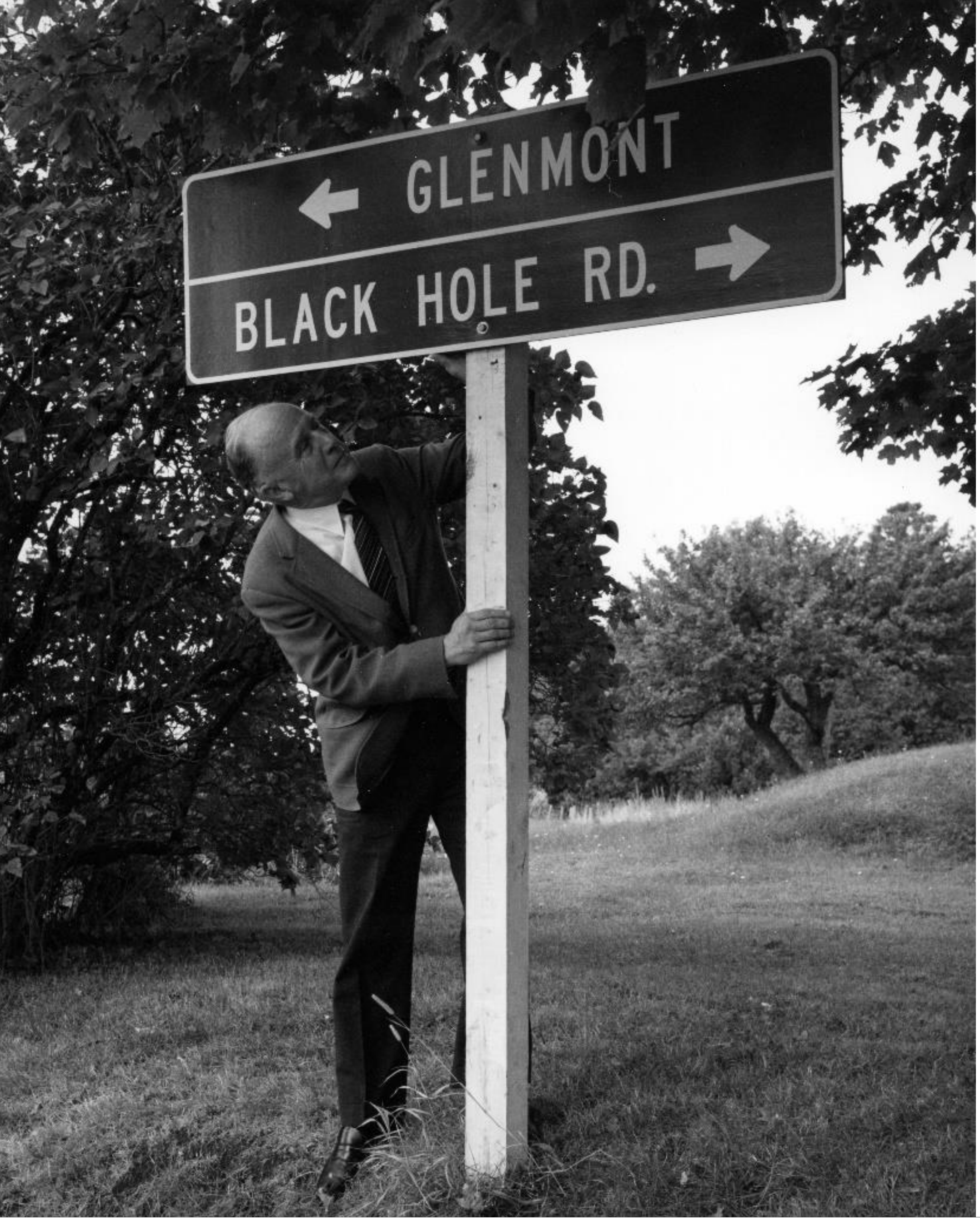

Part I. Black holes and General Relativity

Vítor Cardoso
(Técnico Lisboa)

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Origins of the Classical Definition

16. Hence, according to article 10, if the semi-diameter of a sphære of the same density with the sun were to exceed that of the sun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its surface a greater velocity than that of light, and consequently, supposing light to be attracted by the same force in proportion to its vis inertiæ, with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity.

1919. May 29 eclipse confirms that gravity “bends” light

REVOLUTION IN SCIENCE.

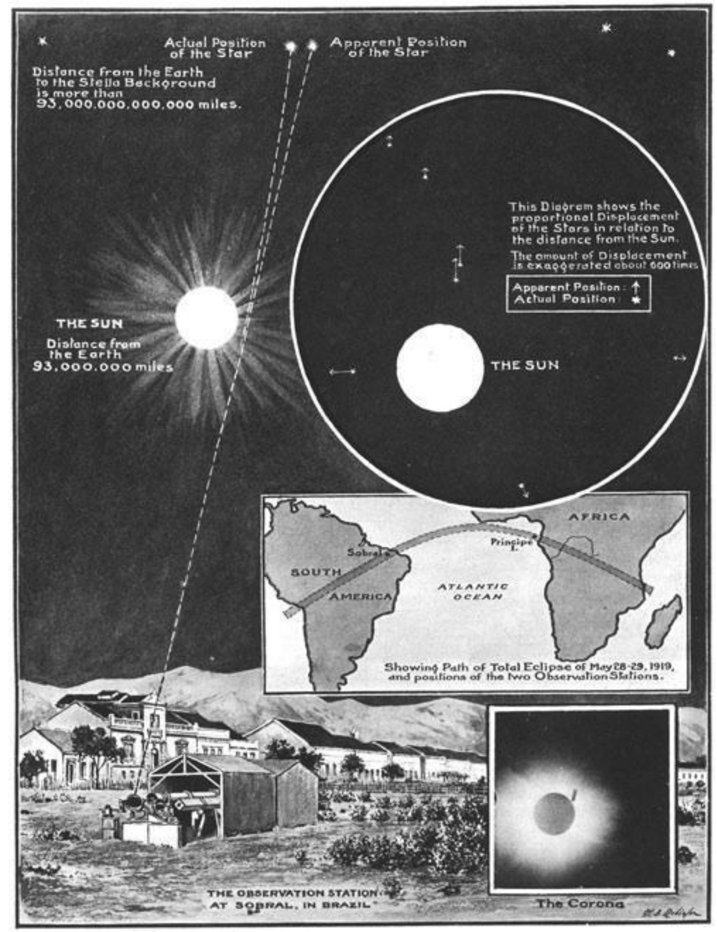
NEW THEORY OF THE UNIVERSE.

NEWTONIAN IDEAS OVERTHROWN.

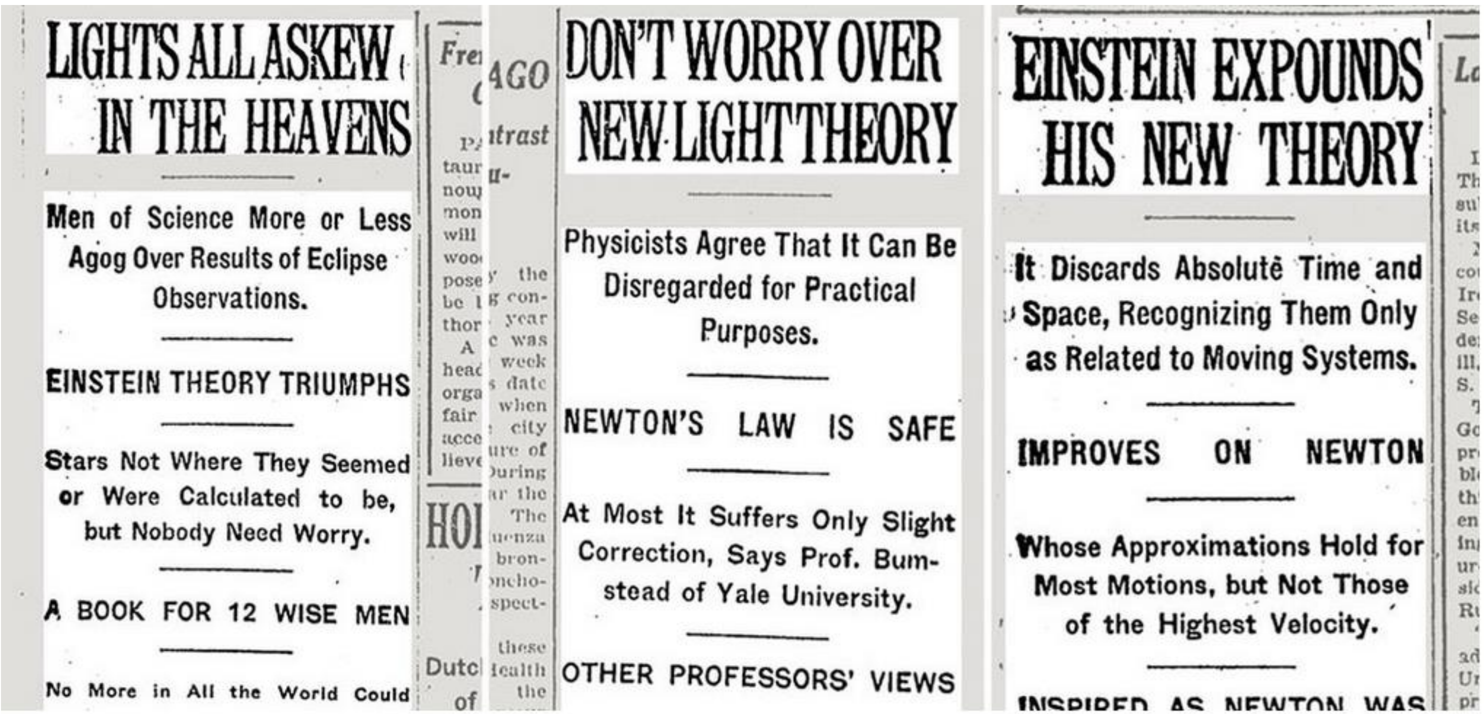
Yesterday afternoon in the rooms of the Royal Society, at a joint session of the Royal and Astronomical Societies, the results obtained by British observers of the total solar eclipse of May 29 were discussed.

The greatest possible interest had been aroused in scientific circles by the hope that rival theories of a fundamental physical problem would be put to the test, and there was a very large attendance of astronomers and physicists. It was generally accepted that the observations were decisive in the verifying of the prediction of the famous physicist, Einstein, stated by the President of the Royal Society as being the most remarkable scientific event since the discovery of the predicted existence of the planet Neptune. But there was differ-

‘Times of London‘, Nov 7 1919



'Illustrated London News‘, Nov 22 1919


LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could

DON'T WORRY OVER NEW LIGHT THEORY

Physicists Agree That It Can Be Disregarded for Practical Purposes.

NEWTON'S LAW IS SAFE

At Most It Suffers Only Slight Correction, Says Prof. Bumstead of Yale University.

OTHER PROFESSORS' VIEWS

EINSTEIN EXPOUNDS HIS NEW THEORY

It Discards Absolute Time and Space, Recognizing Them Only as Related to Moving Systems.

IMPROVES ON NEWTON

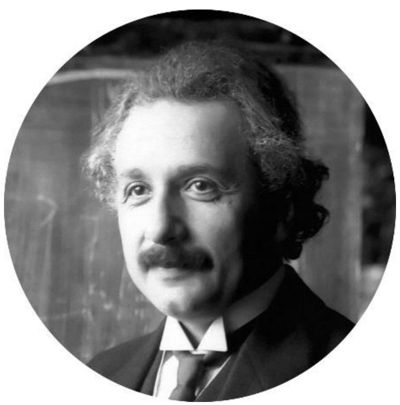
Whose Approximations Hold for Most Motions, but Not Those of the Highest Velocity.

INSPIRED AS NEWTON WAS

Articles in The Times from Nov. 10, 1919, left; Nov. 16, 1919, center; and Dec. 3, 1919.

“I made at once by good luck a search for a full solution. A not too difficult calculation gave the following result: ...”

K. Schwarzschild to A. Einstein
(Letter dated 22 December 1915)



Solution re-discovered by many others:

J. Droste, May 1916 (part of PhD thesis under Lorentz):
Same coordinates, more elegant

P. Painlevé, 1921, A. Gullstrand, 1922: P-G coordinates
(not realized solution was the same)

...and many others

Some History

1784	John Michell and Pierre-Simon Laplace (1796) propose the existence of “dark stars”… that lock in light (in Newtonian mechanics)
1915	Albert Einstein publishes his first paper on GR
1916	Karl Schwarzschild finds the first nontrivial solution
1930	Subrahmanyan Chandrasekhar suggests that a massive star can collapse into something denser (1930)
1939	Oppenheimer and Snyder predict that massive stars can collapse eternally
1963	Roy Kerr finds solution for rotating BHs, Schmidt identifies first quasar
1964	UHURU orbiting X-ray obs. identifies 300 x-ray stars, Cygnus X-1
1967	John Wheeler coins the term “black hole”
1970-1990s	Too much to describe (Hawking radiation, uniqueness, BH formation…)
2015	EH Telescope and GRAVITY missions track region near horizon
2016	LIGO measures gravitational waves from BHs

Long, complex path to correct interpretation

Eddington

Lemaître

Oppenheimer

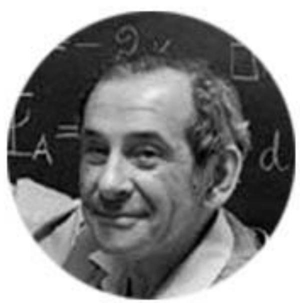
Snyder

Wheeler

Finkelstein

Kruskal

Penrose



Israel

Carter

Hawking

Uniqueness: the Kerr solution

Theorem (Carter 1971; Robinson 1975; Chrusciel and Costa 2012):
A stationary, asymptotically flat, vacuum BH solution must be Kerr

$$\begin{aligned} ds^2 &= \frac{\Delta - a^2 \sin^2\theta}{\Sigma} dt^2 + \frac{2a(r^2+a^2-\Delta)\sin^2\theta}{\Sigma} dt d\phi \\ &\quad - \frac{(r^2+a^2)^2 - \Delta a^2 \sin^2\theta}{\Sigma} \sin^2\theta d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 \end{aligned}$$

$$\Sigma = r^2 + a^2\cos^2\theta\,, \quad \Delta = r^2 + a^2 - 2Mr$$

Describes a rotating BH with mass M and angular momentum J=aM, iff a<M

"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's equations of general relativity provides the *absolutely exact representation* of untold numbers of black holes that populate the universe."

S. Chandrasekhar, The Nora and Edward Ryerson lecture, Chicago April 22 1975

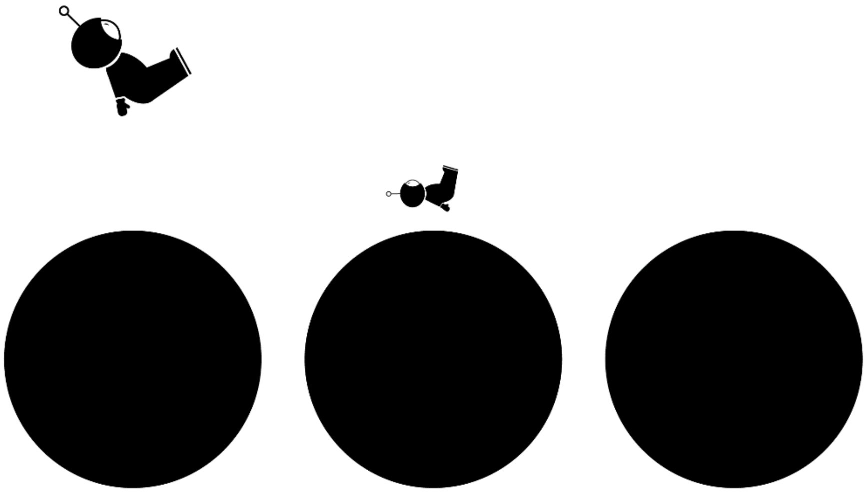
Black holes have no hair

J. A. Wheeler 1971

One star of matter and another of anti-matter produce the same BH.

