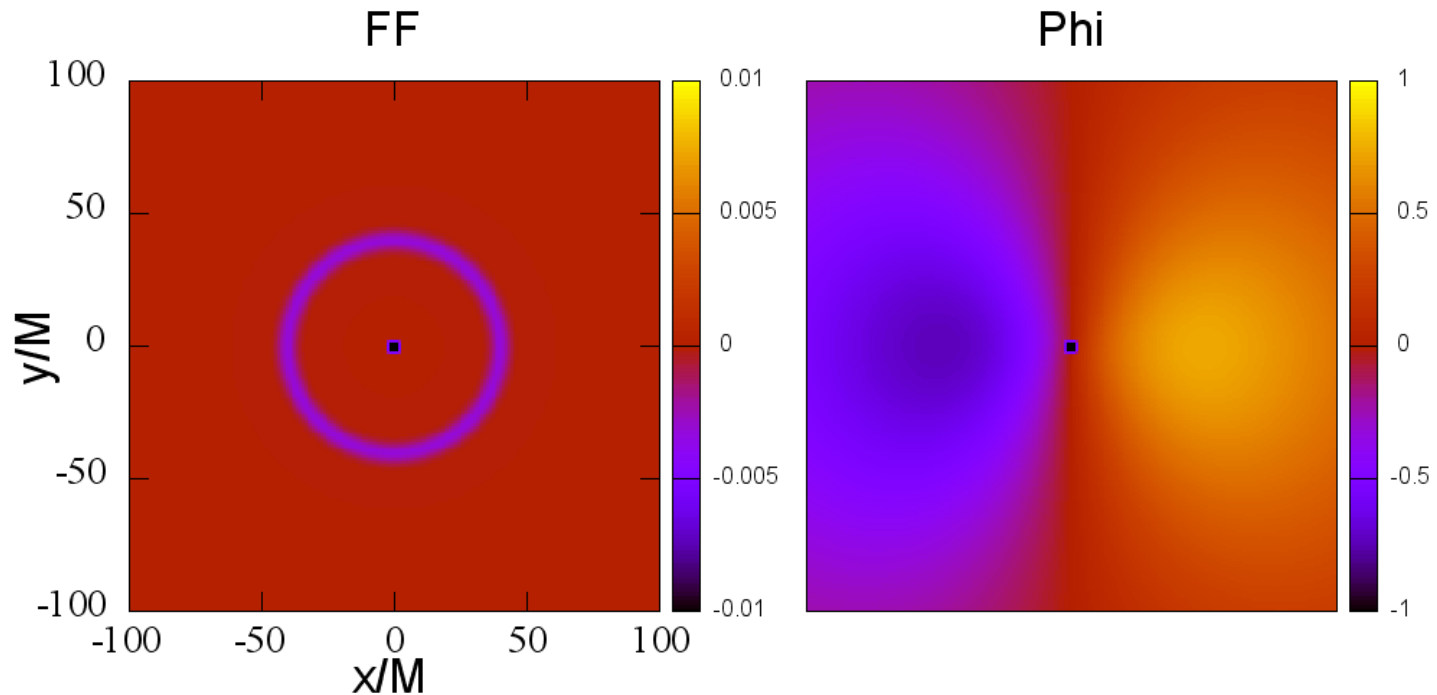
Boskovic+ PRD99:035006 (2019); Ikeda+ PRL122:081101 (2019)



Couplings to Standard Model

$$\mathcal{L} = \frac{R}{k} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}g^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi - \frac{\mu_S^2}{2}\Psi\Psi - \frac{k_{\text{axion}}}{2}\Psi * F^{\mu\nu}F_{\mu\nu}$$

Boskovic+ PRD99:035006 (2019); Ikeda+ PRL122:081101 (2019)



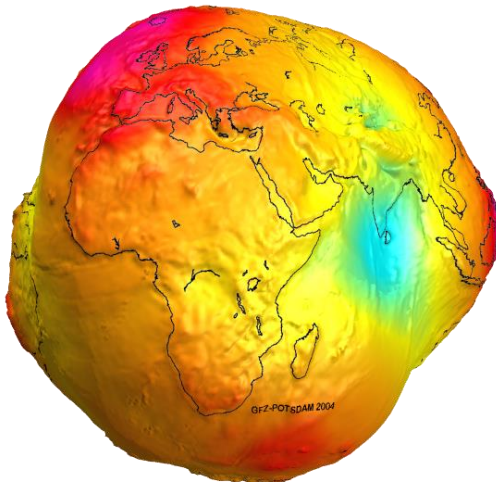
Black holes have no hair

Theorem a:

Isolated, stationary, regular BHs in the Einstein-Klein-Gordon with a *time-independent boson* are described by Kerr family
(impossible to hold the hair)

Theorem b:

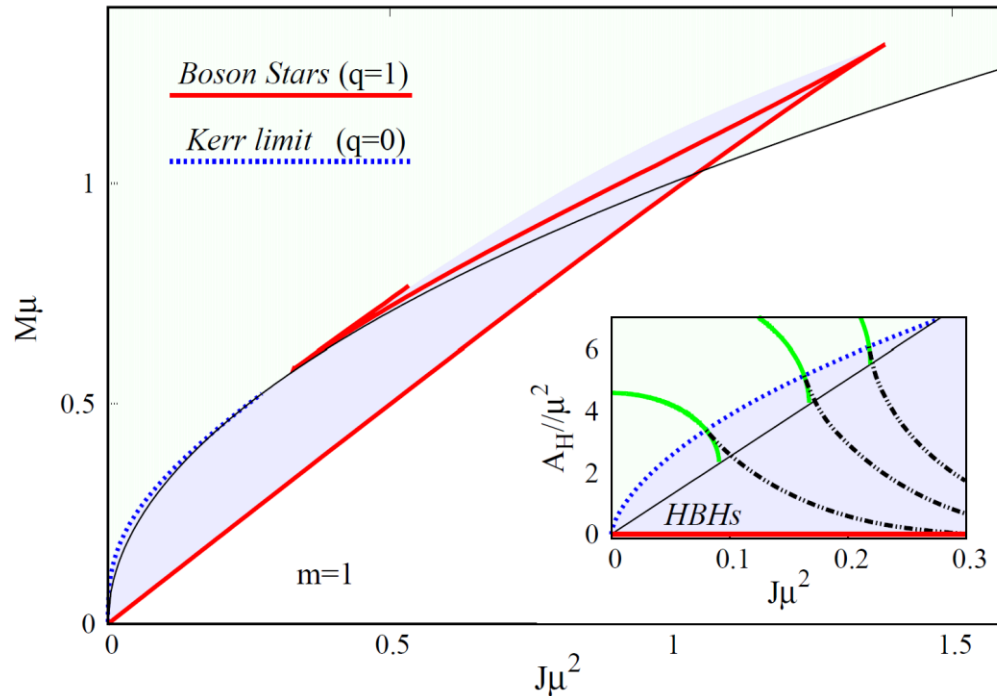
Isolated, stationary, regular BHs in the Einstein-Klein-Gordon theory with *one real scalar* are described by Kerr family
(impossible not to radiate GWs)