BH shares 3 common quantities with progenitor
(when no radiative processes):

mass, rotation (and electric charge)



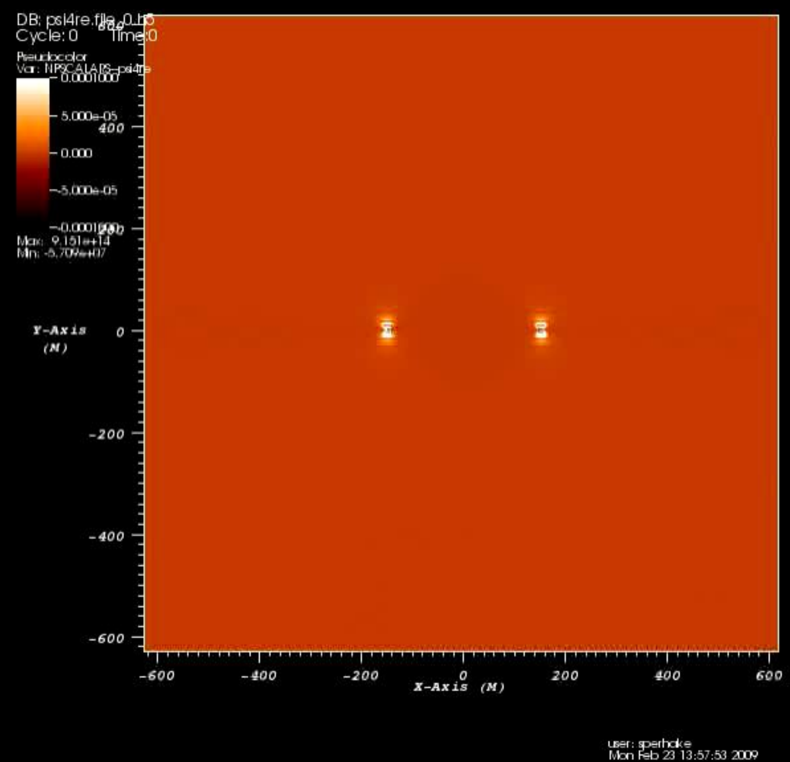
(Weak) Cosmic Censorship violations?

$$M,\ J$$

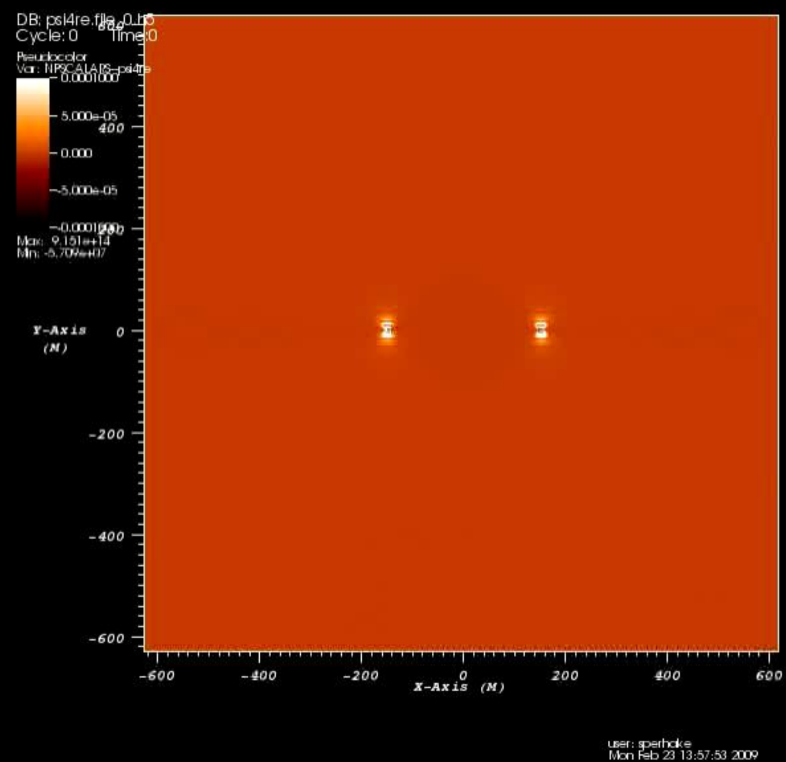
$$\frac{cJ}{GM^2} \leq 1 \qquad \text{or} \qquad a \leq M$$

Black holes have small angular momentum

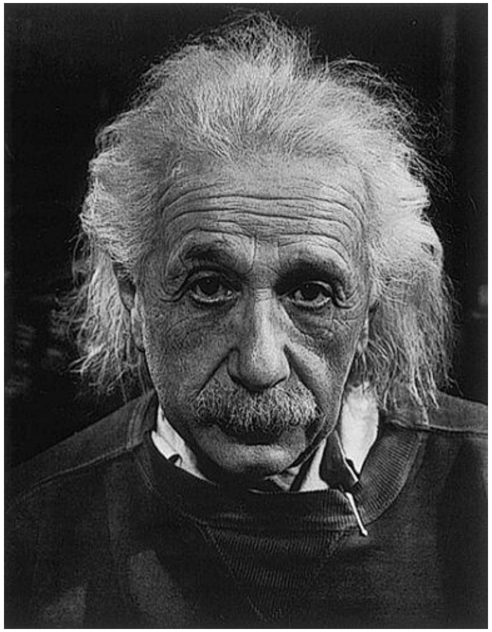
(very compact objects)



Sperhake + PRL103:131102 (2009)



Sperhake + PRL103:131102 (2009)



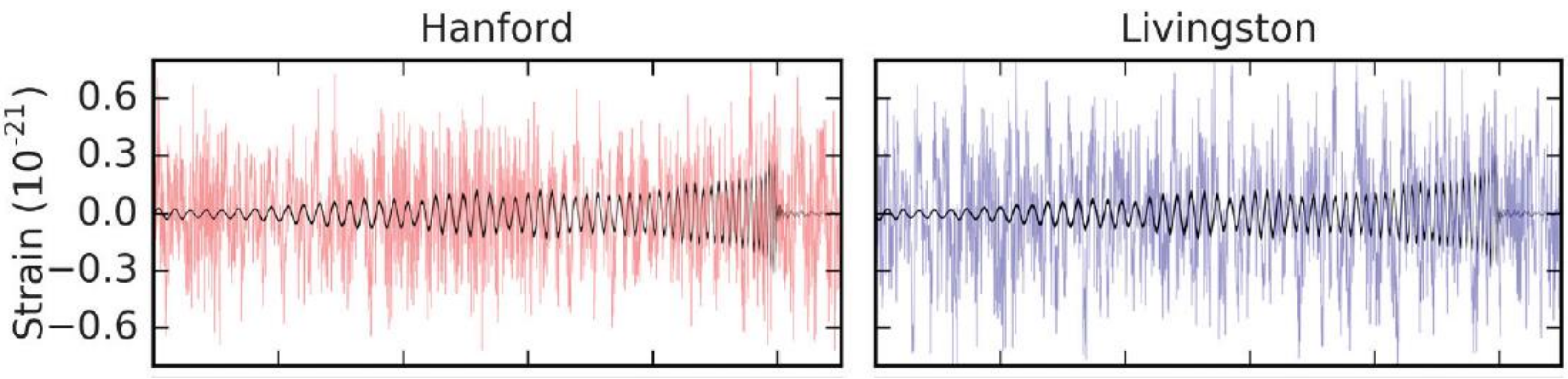
In 1916, Einstein shows that GWs are a consequence of the linear theory.

$$ds^2 = -c^2 dt^2 + (1 + h_+)dx^2 + (1 - h_+)dy^2 + 2h_\times dx dy + dz^2$$

$$\partial_z^2 h_{+,\times} - \frac{1}{c^2}\partial_t^2 h_{+,\times} = 0$$

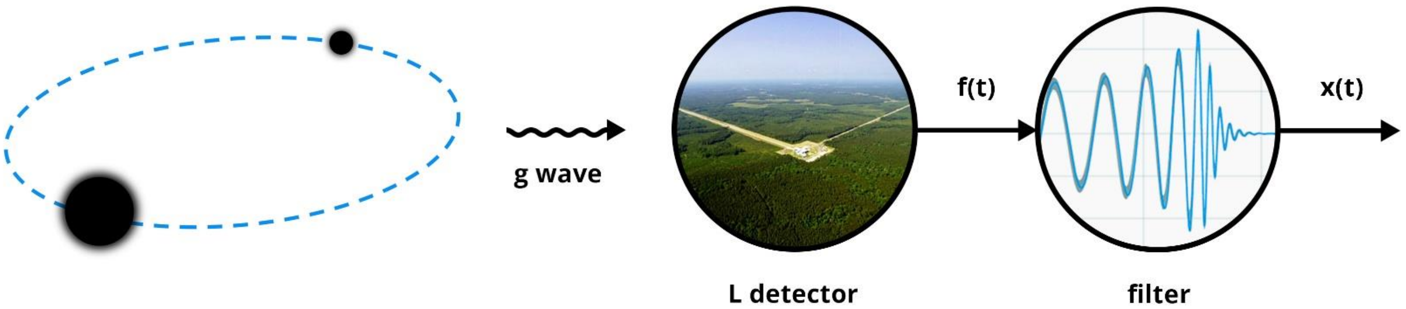
GWs travel at the speed of light

The needle in the haystack problem



The LIGO Collaboration, PRL116:241103 (2016)

Matched-Filtering

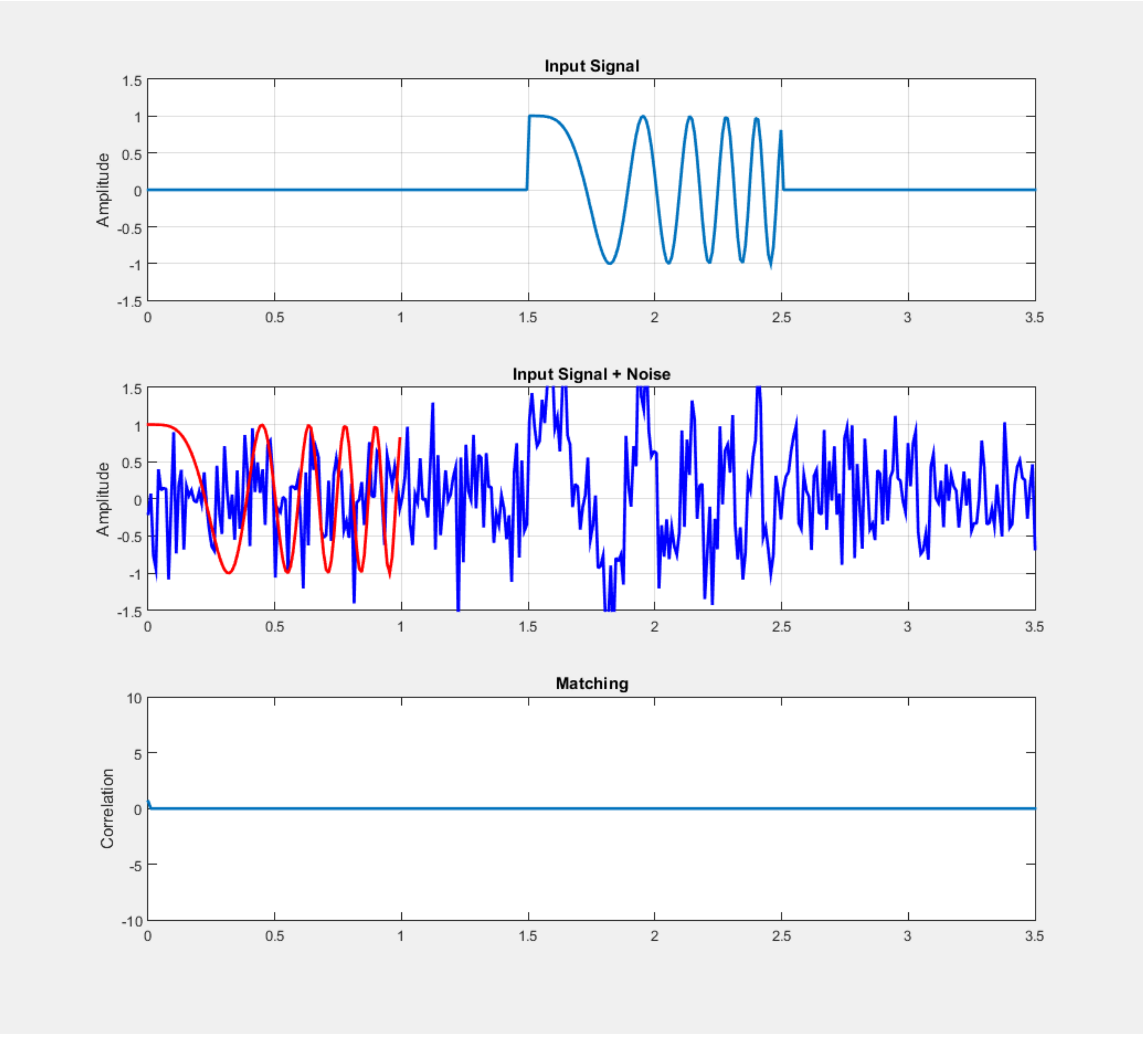


The detector output $f(t) = h(t) + n(t)$

where n(t) is the noise. For stationary Gaussian noise, process signal with filter K(t) against data stream producing number

$$\frac{S}{N} = \frac{\text{expected value of X with signal}}{\text{rms value of X with no signal}} = \frac{\langle X \rangle}{\sqrt{\langle X^2 \rangle_{h=0}}}$$

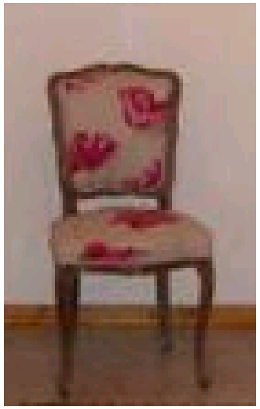
Optimum filter K maximizes SNR and is the signal h itself!



3% Mismatch: 10% lost events!... LIGO used ~250000 templates for CBC searches

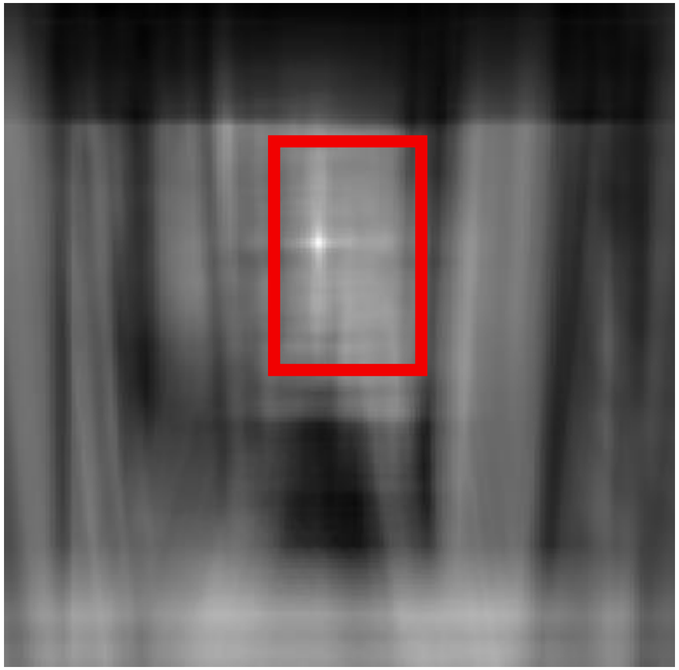
Object recognition...

This is a chair



Find the chair in this image

Output of normalized correlation



Template bank

Problem:

Do not know the intrinsic parameters of signal, *masses, spins, distances*...

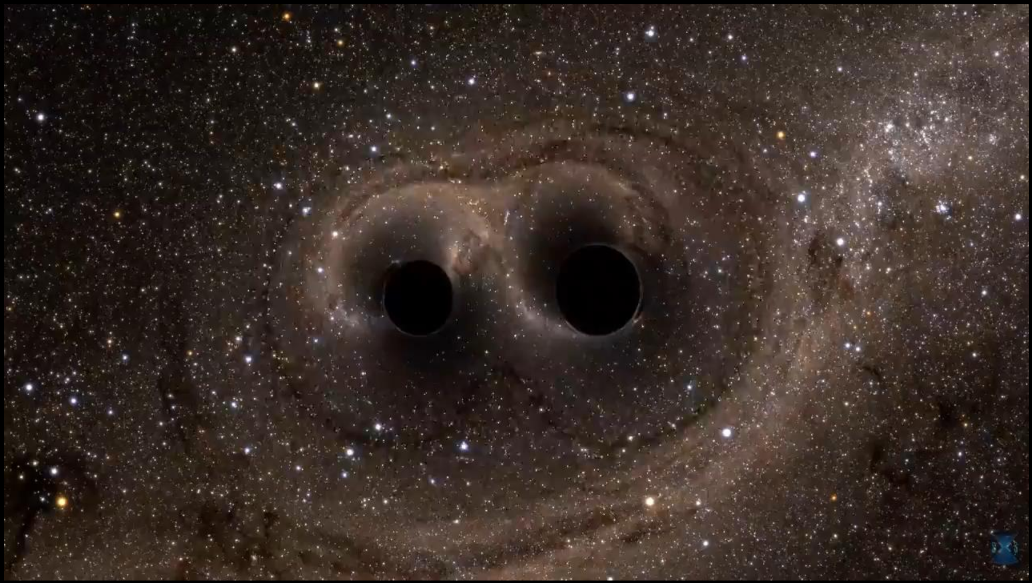
Want to detect any signal in a space of possible signals, all with different phase evolution...