A stationary BH in vacuum is characterized by **mass and spin**

Growing hair with superradiance

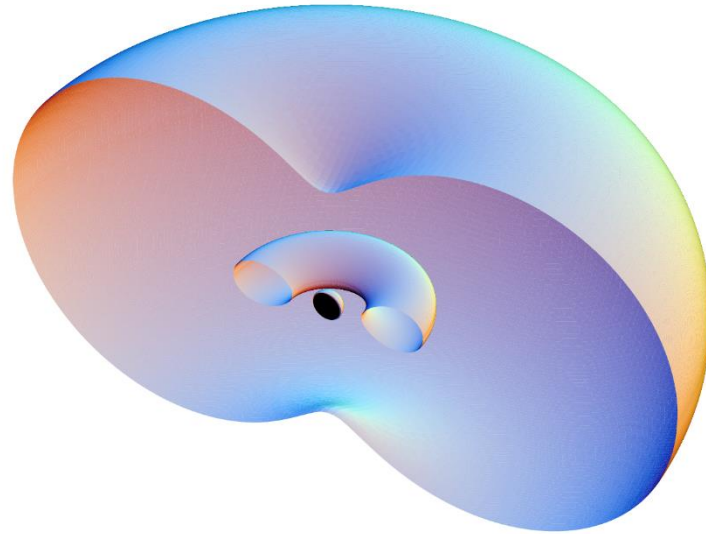
Herdeiro & Radu PRL112 (2014) 221101



Domain of existence of HBHs for $m = 1$ in M - J space. Inset: Area as a function of J along constant M curves. Solid green (dashed black) curves correspond to Kerr (HBHs). For the same M they bifurcate in the (dotted blue) Kerr line.

Growing hair with superradiance

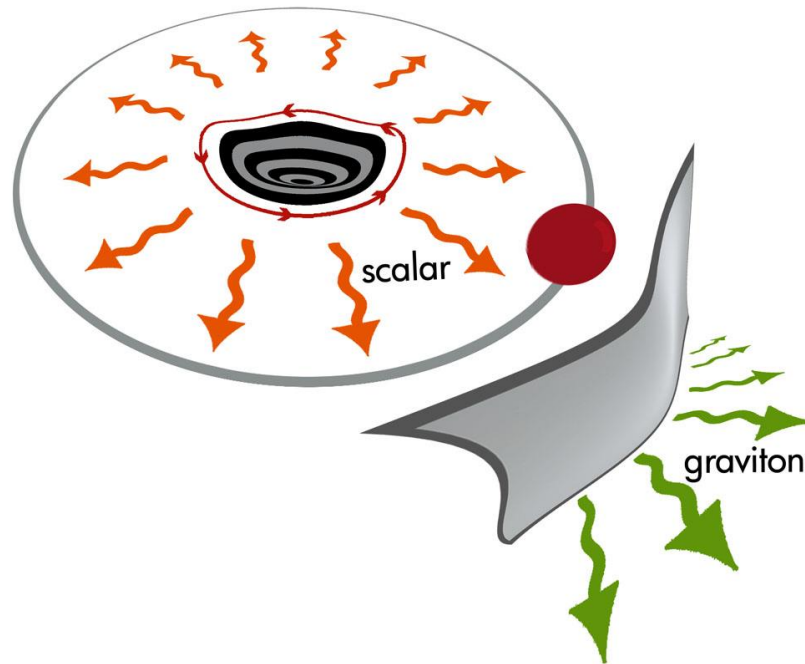
Herdeiro & Radu PRL112 (2014) 221101



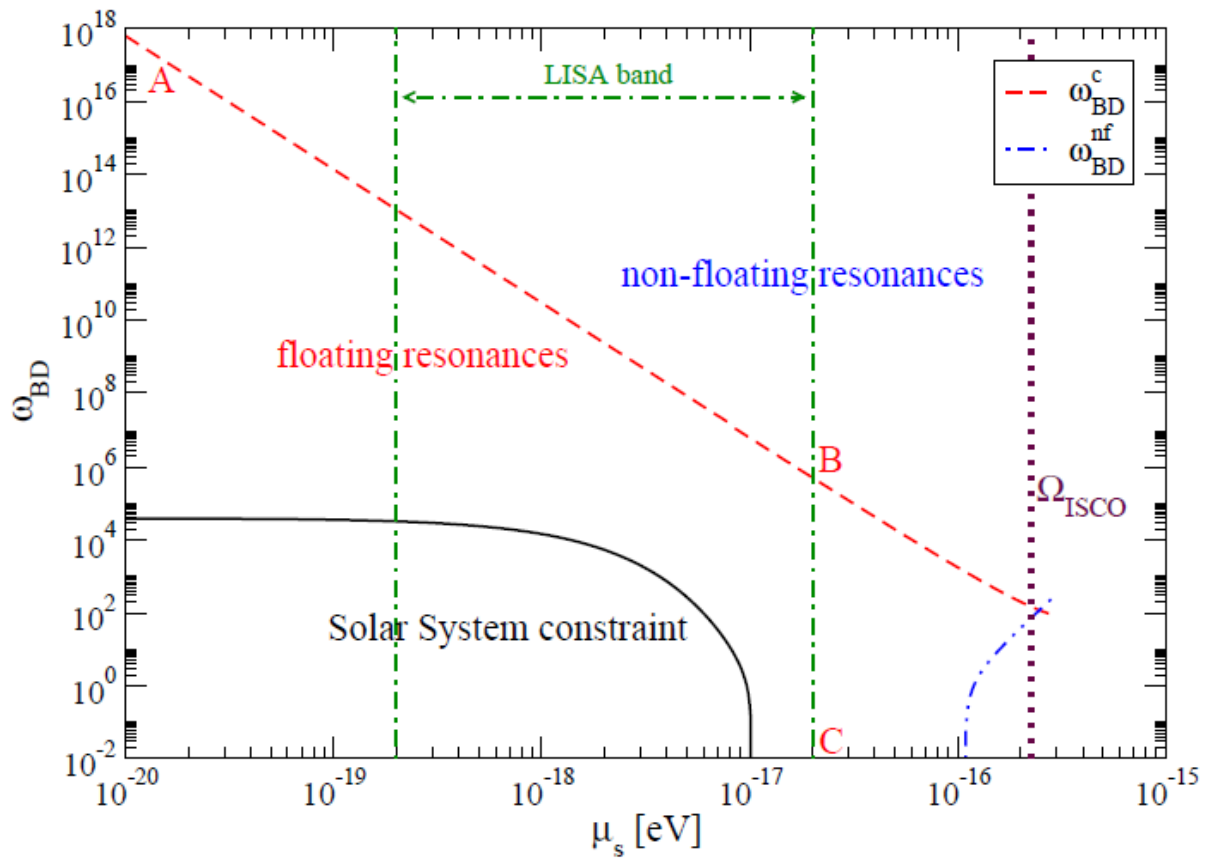
Orbital motion of stars and planets

I. Floating orbits

$$[\square - \mu_s^2] \varphi = \alpha \mathcal{T}$$

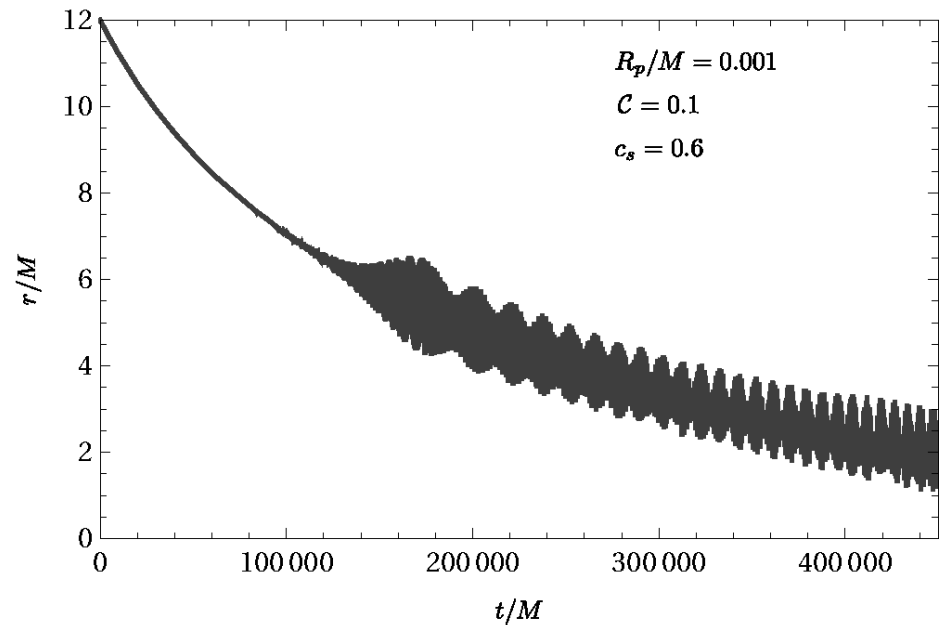
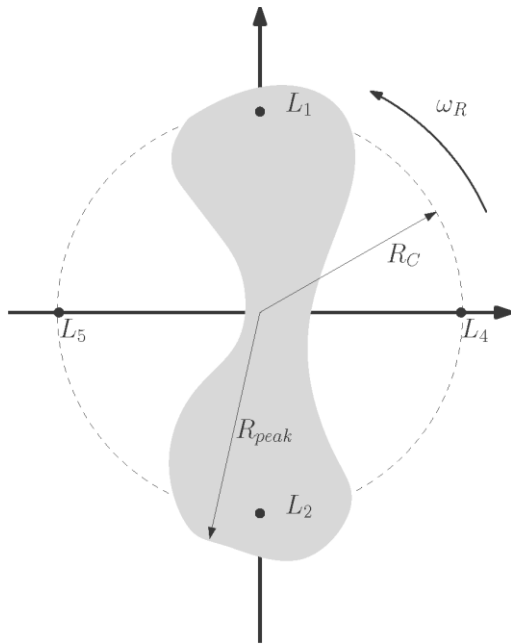


Cardoso + PRL107:241101 (2011); Yunes + PRD81, 084052 (2012);
Fujita + PRD95:044016 (2017); Zhang & Yang, arXiv:1808.02905



Orbital motion of stars and planets

II. Minimal couplings

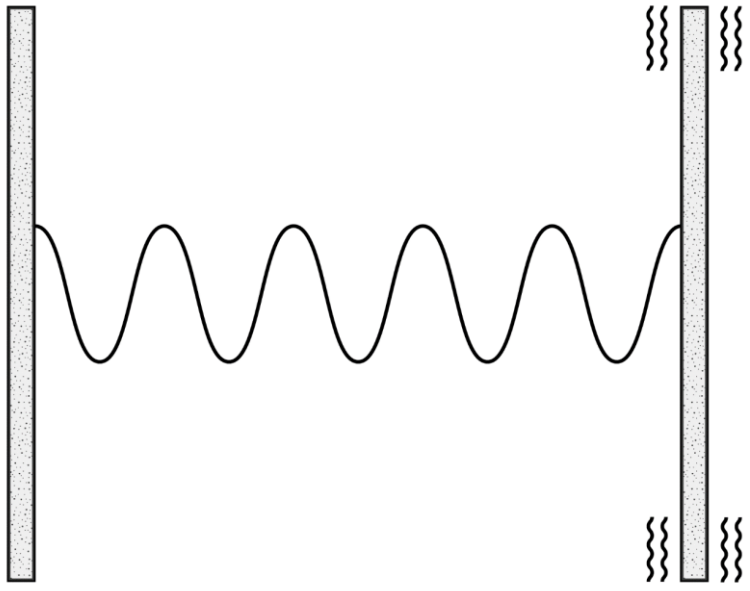


Macedo+ PRD96:083017 (2017); Boskovic+ PRD98:024037 (2018)