And of course, with a finite set of templates!

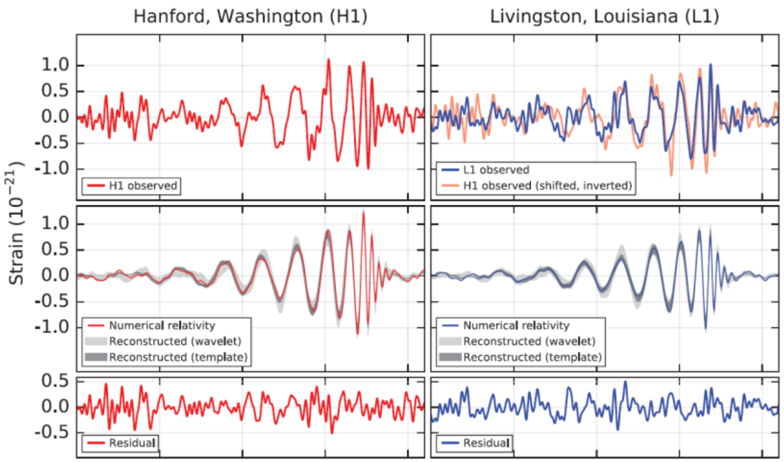
3% Mismatch: 10% lost events!

LIGO uses ~250000 templates for CBC searches

Sathyaprakash & Dhurandhar PRD44:3819 (1991); Owen, PRD53: 6749 (1996)



Gravitational waves from compact objects



Abbott + PRL.116:061102 (2016)

BH seeds, demography...
(how many, where, how?)
See review Barack+ arXiv:1806.05195

What is graviton mass or speed?
See review Barack+ arXiv:1806.05195

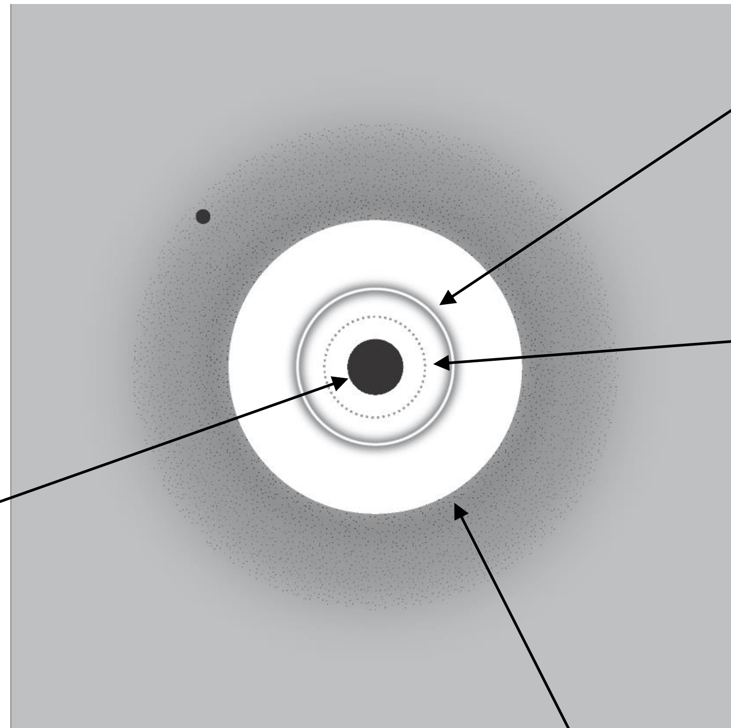
Is it a Kerr black hole?
Can we constrain alternatives?
Berti+ 2005, 2016; Cardoso & Gualtieri CQG33:174001 (2016)

Is the final - or initial - object really a black hole?
Cardoso+ PRL116: 171101 (2016); Cardoso & Pani, Nature Astronomy 1: 586 (2017)

Are there extra radiation channels, corrections to gravity?
Barack+arXiv:1806.05195; Barausse+PRL116:241104(2016);

Can GWs from BHs inform us on fundamental fields/DM?
Barack+arXiv:1806.05195; Arvanitaki+ PRD95: 043001 (2016); Brito+ PRL119:131101 (2017)

Black holes are black



Cardoso & Pani, Nature Astronomy 1: 586 (2017)
Living Reviews in Relativity 22: 1 (2019)
Image: Ana Carvalho

Inspiralling compact objects

$$\text{Binding Energy}: E_b = -\frac{GM\mu}{2L} + \text{other interactions}$$

$$\text{Quadrupole emission}: \dot{E} = -\frac{32}{5}\frac{G\mu^2 L^4 \Omega^6}{c^5} + \text{other emission channels}$$

$$h(f, \text{pars}) = A(f, \text{pars}) e^{i\Psi(f, \text{pars})}$$

$$\Psi = \frac{3}{128}\left(G\mathcal{M}\pi f/c^3\right)^{-5/3}\left(... + \boxed{\alpha_{-4PN}x^{-4} + ... + \alpha_{-1PN}x^{-1}} + 1 + \alpha_{1PN}x + ...\right)$$

New physics or
extra matter

Variation of G

Dipole moment
(electric charge)

Graviton
mass

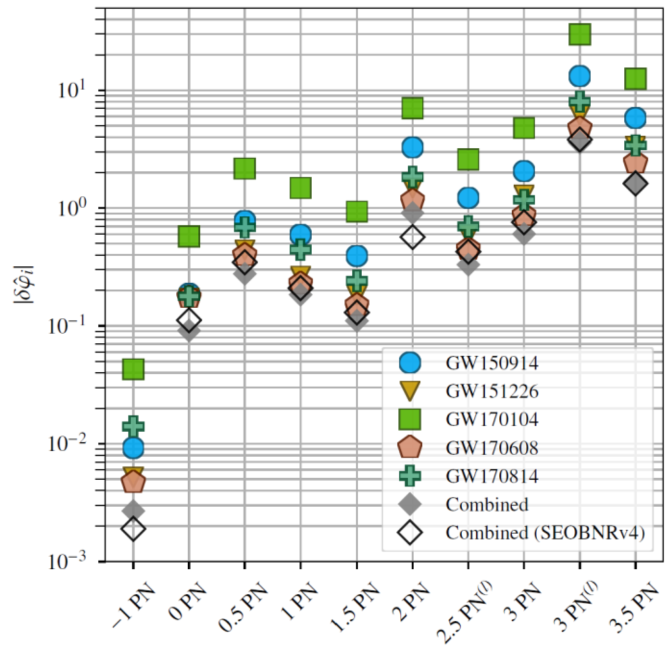
M. Maggiore, Gravitational waves, Volume I
N. Yunes, K. Yagi & F. Pretorius, PRD94:084002 (2016)

Parametrized tests

$$h(f,\mathrm{pars}) = A(f,\mathrm{pars})e^{i\Psi(f,\mathrm{pars})}$$

$$\Psi = \frac{3}{128}\left(G\mathcal{M}\pi f/c^3\right)^{-5/3}\left(\ldots+\alpha_{-4PN}x^{-4}+\ldots+\alpha_{-1PN}x^{-1}+1+\alpha_{1PN}x+\ldots\right)$$

$$x=(\pi M f)^{2/3}\,,\quad M=m_1+m_2\,,\quad \nu=m_1m_2/M^2\,,\quad \mathcal{M}=\nu^{3/5}M$$



LVC arXiv:1903.04467

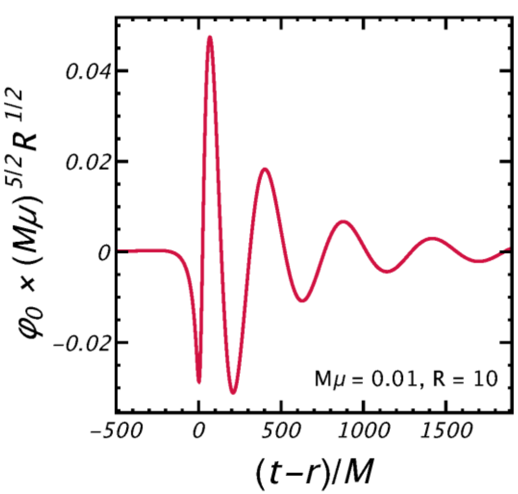
Challenges

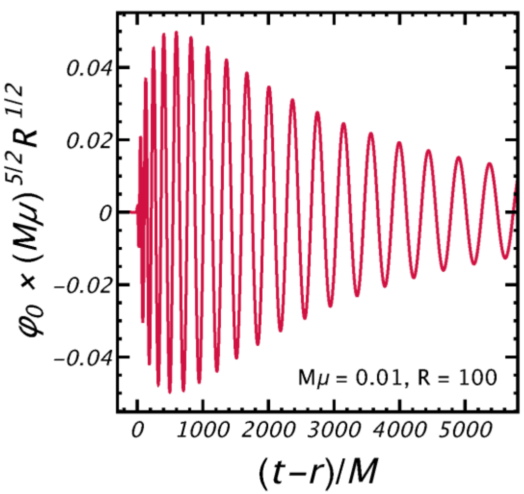
Any specific theory bound to affect all PPN parameters

Some of these - extra dimensions, varying-G, graviton mass, etc, derived with hand-waving arguments, blind to full theory

In other words, we need to know full waveform, and underlying theory

Example: massive graviton monopolar mode





Cardoso, Castro & Maselli, PRL121:251103 (2018);
for higher-dimensional scenarios, see Cardoso, Gualtieri & Moore PRD 100: 124037 (2019)

The postmerger: ringdown and perturbed black holes

Assume your spacetime is *approximately*
that of a Schwarzschild black hole

$$g_{\mu\nu}(x^\nu) = {}^{(0)}g_{\mu\nu}(x^\nu) + h_{\mu\nu}(x^\nu)$$

Still too complex…second order PDEs on 4 variables…

Use background symmetries

Regge and Wheeler Phys.Rev.108: 1063 (1957)
Matthews, J. Soc. Ind. App. Math.10:768 (1962)
Zerilli J. Math. Phys.11:2203 (1970)

First "split" spacetime coordinates $x^\mu = (z^A, y^a)$,
where $z^A = (\theta,\, \phi)\,, \qquad y^a = (t,\, r)$

Introduce metric on unit sphere $ds^2 = \gamma_{AB} dz^A dz^B$

$$\gamma_{AB} \equiv \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix}$$

Denote covariant derivative with respect to γ_{AB} by ∇_A.

Define Levi-Civita tensor $\varepsilon_{AB} \equiv \begin{pmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{pmatrix}$

- Scalar harmonics $\gamma^{AB}\nabla_A\nabla_B Y^{\ell m} = -\ell(\ell+1)Y^{\ell m}$
- Polar vector harmonics $Y_A^{\ell m} \equiv \nabla_A Y^{\ell m}$
- Axial vector harmonics $S_A^{\ell m} \equiv \varepsilon_{AC}\gamma^{BC}\nabla_B Y^{\ell m}$
- Polar rank-two tensor harmonics $Z_{AB}^{\ell m} \equiv \nabla_A\nabla_B Y^{\ell m} + \frac{\ell(\ell+1)}{2}\gamma_{AB}Y^{\ell m}$
- Axial rank-two tensor harmonics $S_{AB}^{\ell m} \equiv \frac{1}{2}(\nabla_B S_A^{\ell m} + \nabla_A S_B^{\ell m})$

$$h_{\mu\nu}^{\text{ax}} = \begin{pmatrix} 0 & 0 & \frac{h_0}{\sin\theta} Y^{\ell m}_{,\phi} & -h_0 \sin\theta Y^{\ell m}_{,\theta} \\ 0 & 0 & \frac{h_1}{\sin\theta} Y^{\ell m}_{,\phi} & -h_1 \sin\theta Y^{\ell m}_{,\theta} \\ \frac{h_0}{\sin\theta} Y^{\ell m}_{,\phi} & \frac{h_1}{\sin\theta} Y^{\ell m}_{,\phi} & 0 & 0 \\ -h_0 \sin\theta Y^{\ell m}_{,\theta} & -h_1 \sin\theta Y^{\ell m}_{,\theta} & 0 & 0 \end{pmatrix}$$

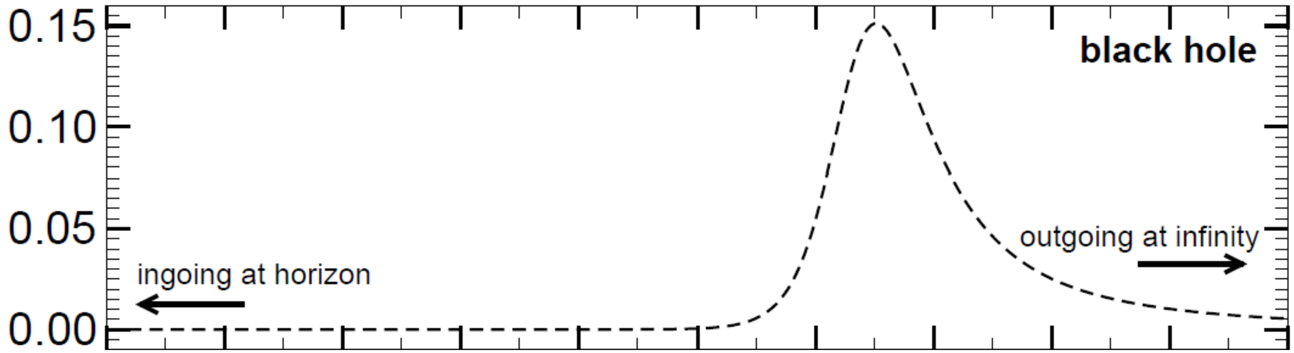
$$h_{\mu\nu}^{\text{pol}} = \begin{pmatrix} -f(r)\, H_0 Y_{\ell m} & -H_1 Y_{\ell m} & 0 & 0 \\ -H_1 Y_{\ell m} & -\frac{1}{f(r)} H_2 Y_{\ell m} & 0 & 0 \\ 0 & 0 & -r^2 K Y_{\ell m} & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta K Y_{\ell m} \end{pmatrix}$$