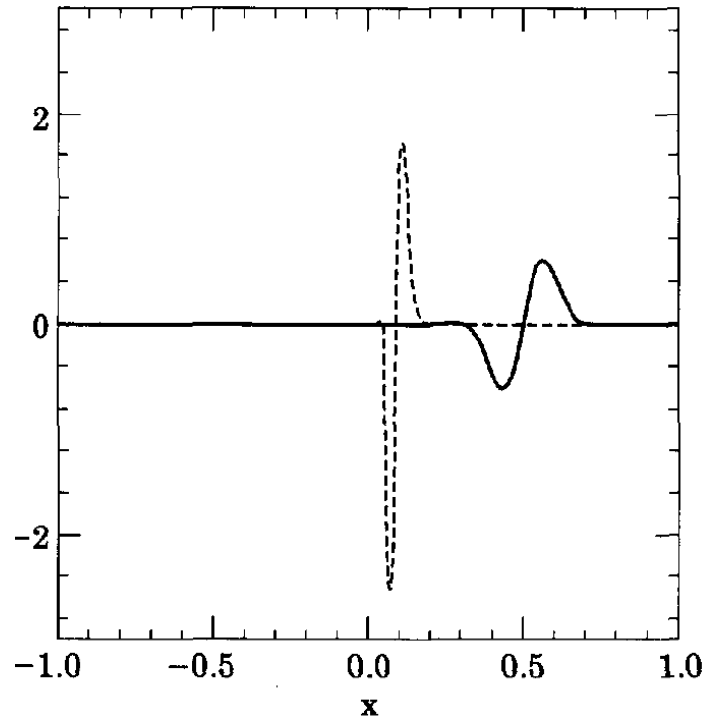
Also Khmelnsky & Rubakov JCAP1402:019 (2014); Blas+ PRL 118 (2017) 261102

Energy extraction from black hole binaries?

1. Superradiance, if individual black holes spin
2. Ergoregions in binaries?
3. Slingshot effect for massless waves
4. Parametric resonance? Fermi-like acceleration?



a.s.c.



Open questions

Can we measure (rotational) superradiance?

Torres+ Nature Physics 13:2017

Penrose process/superradiance

Vicente+ PRD97: 084032 (2018)

Is there a maximum amplification factor?

Do binaries superradiate?

Influence of viscosity?

Coupling to disks and/or magnetic fields?

Plasmas? Other environmental

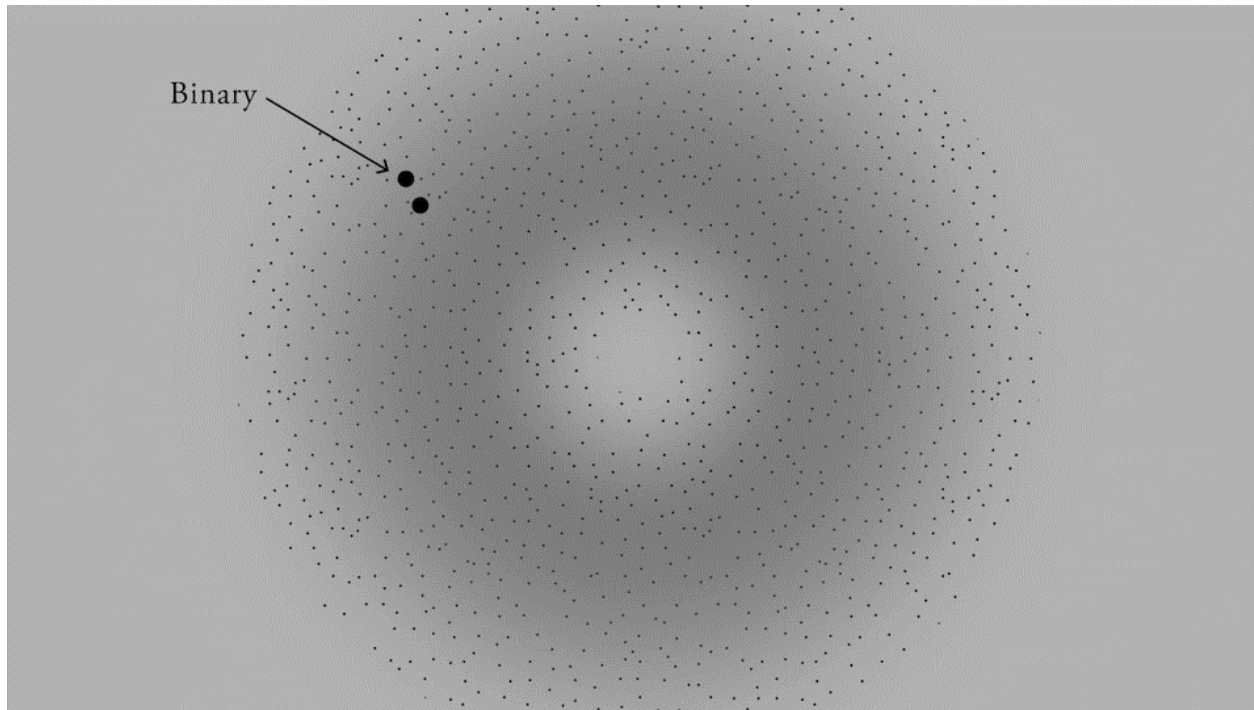
GWs and dark matter I

DM not strong-field phenomenon, but GW observations may reveal a “mundane” explanation in terms of heavy BHs.

Bird + PRL116:201301 (2016)

Inspiral occurs in DM-rich environment and may modify the way inspiral proceeds, given dense-enough media: accretion and gravitational drag play important role.

Eda + PRL110:221101 (2013); Macedo + ApJ774:48 (2013); Cardoso + arXiv 1909.05870; Kavanagh + arXiv 2002.12811; Annulli + arXiv 2009.00012



Small Compton wavelength: heavy DM

Accretion:

Bondi-Hoyle 1944

$$\dot{m}_i = 4\pi G^2 \rho \frac{m_i^2}{(v_i^2 + c_s^2)^{3/2}}$$

Dynamical friction:

Chandrasekhar 1943; Ostriker 1999

$$\mathbf{F}_{d,i} = -G^2 m_i^2 \rho I_d(v_i) \hat{\mathbf{r}}_i$$

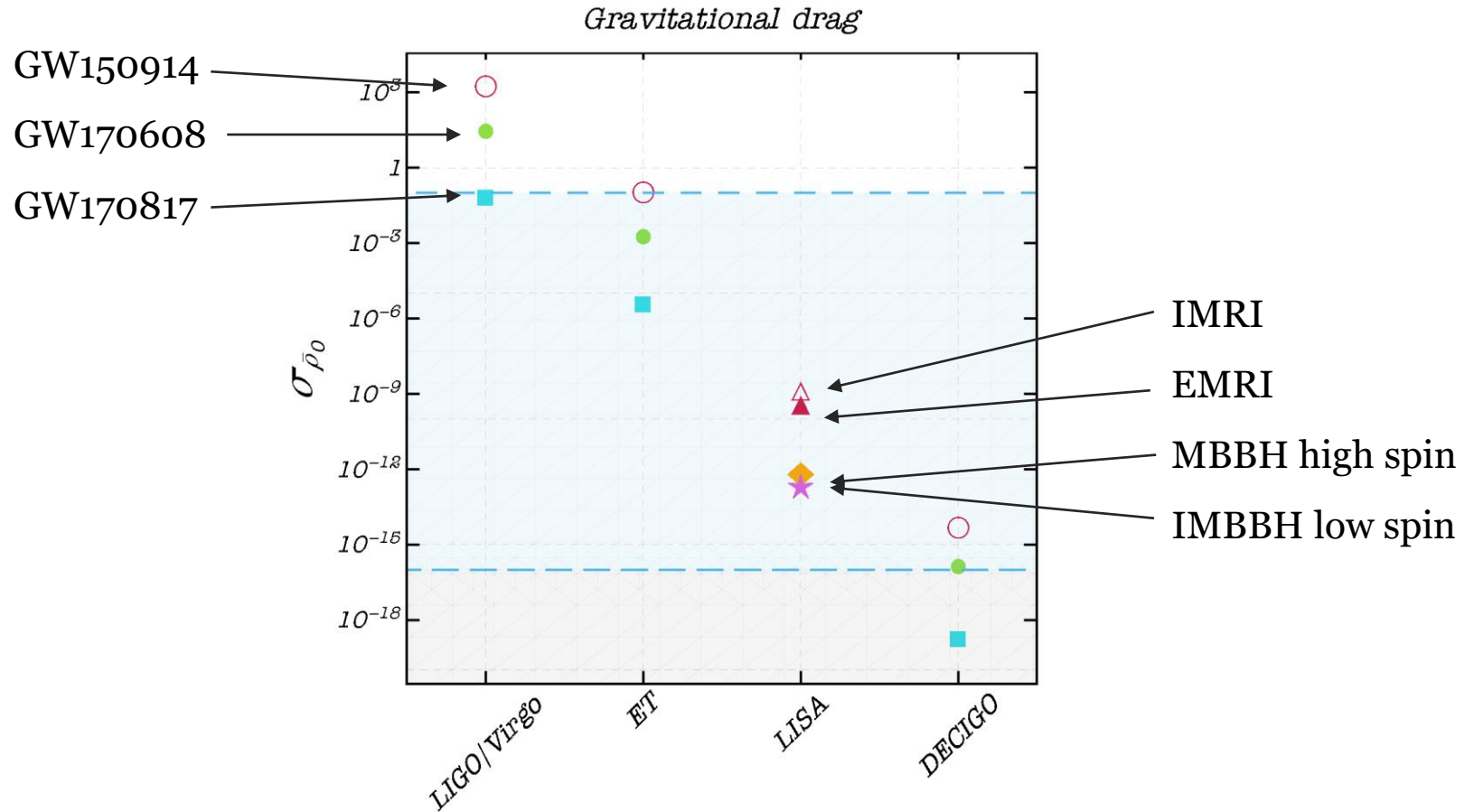
Dephasing (coefficients depend on DM distribution):

Silk and Eda 2013; Macedo+ 2013; Barausse+ 2014; Cardoso & Maselli 2019

$$\Psi(f) = \Psi_{\text{GR}}^{(0)} [1 + (\text{PN corrections}) + \delta_{\Psi_{\text{env}}}]$$

$$\delta_{\Psi_{\text{env}}}^{\text{DF}} \approx -\frac{(1 - 3\eta)R^\beta \rho_0}{\eta^2 \mathcal{M}^{\frac{1}{3}(\beta+5)}} (\pi f)^{\frac{2\beta}{3} - \frac{11}{3}}$$

Small Compton wavelength: heavy DM



Effect is -5.5 PN on GW phase

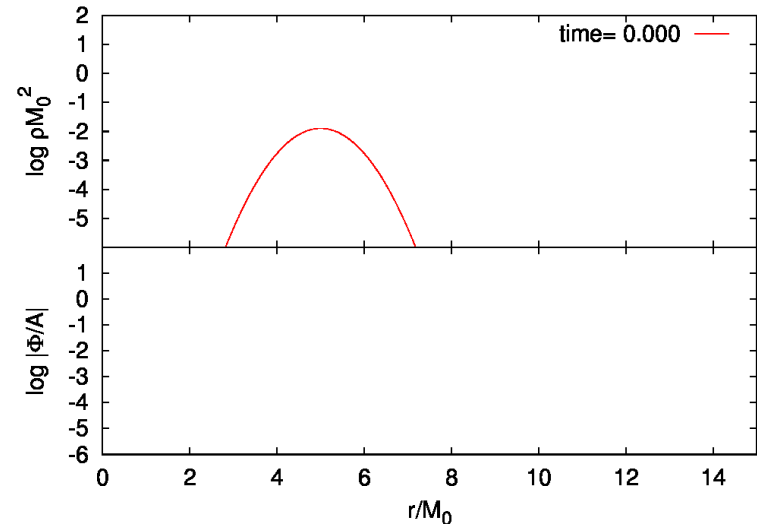
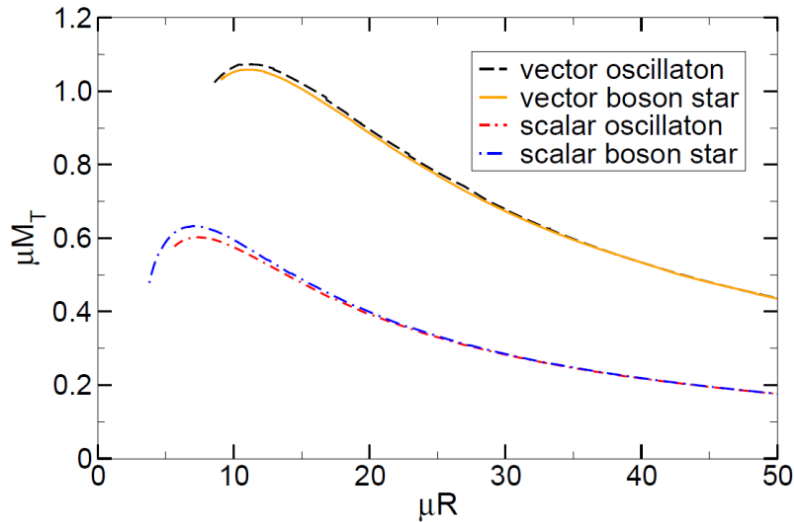
Cardoso & Maselli arXiv 1909.05870