Vacuum

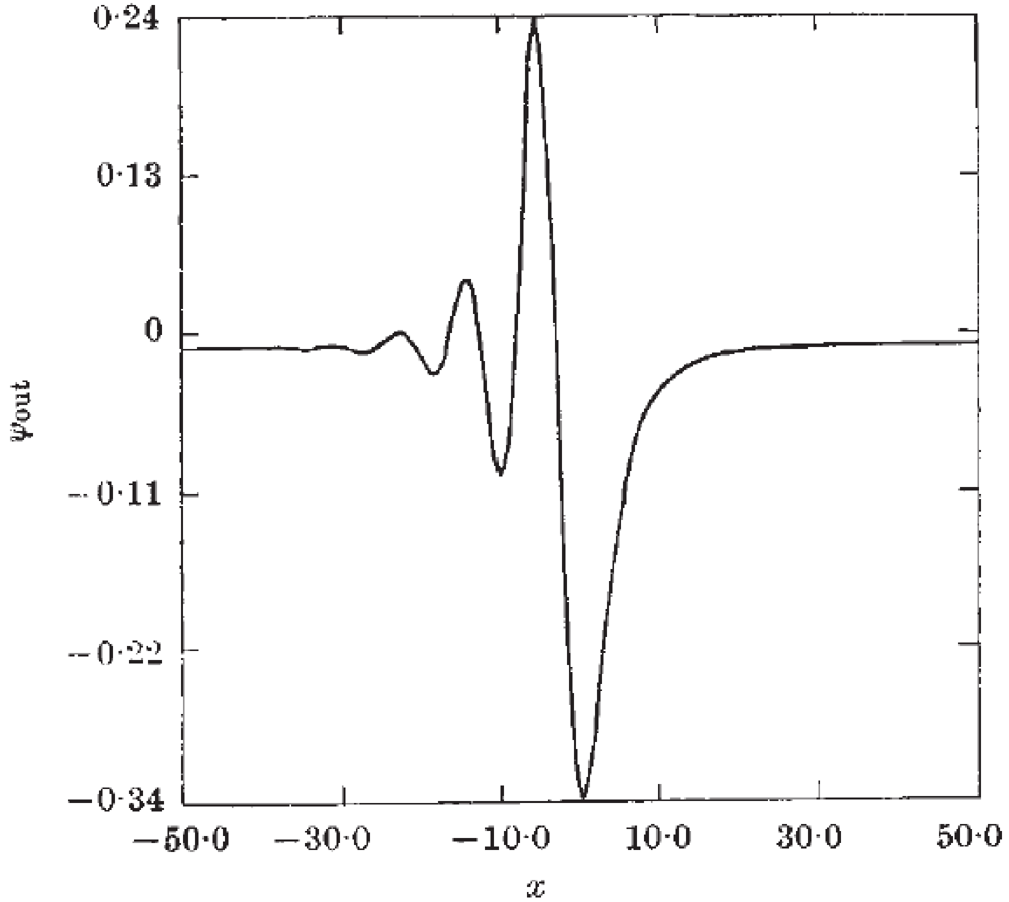
$$\Psi = \frac{1}{r}\left(1 - \frac{2M}{r}\right) h_1$$

$$0 = \left(1 - \frac{2M}{r}\right)^2 \frac{\partial^2 \Psi}{\partial r^2} + \frac{2M}{r^2}\left(1 - \frac{2M}{r}\right)\frac{\partial \Psi}{\partial r} - \frac{\partial^2 \Psi}{\partial t^2} - V\Psi$$

$$V = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right)$$



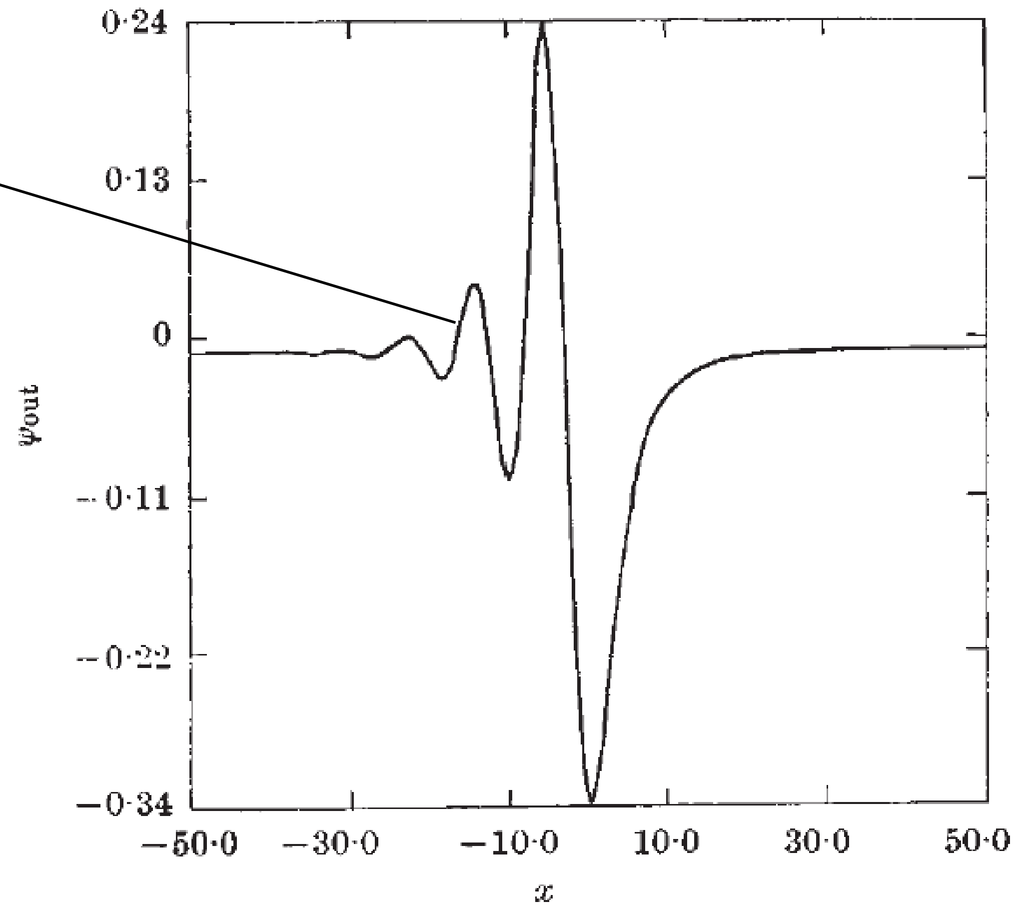
Hair loss: the characteristic modes of black holes



C.V.Vishveshwara, Nature 227: 938 (1970)
Data and routines at blackholes.ist.utl.pt

Experiment repeated:
same decay timescale and ringing for different initial conditions;

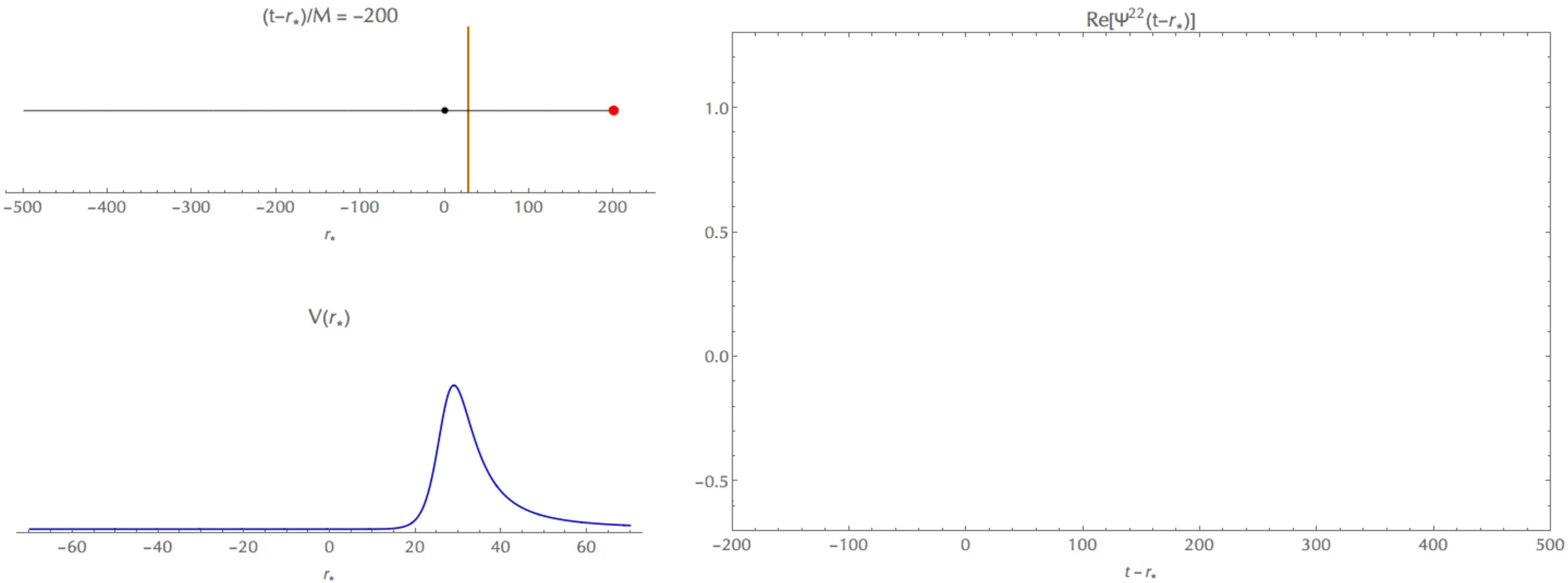
Universal ringdown



C.V.Vishveshwara, Nature 227: 938 (1970)
Data and routines at blackholes.ist.utl.pt

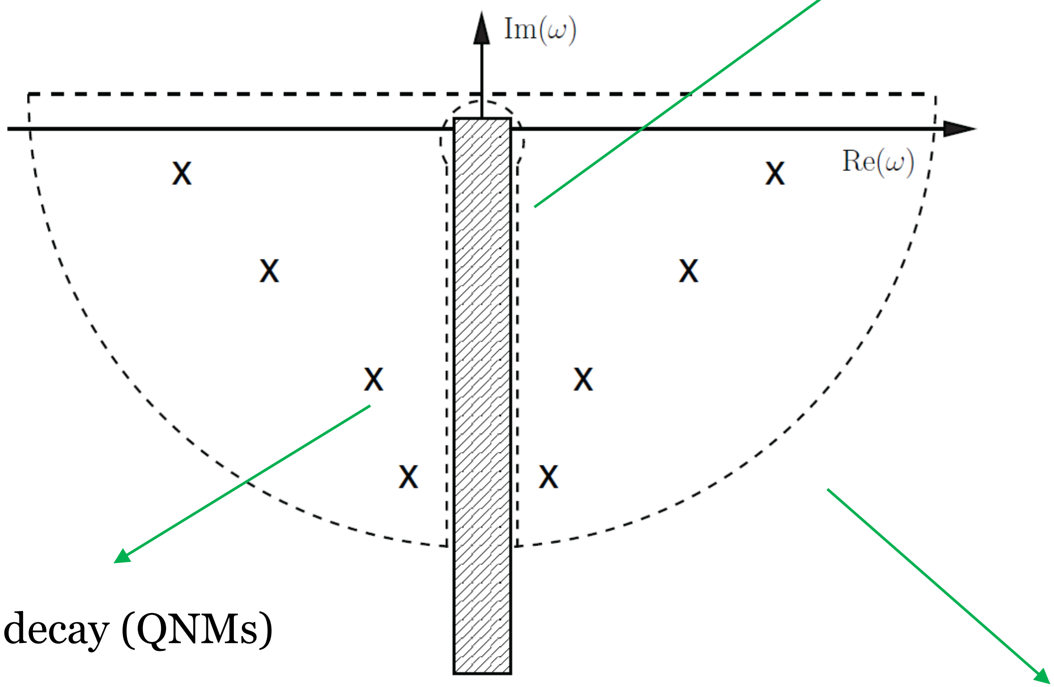
Excitation of BH ringdown and the light ring

$\mathcal{E} = 1.5,\ \mathcal{J} = 0$



After harmonic decomposition $\dfrac{\mathrm{d}^2\Psi}{\mathrm{d}r_*^2} + (\omega^2 - V)\Psi = I(\omega, r)$

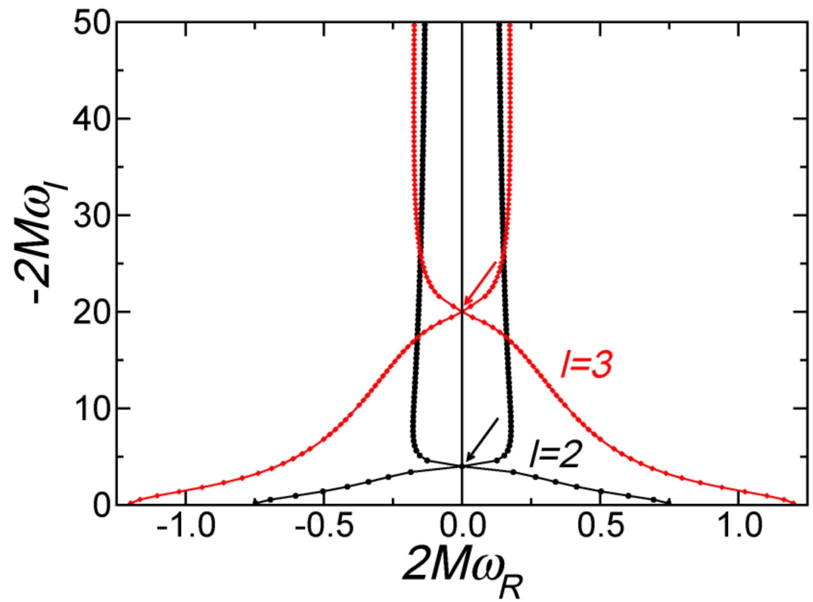
Branch cut: power-law tails



Poles: exponential decay (QNMs)

Quarter circles: prompt response

Leaver PRD34 1986



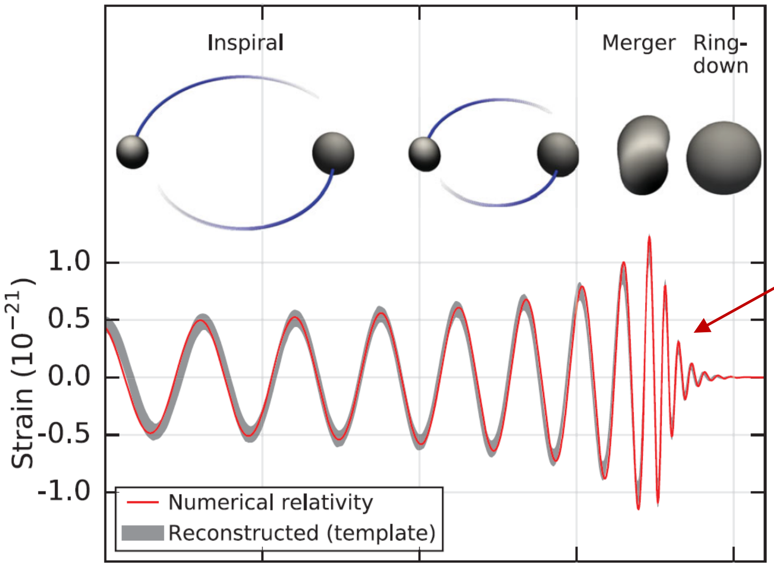
$$f = \omega_R/2\pi = 1.207 \left(\frac{10\ M_\odot}{M}\right) \text{kHz}$$

$$\tau = 1/|\omega_I| = 0.5537 \left(\frac{M}{10\ M_\odot}\right) \text{ms}$$

Berti+ PRD73: 064030 (2006); Berti + CQG 26: 163001 (2009)

"After the advent of gravitational wave astronomy, the observation of these resonant frequencies might finally provide direct evidence of BHs with the same certainty as, say, the 21 cm line identifies interstellar hydrogen" (S. Detweiler ApJ239:292 1980)

BH spectroscopy: testing the Kerr nature

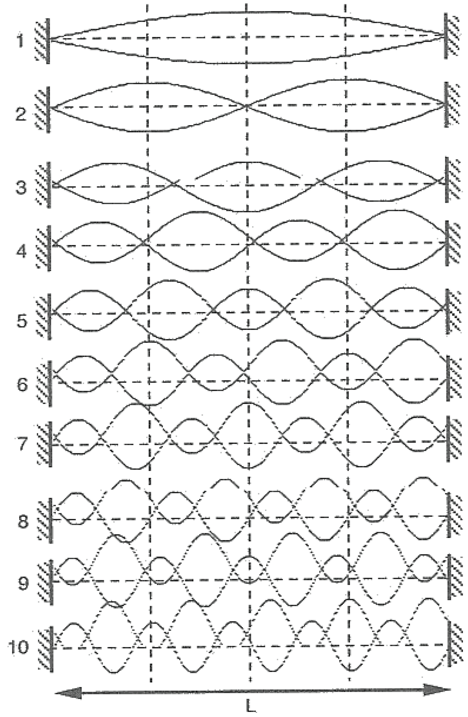


Ringdown:

$$h = \sum_k A_k e^{-\omega_I^{(k)} t} \sin\left(\omega_R^{(k)} t\right)$$

Berti, Cardoso and Will PRD73: 064030 (2006); Berti, Cardoso and Starinets, CQG 26: 163001 (2009)
Data and routines at ***blackholes.ist.utl.pt*** *and* ***https://pages.jh.edu/~eberti2/***

Tests of the no-hair hypothesis



$$f = \frac{nv}{2L}\,, \quad n = 1,\, 2,\, 3\ldots$$

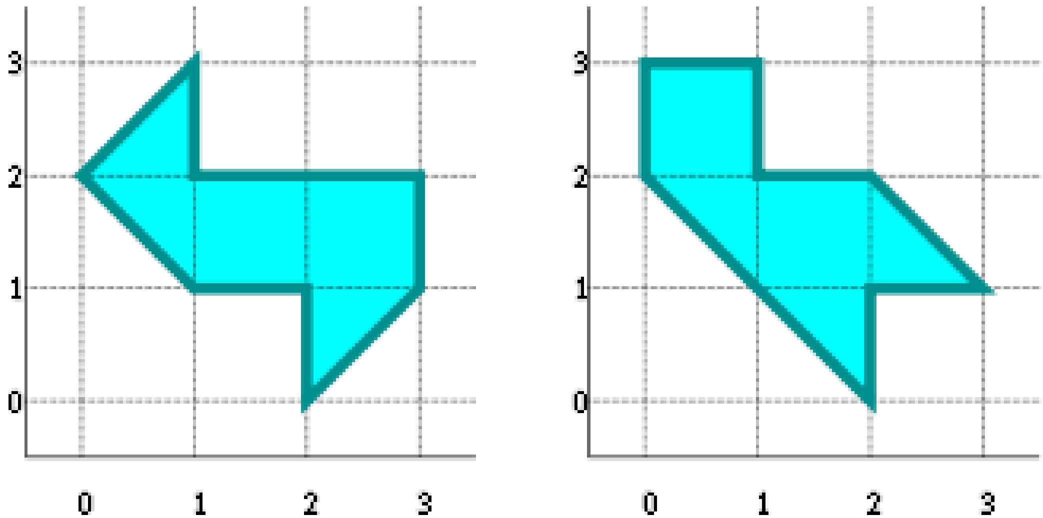
Measure fundamental mode, determine length L.
Measure first overtone, test if it's a string…

“Can one hear the shape of a drum?”