Also Eda + PRL 110 (2013) 221101; Macedo+ApJ774 (2013) 48

Large Compton wavelength: ultralight DM

For motivation see Robles & Matos 2012; Hui, Ostriker & Witten 2016

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi^* - \frac{\mu^2}{2} \Phi^2 \right)$$

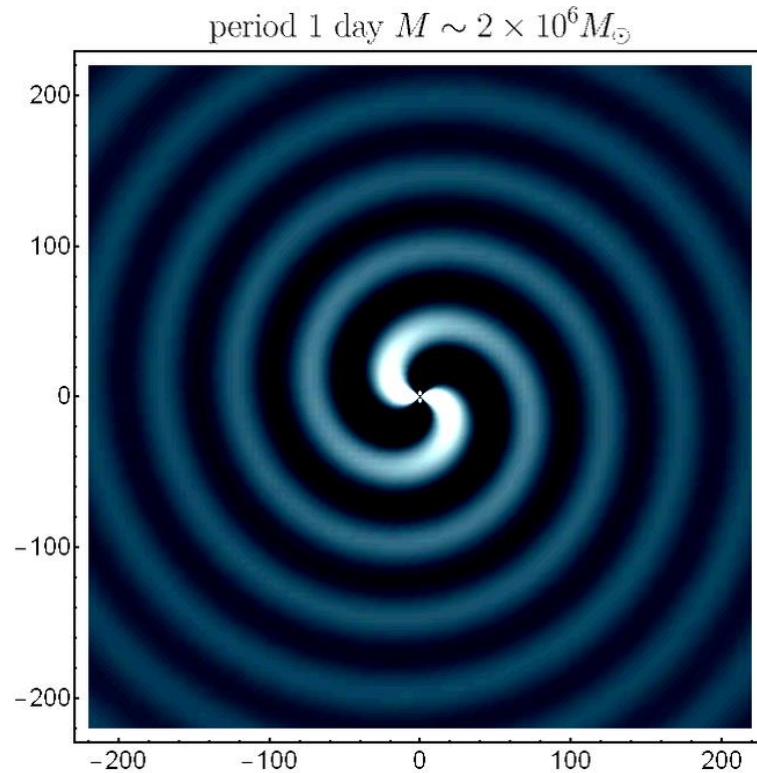


$$\frac{M_{\max}}{M_\odot} = 8 \times 10^{-11} \frac{\text{eV}}{m_B c^2}$$

Universal profiles and universal dynamics!

Large Compton wavelength: ultralight DM

$$\nabla^2 \delta\Psi_p = 2\mu (\mu U_0 + \gamma) \delta\Psi_p + 2\mu^2 \Psi_0 \delta U_p$$
$$\nabla^2 \delta U_p = 4\pi (2\mu^2 \Psi_0 \delta\Psi_p + P)$$



Large Compton wavelength: ultralight DM

$$\nabla^2 \delta\Psi_p = 2\mu (\mu U_0 + \gamma) \delta\Psi_p + 2\mu^2 \Psi_0 \delta U_p$$

$$\nabla^2 \delta U_p = 4\pi (2\mu^2 \Psi_0 \delta\Psi_p + P)$$

$$\begin{aligned} \dot{E}^{\text{rad}} &= 128\pi^3 (\mu^2 m_p \Psi_0)^2 (1 + (-1)^m)^2 \\ &\times \sum_{m=1}^{+\infty} \left(\frac{Y_m^m(\pi/2, 0)}{\Gamma(m + 3/2)} \frac{m^{m-1} (M\omega_{\text{orb}})^{m/3}}{2^{m+1} \omega_{\text{orb}}} \right)^2 \end{aligned}$$

$$\Upsilon(f) = \Upsilon_{\text{GR}}^{(0)} [1 + (\text{PN corrections}) + \delta\Upsilon]$$

$$\delta\Upsilon = \frac{16\mu^4 \Psi_0^2}{51\pi^3 f^4} \sim 10^{-24} \left[\frac{\mu}{10^{-22} \text{ eV}} \right]^4 \left[\frac{10^{-4} \text{ Hz}}{f} \right]^4 \left[\frac{M_{\text{NBS}} \mu}{0.01} \right]^4$$

-6 PN on GW phase

Conclusions: exciting times!

Gravitational wave astronomy *will* become a precision discipline, mapping compact objects throughout the entire visible universe.