Mark Kac, American Mathematical Monthly, *1966*

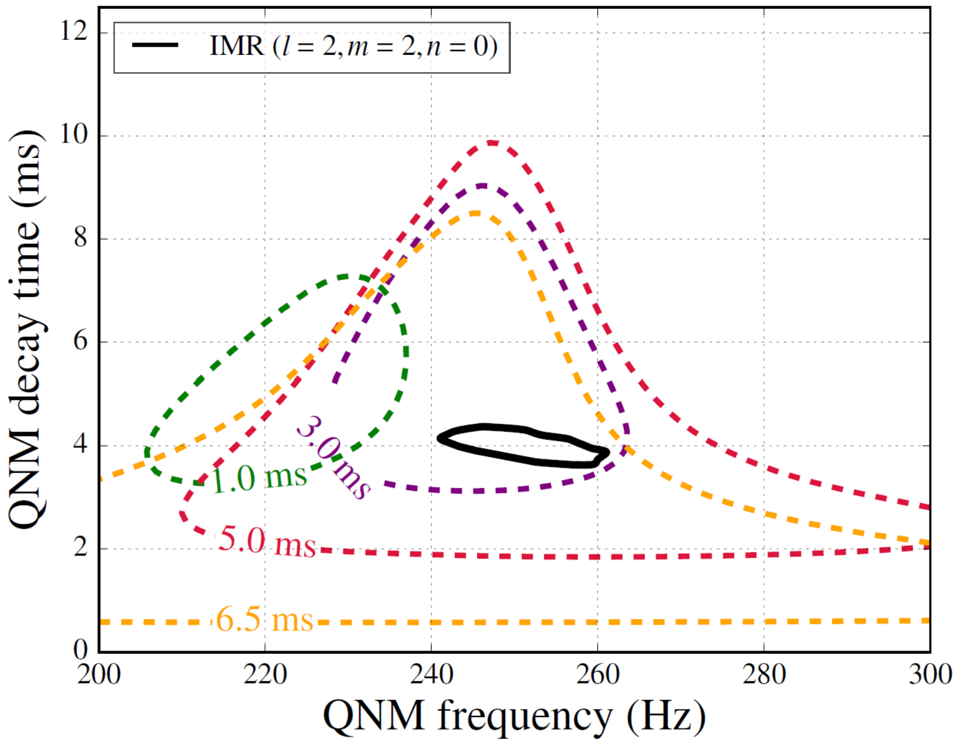
$$N(\lambda) = \sum_{\lambda_n < \lambda} 1 \sim \frac{|\Omega|}{2\pi} \lambda$$

H. Weyl 1911



Gordon, Webb & Wolpert, Inventiones Mathematicae 1992

One-mode tests

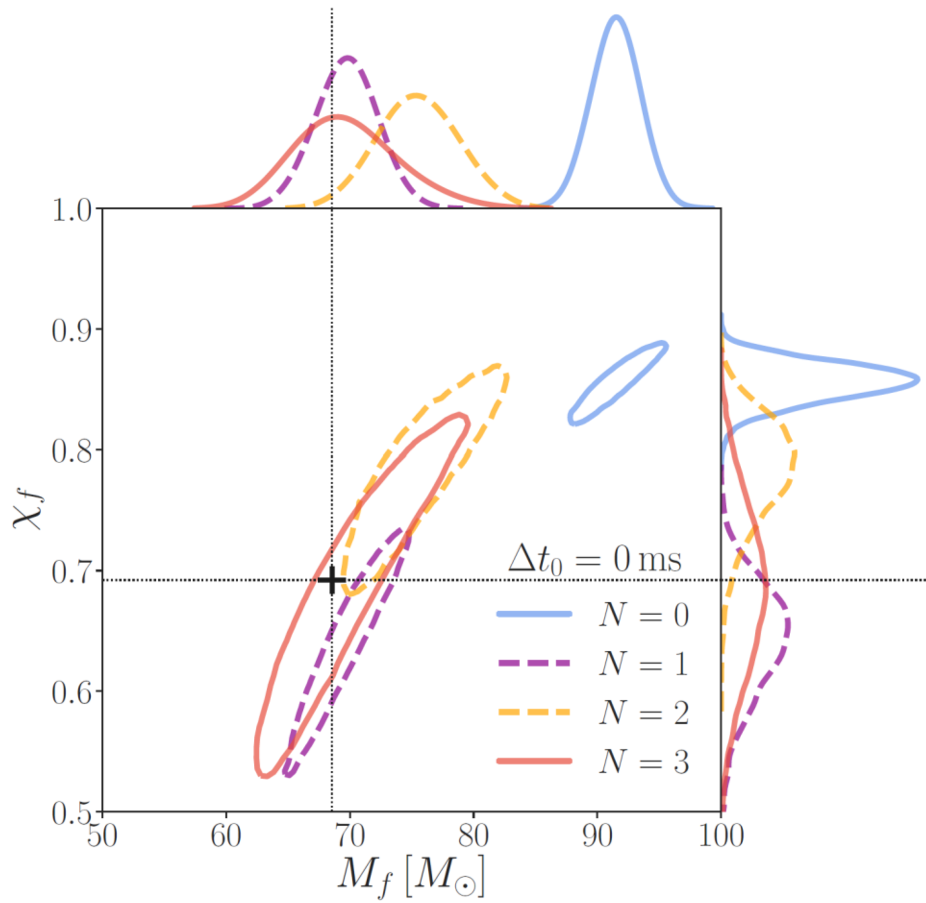


90% posterior distributions.

Black solid is 90% posterior of QNM as derived from the posterior mass and spin of remnant.

LSC PRL116:221101 (2016); see Bhaibav+PRD97:044048 (2019); Isi+ PRL123:111102 (2019)

The importance of overtones



Giesler+ *arXiv:1903:08284*

Baibhav + PRD97:044048 (2018); Brito+ PRD98: 084038 (2019)

Extra couplings with spectroscopy

Example: BH charge

(mini-charged DM models predict heavy, fractional "electrons" and RN geometry: Rujula + 1990; Perl+ 1997; Holdom 1986; Sigurdson + 2004)

$$\mathcal{L} = \sqrt{-g}\left(\frac{R}{16\pi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + 4\pi e j^{\mu}_{\rm em}A_{\mu} + 4\pi e_h j^{\mu}_{h}B_{\mu} + 4\pi\epsilon e j^{\mu}_{h}A_{\mu}\right)$$

Ringdown bound $\frac{Q}{M} \lesssim 0.1\sqrt{\frac{100}{\rho}}$

Or 1 electron per 10^(19) neutrons. Bound can be generalized to other theories, provided spectra is known.

Cardoso + JCAP 1605: 054 (2016)
Blázquez-Salcedo + PRD94:104024 (2016)

Results not affected by environment

i. GWs are redshifted and lensed in “usual”, EM way
(use geometric optics)

ii. GWs do not couple to perfect, homogeneous fluids

iii. Viscosity: $L_{att} = \frac{c^6}{32\pi\eta\, G} = 10^{18}\,\frac{1poise}{\eta}$ light years

iv. Medium of oscillators $L_{att} = \frac{1}{n\sigma} = 10^{28}$ light years or so
(if all our galaxy consists of BHs of roughly 10 solar masses)

Kip Thorne, Gravitational Radiation (Les Houches 1984); Grishchuk, Polnarev 1980; Barausse+ PRD89:104059 (2014); Annulli+ PRD99:044038 (2019)

Environment: ringdown properties

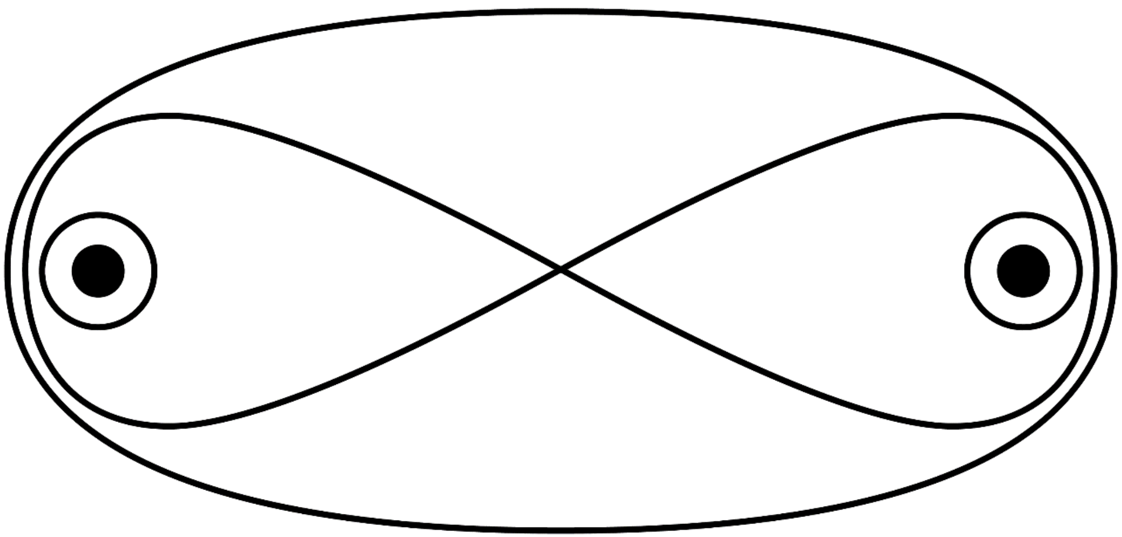
Correction	$\lvert\delta_R\rvert[\%]$	$\lvert\delta_I\rvert[\%]$
spherical near-horizon distribution	0.05	0.03
ring at ISCO	0.01	0.01
electric charge	10^{-5}	10^{-6}
magnetic field	10^{-8}	10^{-7}
gas accretion	10^{-11}	10^{-11}
DM halos	$10^{-21}\rho_3^{\mathrm{DM}}$	$10^{-21}\rho_3^{\mathrm{DM}}$
cosmological effects	10^{-32}	10^{-32}

Barausse + PRD89: 104059 (2014)

Binaries: molecular spectroscopy?

$$ds^2 = -\frac{dt^2}{U^2} + U^2\left(d\rho^2 + \rho^2 d\phi^2 + dz^2\right)$$

$$U(\rho,\, z) = 1 + \frac{M}{\sqrt{\rho^2 + (z-a)^2}} + \frac{M}{\sqrt{\rho^2 + (z+a)^2}}$$



Chandrasekhar PRSLA421:227 (1989); Assumpção+ PRD98: 064036(2018)

Gravitational molecules: a toy model

Change to prolate confocal elliptic coordinates

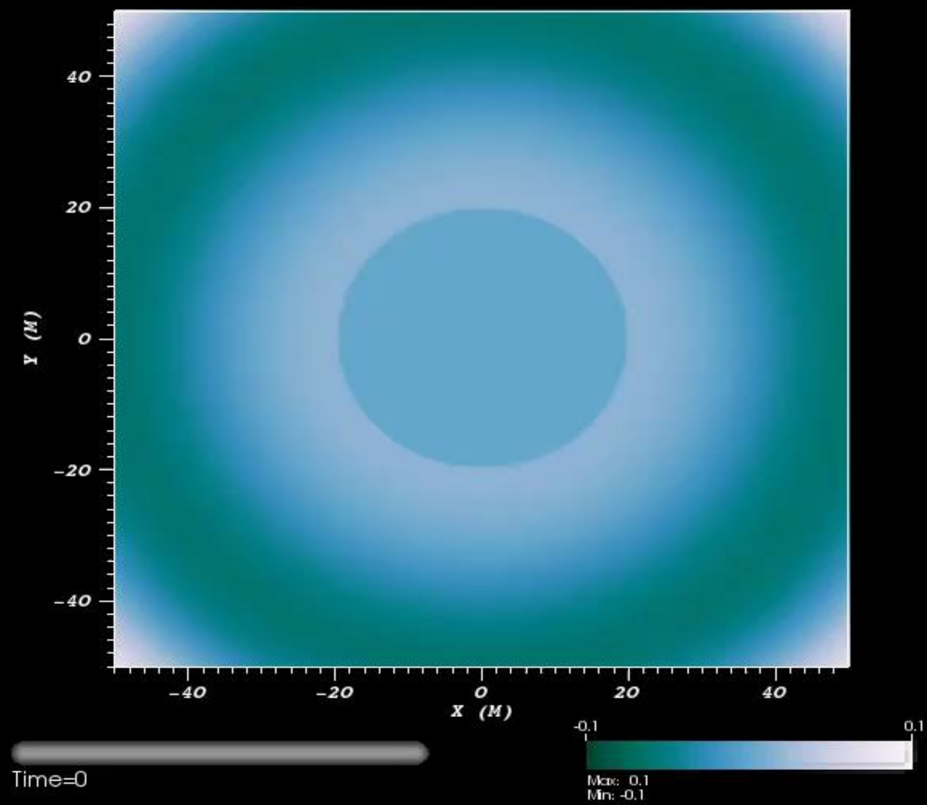
$$\rho^2 + (a-z)^2 = a^2(\chi+\eta)^2$$
$$\rho^2 + (a+z)^2 = a^2(\chi-\eta)^2$$

$$\partial_\eta\left((1-\eta^2)\partial_\eta S\right) + \left(-a^2\omega^2\eta^2 - \frac{m^2}{1-\eta^2} + \Lambda\right) S = 0$$
$$\partial_\chi\left((\chi^2-1)\partial_\chi R\right) + \left(a^2\omega^2\chi^2 + 8Ma\chi\omega^2 - \frac{m^2}{\chi^2-1} - \Lambda\right) R = 0$$

Klein-Gordon equation is identical to Schrodinger
for Di-Hydrogen ionized molecule

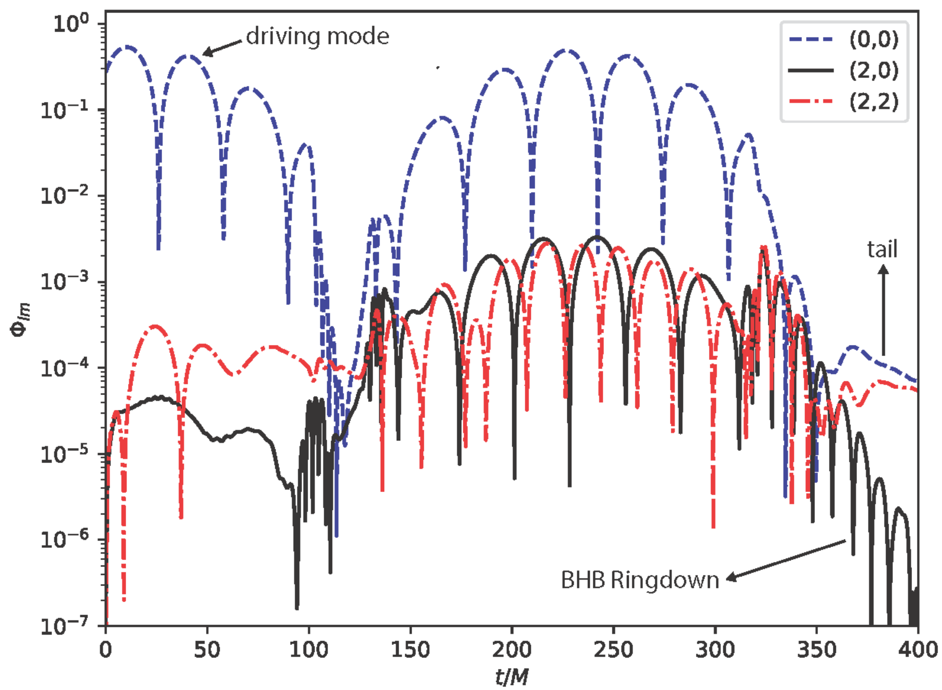
Bernard+ PRD100: 044002 (2019)
for Hydrogen molecule see Burrau M7: 1 (1928); Wilson PRSLA118:635 (1929); Hylleraas ZfP71: 739 (1931)

Gravitational molecules: a real BH binary



Bernard + PRD100: 044002 (2019); arXiv:1905.05204

Gravitational molecules: a real BH binary



$$T = (1.03 \pm 0.04)\, L + (8 \pm 1)\, M$$

May global BHB modes be resonantly excited?

Bernard + PRD100: 044002 (2019); arXiv:1905.05204

Open questions

How do different sources excite ringdown?

What are nonlinear effects in ringdown?

Are power-law tails excited in nonlinear simulations?

Precise calculation of QNMs of binaries

Impact of environment on GW emission and propagation?

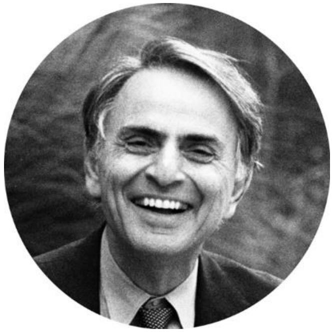
Prescription to solve two-body problem (more) generically?

The evidence for black holes

1. BH exterior is pathology-free, interior is not.

2. Quantum effects not fully understood. Non-locality to solve information paradox? Hard-surface to quantize BH area *(Bekenstein & Mukhanov 1995)*

3. Tacitly assumed quantum effects at Planck scales. Planck scale could be significantly lower *(Arkani-Hamed+ 1998; Giddings & Thomas 2002)*. Even if not, many orders of magnitude standing, surprises can hide *(Bekenstein & Mukhanov 1995)*.



“*Extraordinary claims require extraordinary evidence.*”
Carl Sagan

4. Dark matter exists, and interacts gravitationally. Are there compact DM clumps?

5. Physics is experimental science. We can test exterior. Aim to quantify evidence for horizons. Similar to quantifying equivalence principle.

"Plus un fait est extraordinaire, plus il a besoin d'être appuyé de fortes preuves; car, ceux qui l'attestent pouvant ou tromper ou avoir été trompés, ces deux causes son d'autant plus probables que la réalité du fait l'est moins en elle-même…."

Laplace, Essai philosophique sur les probabilities 1812

"No testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous than the fact which it endeavors to establish."

David Hume, An Enquiry concerning
human understanding 1748

"Extraordinary claims require extraordinary evidence."

Carl Sagan

Some challenges

i. Are there alternatives?

ii. Do they form dynamically under reasonable conditions?

iii. Are they stable?

iv. How do they look like? Is GW or EM signal similar to BHs?

v. Observationally, how close do we get to horizons?

i. Alternatives

Boson stars, fermion-boson stars, oscillatons
Kaup 1968; Ruffini, Bonazzolla 1969; Colpi + 1986; Okawa+ 2014; Brito + 2015

Anisotropic stars
Bowers, Liam 1974; Dev, Gleiser 2000; Raposo + arXiv:1811.07917

Wormholes
Morris, Thorne 1988; Visser 1996; Damour and Solodukhin 2007; Maldacena+ 2017

Gravastars
Mazur, Mottola 2001

Fuzzballs, Superspinars, collapsed polymers, 2-2 holes
Mathur 2000; Gimon, Horava 2009; Brustein, Medved 2016;Holdom, Ren 2016

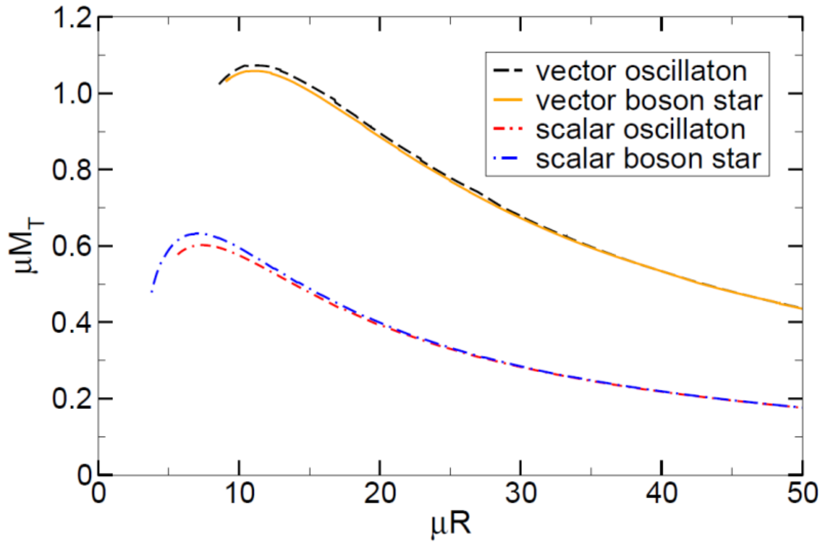
…

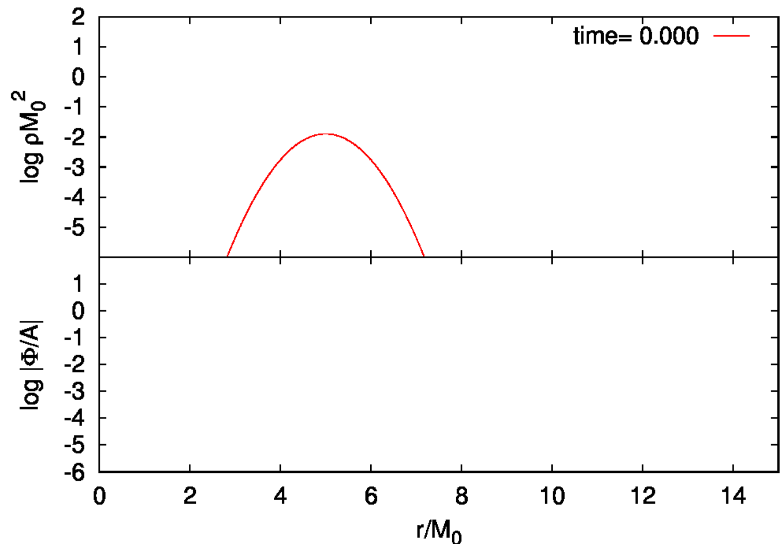
Bekenstein-Mukhanov proposal for BH area quantization
Bekenstein and Mukhanov (1995)

ii. Formation

Boson stars, fermion-boson stars, oscillatons

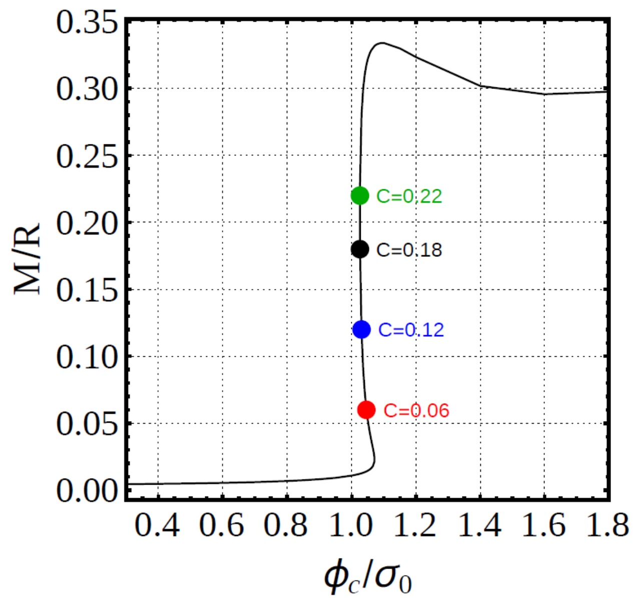
(Kaup ‘68; Ruffini, Bonazzolla ‘69; Colpi+ 1986; Tkachev ’91; Okawa+ 2014; Brito+ 2015)

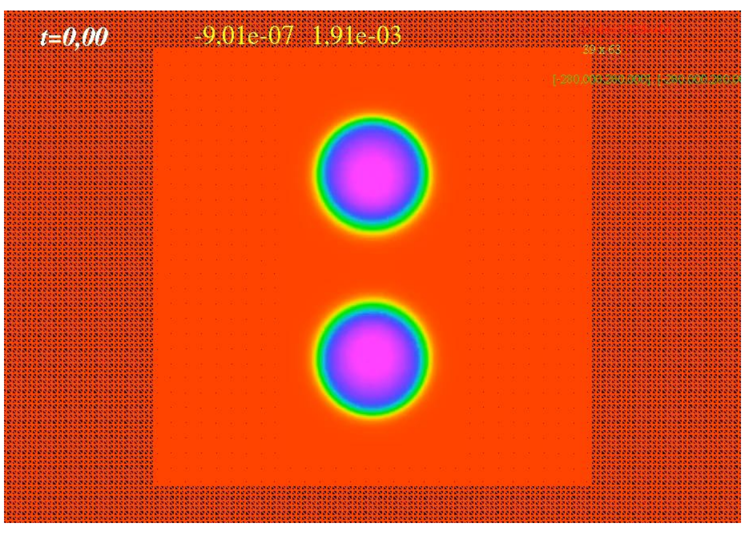


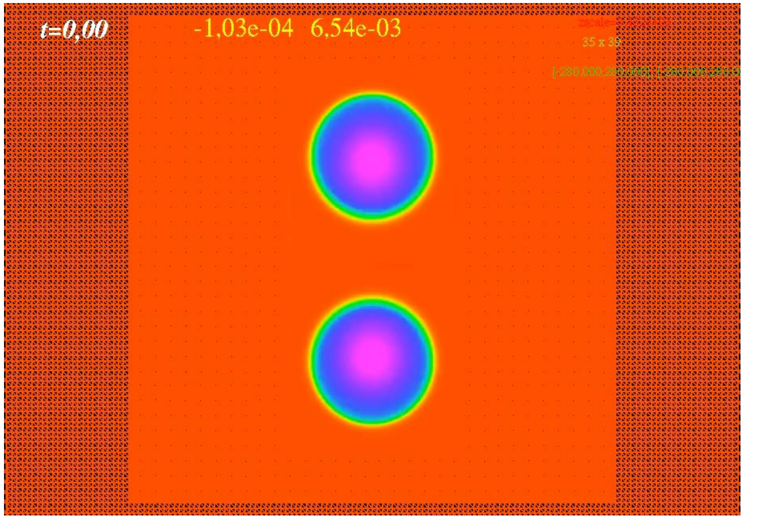


$$\frac{M_{\max}}{M_\odot} = 8 \times 10^{-11} \frac{\text{eV}}{m_B c^2}$$

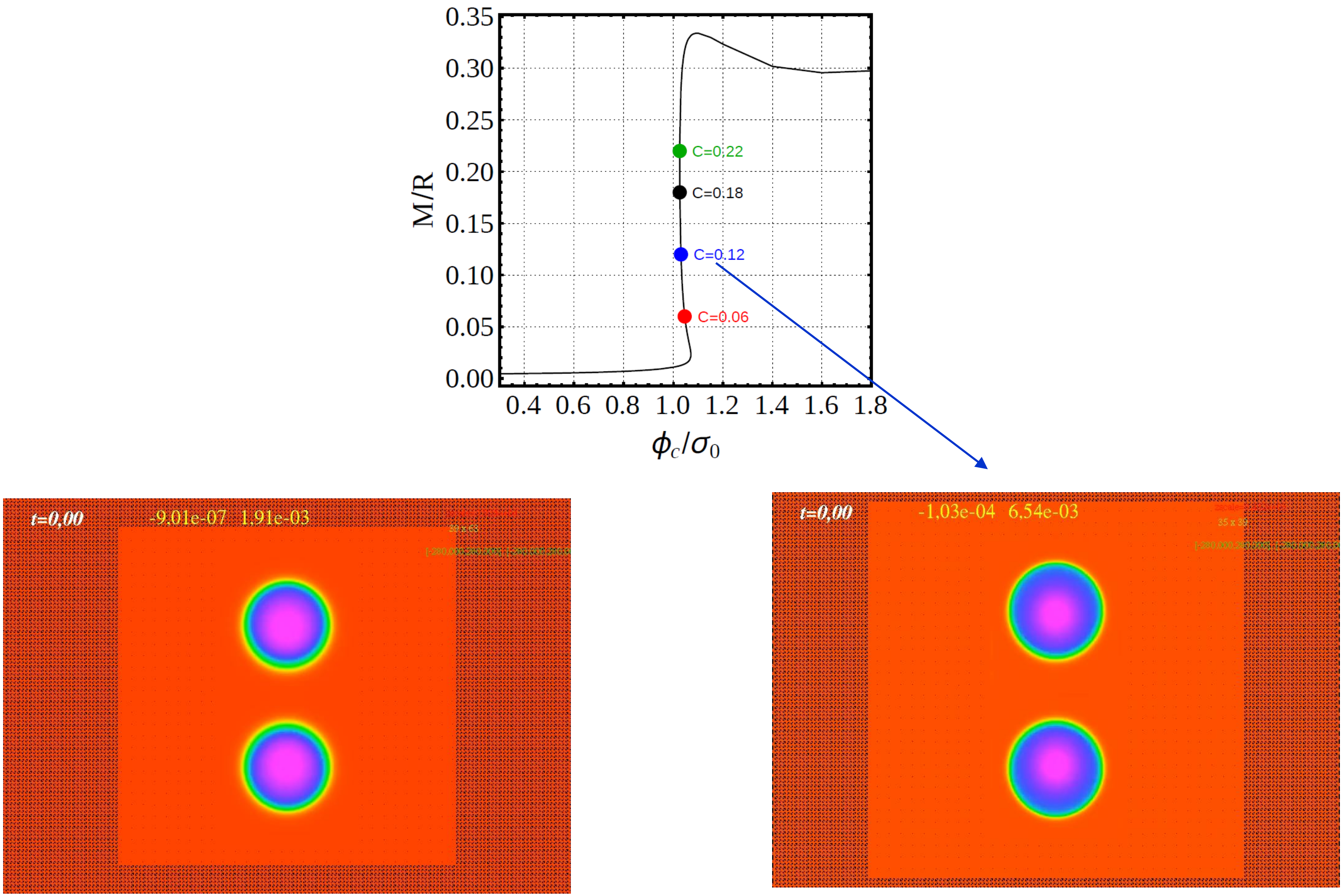
Challenge: repeat for anisotropic stars, wormholes, gravastars, etc.