Black holes remain the most outstanding object in the universe. They respond in simple way to external perturbations, and may serve as seeds for atoms of fundamental light fields. Black hole and BH binary spectroscopy is an exciting tool to understand the content of our universe

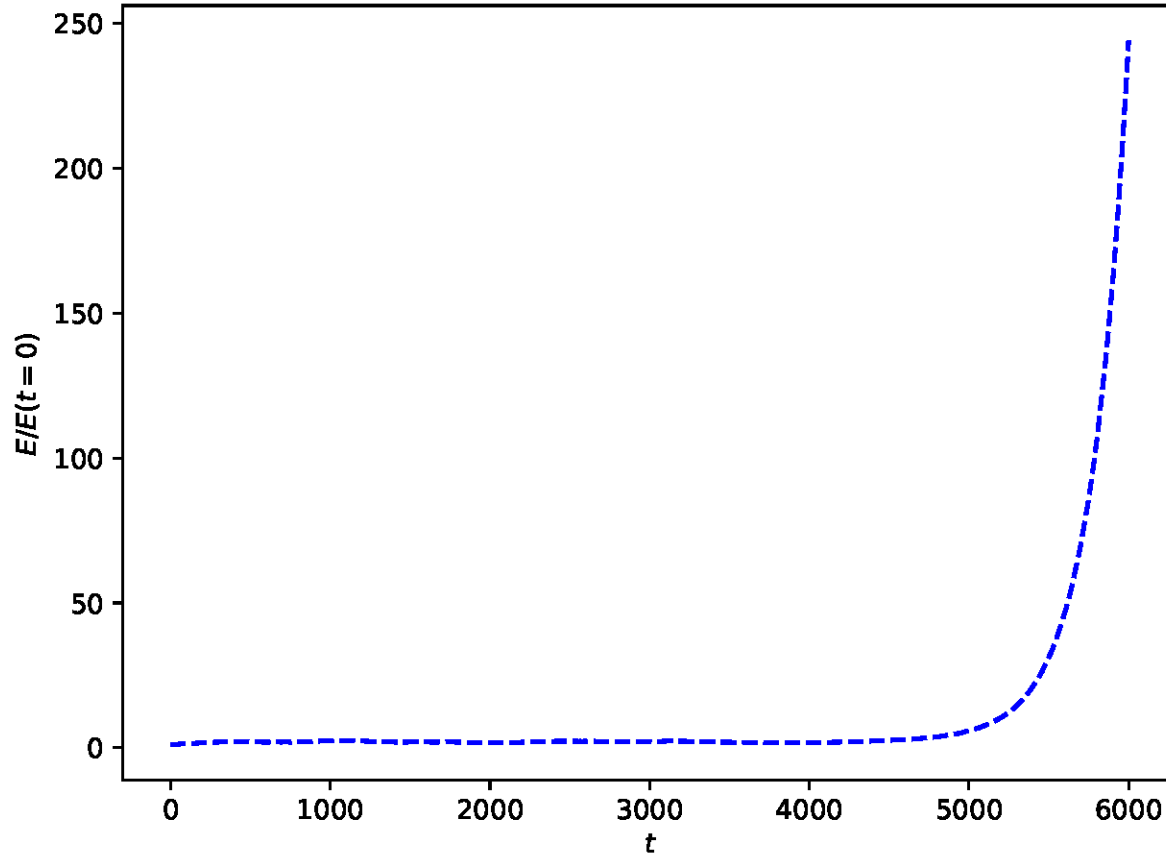
“After the advent of gravitational wave astronomy, the observation of these resonant frequencies might finally provide direct evidence of BHs with the same certainty as, say, the 21 cm line identifies interstellar hydrogen”

(S. Detweiler ApJ 239:292 1980)

Thank you



Energy extraction from binaries?

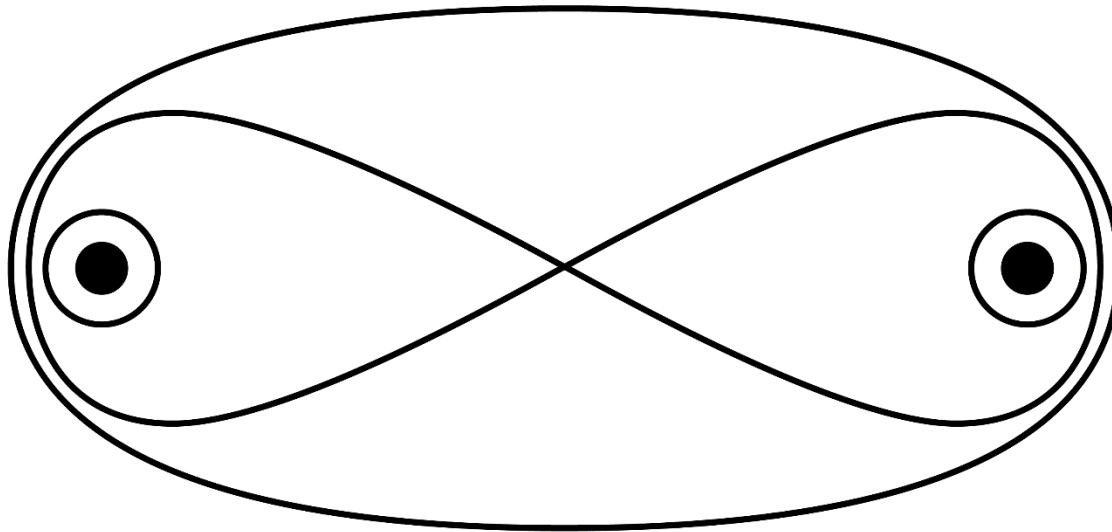


Bernard + arXiv:1905.05204 (and work in progress)

Gravitational molecules: a toy model

$$ds^2 = -\frac{dt^2}{U^2} + U^2 (d\rho^2 + \rho^2 d\phi^2 + dz^2)$$

$$U(\rho, z) = 1 + \frac{M}{\sqrt{\rho^2 + (z - a)^2}} + \frac{M}{\sqrt{\rho^2 + (z + a)^2}}$$



Gravitational molecules: a toy model

Change to prolate confocal elliptic coordinates

$$\rho^2 + (a - z)^2 = a^2(\chi + \eta)^2$$

$$\rho^2 + (a + z)^2 = a^2(\chi - \eta)^2$$

$$\partial_\eta \left((1 - \eta^2) \partial_\eta S \right) + \left(-a^2 \omega^2 \eta^2 - \frac{m^2}{1 - \eta^2} + \Lambda \right) S = 0$$

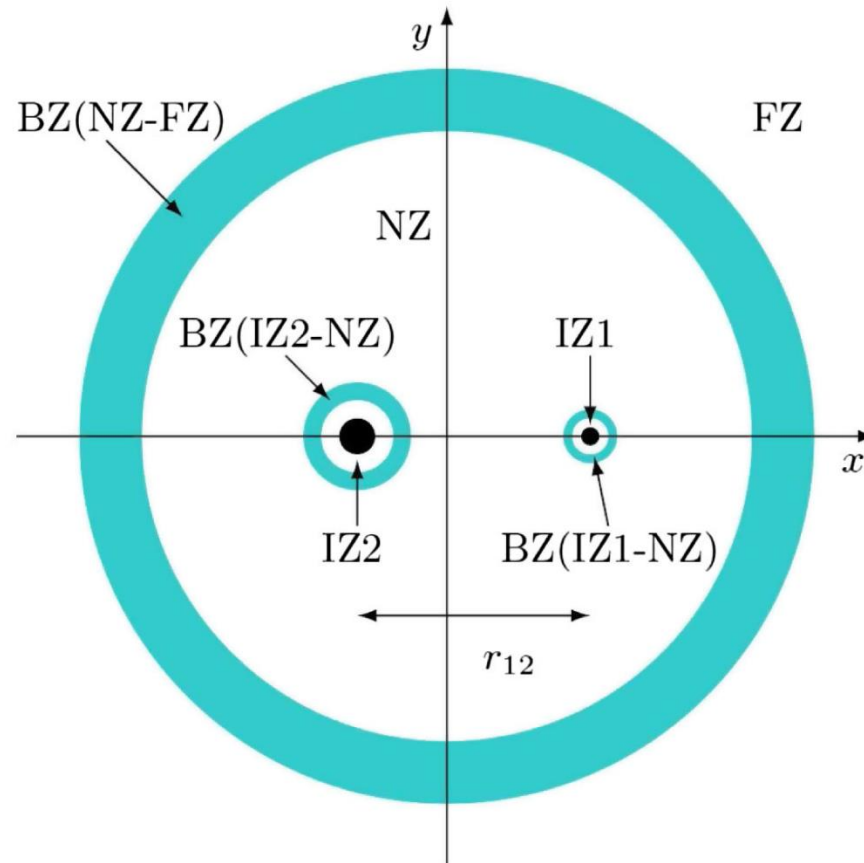
$$\partial_\chi \left((\chi^2 - 1) \partial_\chi R \right) + \left(a^2 \omega^2 \chi^2 + 8Ma\chi\omega^2 - \frac{m^2}{\chi^2 - 1} - \Lambda \right) R = 0$$

Klein-Gordon equation is identical to Schrodinger
for Di-Hydrogen ionized molecule!

Bernard+ (2019)

*for Hydrogen molecule see Burrau M7: 1 (1928);
Wilson PRSLA118:635 (1929); Hylleraas ZfP71: 739 (1931)*

Gravitational molecules: a real BH binary



Mundim+ PRD89: 084008 (2014); Bernard + (2019)

GWs and dark matter

I

Dark matter is not a strong-field phenomenon, but GW observations may reveal a more “mundane” explanation in terms of heavy BHs

Bird + PRL116:201301 (2016)

II

Inspiral occurs in dark-matter rich environment and may modify the way inspiral proceeds, given dense-enough media: accretion and gravitational drag play important role.

Eda + PRL110:221101 (2013); Macedo + ApJ774:48 (2013)

DM II

Inspiral occurs in dark-matter rich environment and may modify the way inspiral proceeds, given dense-enough media: accretion and gravitational drag play important role.

Eda + PRL110:221101 (2013); Macedo + ApJ774:48 (2013); Barausse+PRD 2014

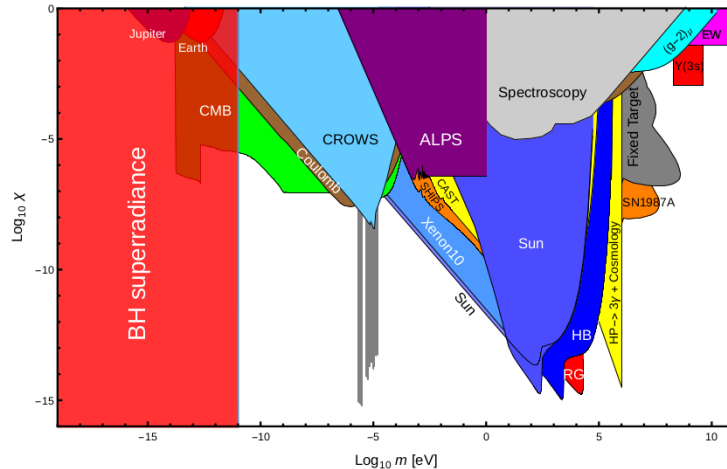
Self-gravity:

$$\rho_0 = 10^3 M_\odot \text{pc}^{-3} \sim 10^4 \text{GeVcm}^{-3}$$
$$\frac{M_{\text{inside } r}^{\text{DM}}}{M_{\text{BH}}} = 10^{-19} \left(\frac{M_{\text{BH}}}{10^6 M_\odot} \right)^2 \left(\frac{r}{100M} \right)^3 \frac{\rho_{\text{DM}}}{\rho_0}$$

Accretion:

$$\dot{M}_{\text{BH}} = \frac{16\pi G^2 M_{\text{BH}}^2 \rho_{\text{DM}}}{v_{\text{DM}} c^2} \quad (\dot{M} = \sigma \rho v)$$
$$\frac{\Delta M_{\text{BH}}}{M_{\text{BH}}} = 10^{-16} \left(\frac{M_{\text{BH}}}{10^6 M_\odot} \right) \frac{\rho_{\text{DM}}}{\rho_0} \frac{T}{1 \text{ year}} \left(\frac{\sigma_v}{220 \text{ Km/s}} \right)^{-1}$$

DM III. Light fields



→ Cardoso+ 2018, adapted from Sigl (2017) and Jaeckel arXiv:1303.1821

Interesting as effective description; proxy for more complex interactions; arise as interesting extensions of GR* (*BD or generic ST theories, $f(R)$, etc.*)

Bosons do exist (Higgs) and lighter versions may as well
*Peccei-Quinn (interesting because not invented to solve DM problem),
 axiverse (moduli and coupling constants in string theory)*

$$\mathcal{L} = \frac{R}{k} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Psi \partial_\nu \Psi - \frac{\mu_S^2}{2} \Psi \Psi - \frac{k_{\text{axion}}}{2} \Psi * F^{\mu\nu} F_{\mu\nu}$$

...and one or more could be a component of DM. *D. Marsh, Phys. Repts. 2016*