Palenzuela+ PRD96:104058(2017)

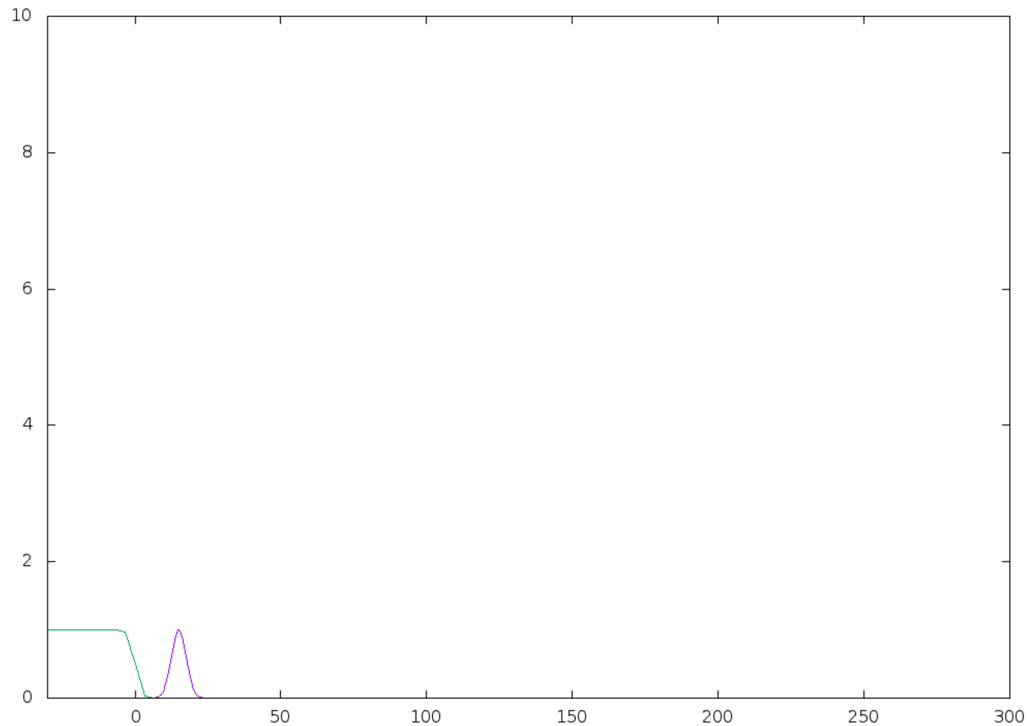


Palenzuela+ PRD96:104058(2017)

iiia. Stability of objects with ergoregions

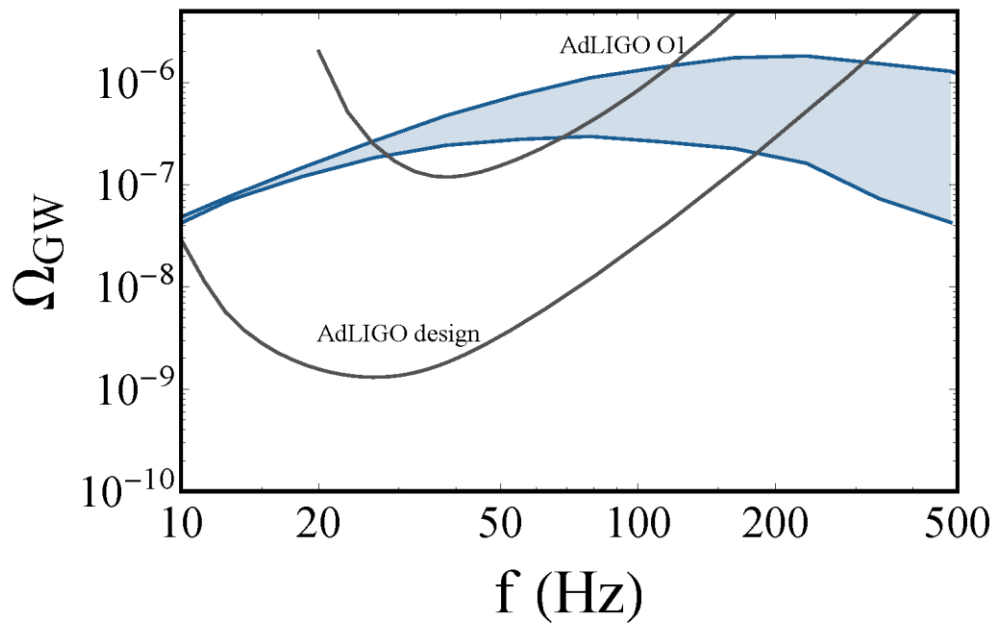
AS flat, horizonless spacetimes with ergoregions are linearly unstable

Friedmann Comm. Math.Phys.63:243 (1978); Moschidis Comm. Math. Phys. 358: 437 (2016)



Vicente & Cardoso PRD97:084032 (2018); Brito+ Lect. Notes Phys 906 (2015)

Stochastic background of GWs



Blue bands bracket population models, from optimistic to pessimistic

Barausse+ CQG35:20LT01 (2018)

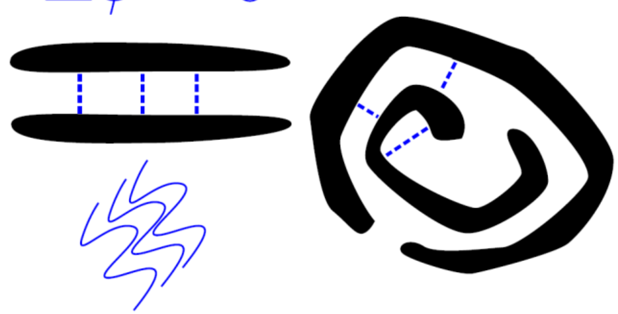
iiib. Stability of objects with photospheres

Static objects: *No uniform decay estimate with faster than logarithmic decay can hold for axial perturbations of ultracompact objects.*

Keir CQG33: 135009 (2016); Cardoso + PRD90:044069 (2014)

$$\mathcal{E}^{(N)}_{\text{local}}(t) \lesssim \frac{1}{(\log(2+t))^2}\mathcal{E}^{(N)}_{(2)}(0)$$

$$\Box\phi = 0$$



Burq, Acta Mathematica 180: 1 (1998)

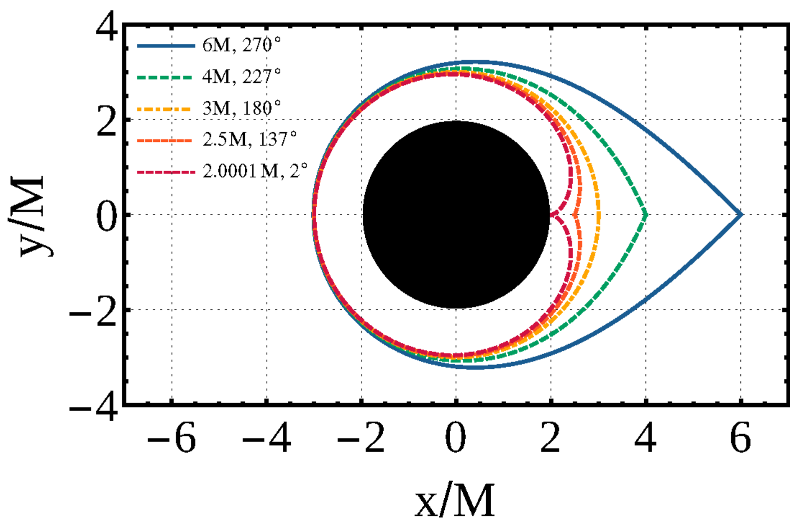
iv. EM constraints

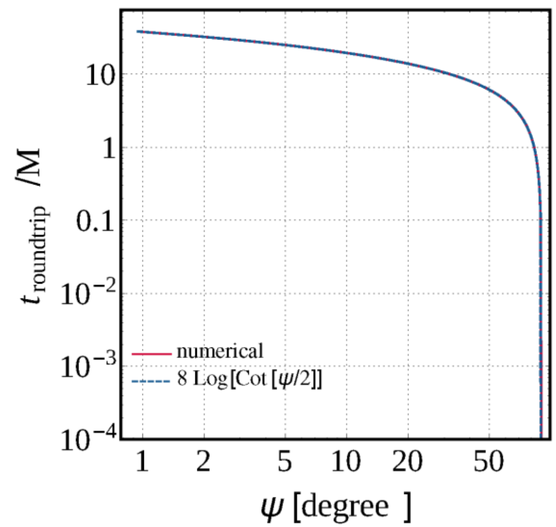
$$r = 2M\,(1+\epsilon)$$

$\epsilon \lesssim 10^{-5}$ Absence of transients from tidal disruptions

$\epsilon \lesssim 10^{-35}$ Dark central spot on SgrA

Carballo-Rúbio, Kumar, PRD97:123012 (2018), Broderick, Narayan CQG24:659 (2007)

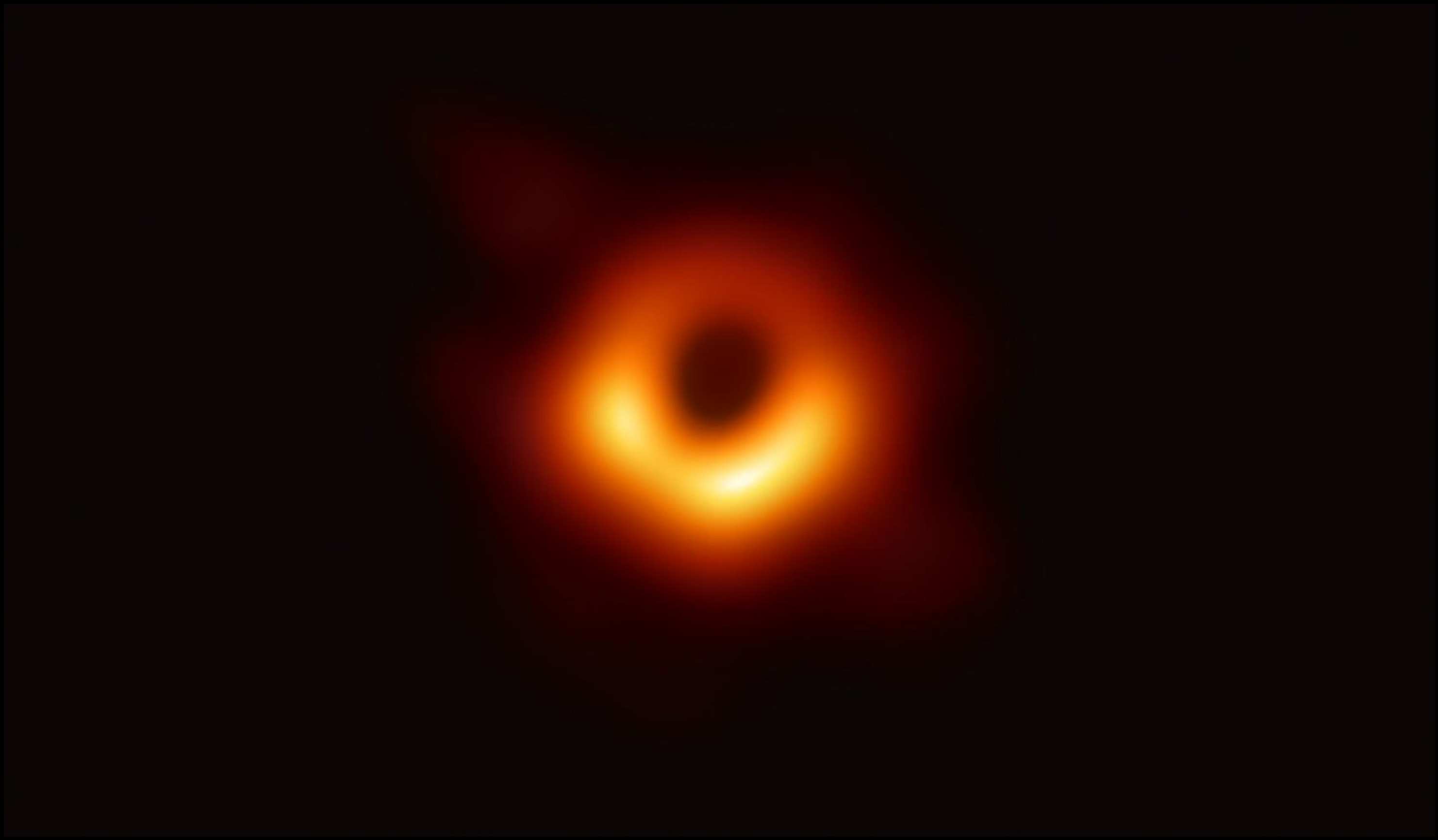




Lensing has to be properly included, as well as emission into other channels

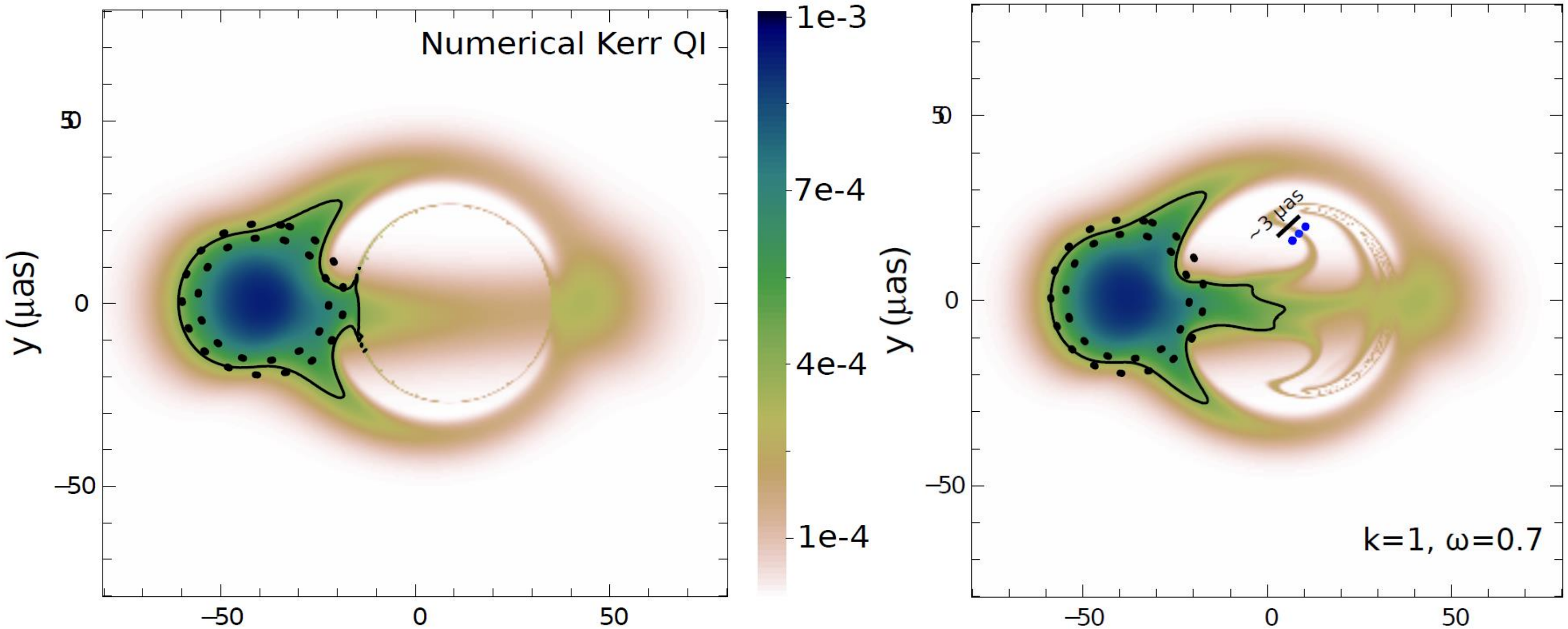
Cardoso, Pani Nature Astronomy 1 (2017); Living Reviews in Relativity 22: 1 (2019)

Images of black holes?



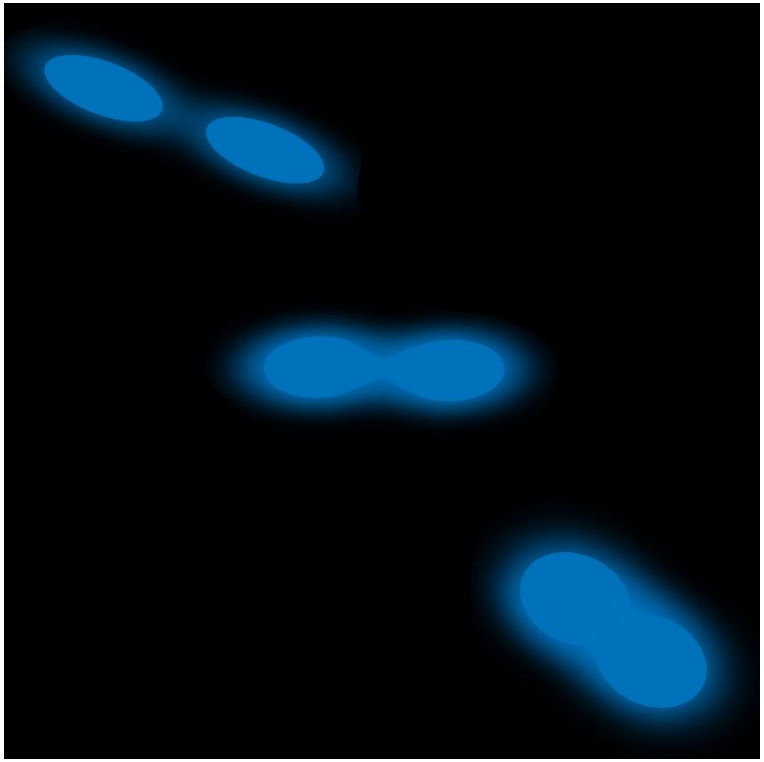
EHT Collaboration ApJL 875: 1 (2019)

Shadows



Vincent+ CQG 33:105015 (2016)

iv. GW signal



Nature of inspiralling objects is encoded

(a) in way they respond to own field (multipolar structure)

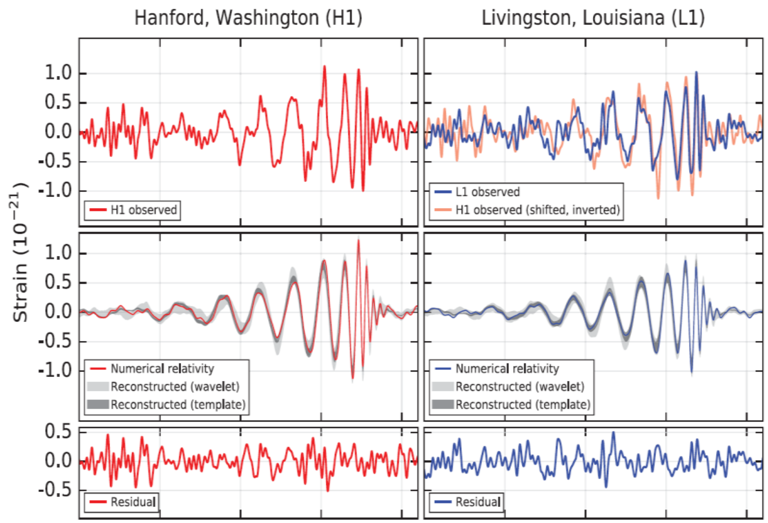
(b) in way they respond when acted upon by external field of companion – through their tidal Love numbers (TLNs), and

(c) on amount of radiation absorbed, i.e., tidal heating

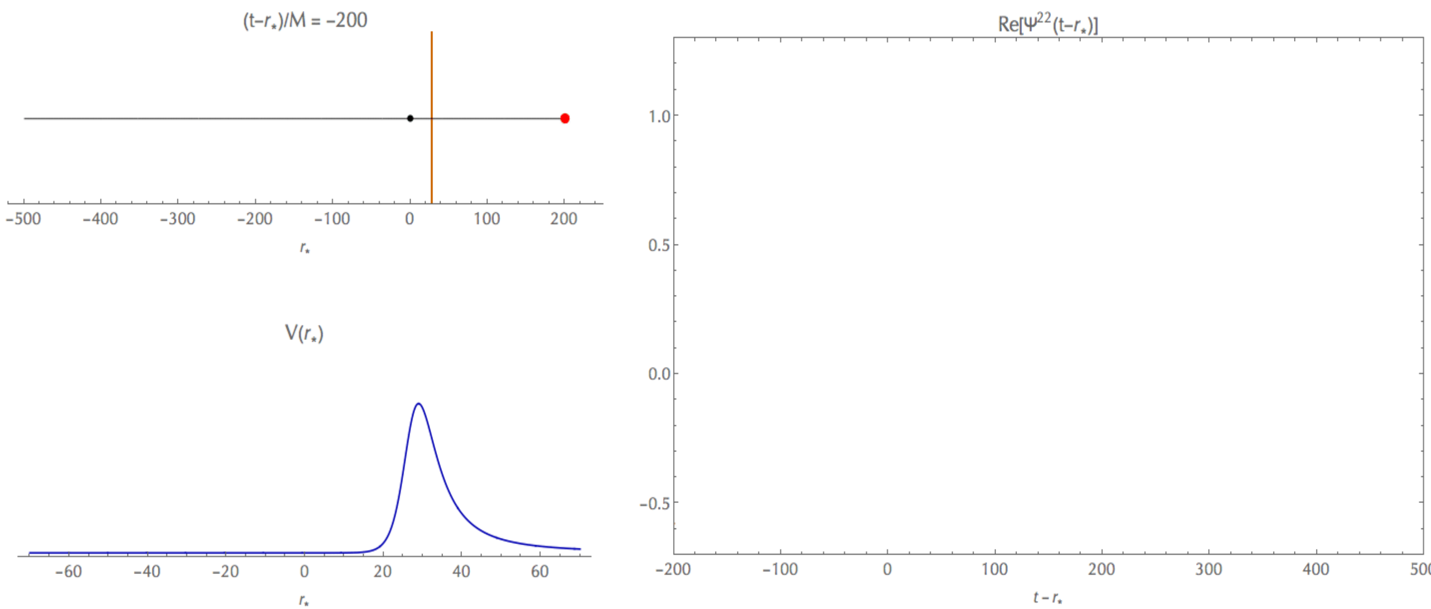
$$\tilde{h}(f) = \mathcal{A}(f) e^{i(\psi_{\mathrm{PP}} + \psi_{\mathrm{TH}} + \psi_{\mathrm{TD}})}$$

Cardoso + PRD95:084014 (2017); Sennett + PRD96:024002 (2017)
Maselli+ PRL120:081101 (2018)

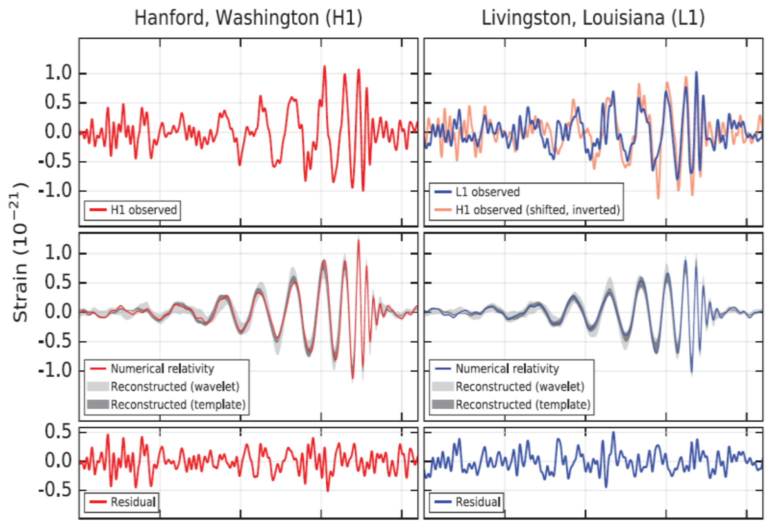
Post-merger



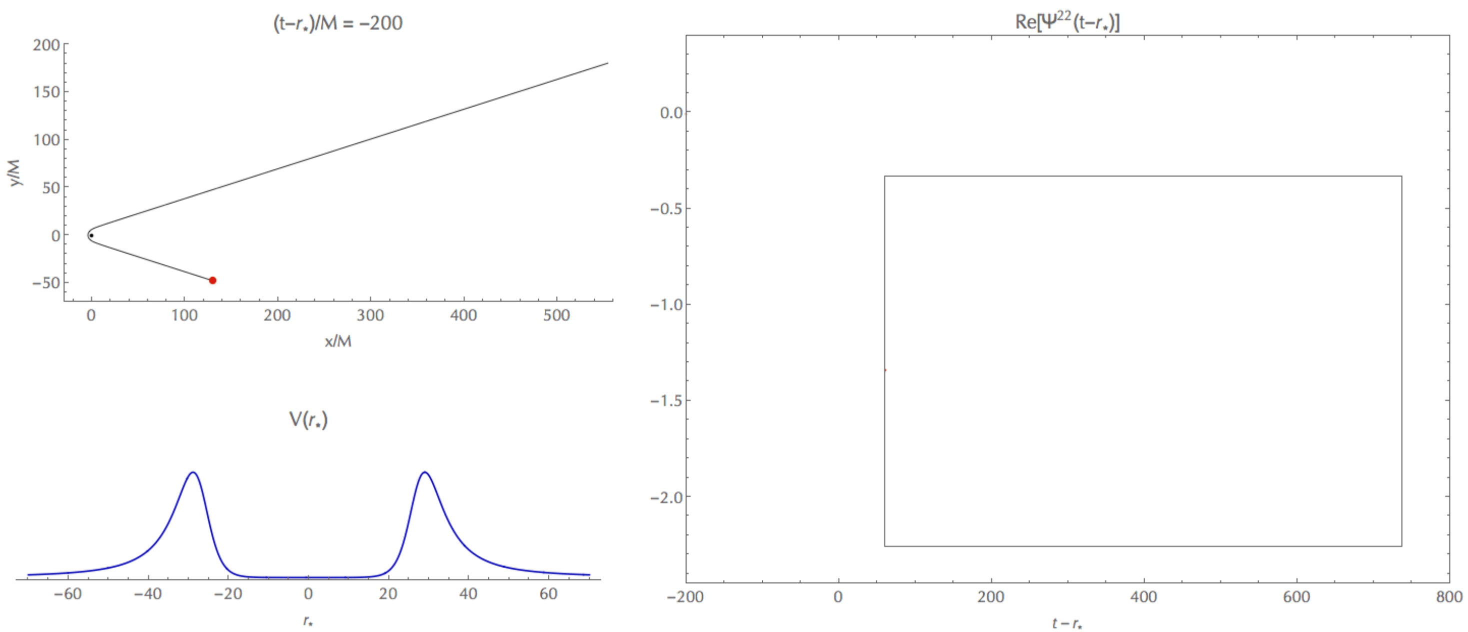
$\mathcal{E} = 1.5$, $\mathcal{J} = 0$



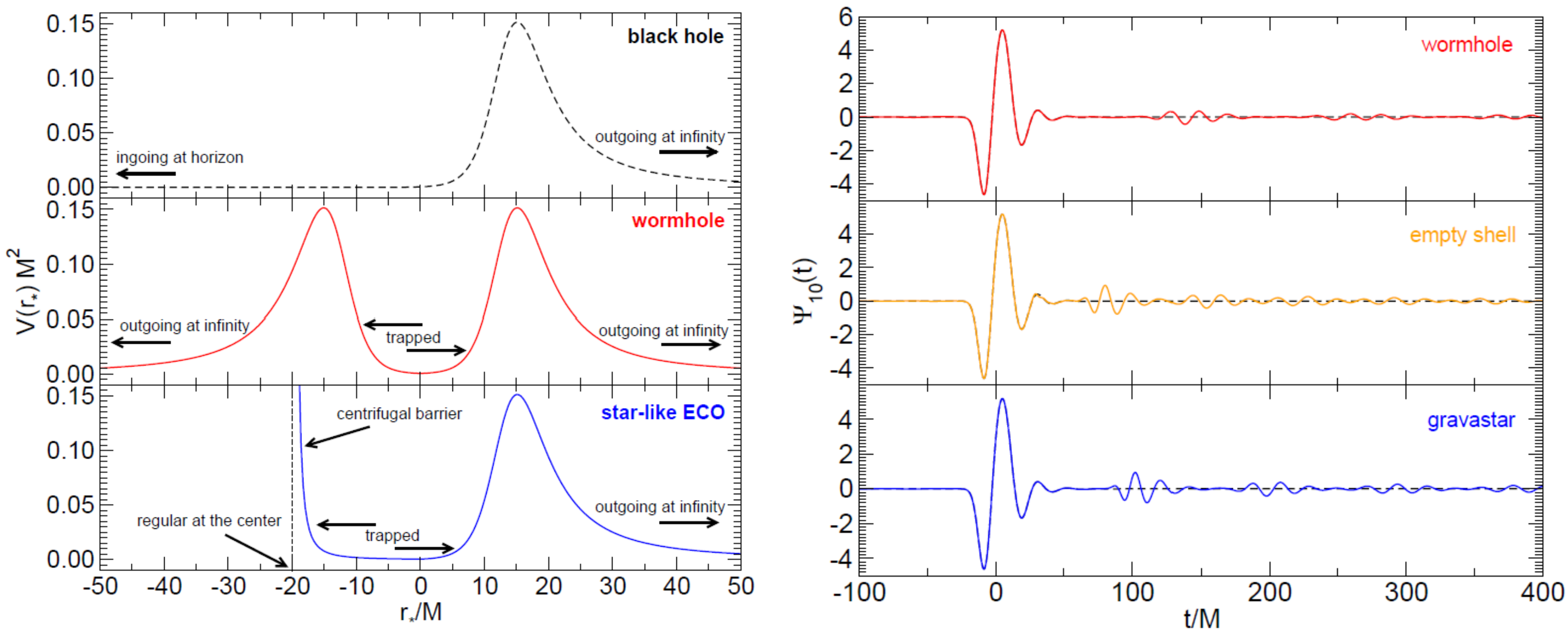
Post-merger



$\mathcal{E} = 1.5$, $r_{min}=4.3M$, $r_0-2M = 10^{-6}M$



Echoes



Cardoso + PRL116:171101 (2016); Cardoso and Pani, Nature Astronomy 1: 2017
Cardoso and Pani, Living Reviews in Relativity, to appear

Echoes and BH transfer functions

$$\frac{d^2\psi}{dz^2} + \left(\omega^2 - V\right)\psi = \mathcal{S}$$
$$\psi_{\rm ECO} \sim e^{-i\omega z} + \mathcal{R}e^{i\omega z - 2i\omega z_0}$$

The signal can be expressed as the one which would arise from a BH, with an appropriate transfer function K

$$\psi_{\rm ECO}^{\infty} = \psi_{\rm BH}^{\infty} + \mathcal{K}e^{2i\omega z}\psi_{\rm BH}^{r_+}$$
$$\mathcal{K} = \frac{\mathcal{R}e^{-2i\omega z_0}}{B_{\rm out} - B_{\rm in}\mathcal{R}e^{-2i\omega z_0}}$$

The expansion as a geometric series yields a series of echoes!

$$\mathcal{K} = \frac{\mathcal{R}e^{-2i\omega z_0}}{B_{\rm out}} \sum_{n=1}^{\infty} \left(\frac{B_{\rm in}\mathcal{R}}{B_{\rm out}}\right)^{n-1} e^{-2i\omega(n-1)z_0}$$

Mark+ PRD96: 084002 (2017)

Echoes and Dyson series

Express instead the problem in a flat spacetime background, treating the potential V as a perturbation

$$\psi = \psi_0 + \int_{z_0}^{\infty} g(z,z')\, V(z')\psi(z')\, dz'$$

$$g(z,z') = \frac{e^{i\omega|z-z'|} + \mathcal{R}e^{-2i\omega z_0}\, e^{i\omega(z+z')}}{2i\omega}$$

g is Green function for free wave operator, with previous BCs, and psi_0 is free wave amplitude. Solution is Dyson series

$$\psi = \sum_{k=1}^{\infty} \int_{z_0}^{\infty} g(z,z_1)\cdots g(z_{k-1},z_k) V(z_1)\cdots V(z_{k-1}) \mathcal{S}(z_k) dz_1 \cdots dz_k$$

The expansion as a geometric series yields a series of echoes!

$$\psi = \psi_o + \sum_{n=1}^{\infty} \psi_n$$

Correia, Cardoso PRD97: 084030 (2018)

v. The evidence for black holes

	Constraints		Source
	$\epsilon(\lesssim)$	$\frac{\nu}{\nu_\infty}(\gtrsim)$	
1.	$\mathcal{O}(1)$	1.4	Sgr A* & M87
2.	$\mathcal{O}(0.01)$	10	GW140915
3.	$10^{-4.4}$	158	All with $M > 10^{7.5} M_\odot$
4.	10^{-14}	10^7	Sgr A*
5.	10^{-40}	10^{20}	All with $M < 100 M_\odot$

	Effect and caveats
1.	Uses detected structure in "shadow" of SgrA and M87.
	Spin effects are poorly understood; systematic uncertainties not quantified.
2.	Uses same ringdown as BH and lack of echoes.
	?
3.	Lack of optical/UV transients from tidal disruption events.
	Assumes: all objects are horizonless, have a hard surface, spherical symmetry, and isotropy.
4.	Uses absence of relative low luminosity from Sgr A*, compared to disk.
	Spin effects and interaction of radiation with matter poorly understood; assumes spherical symmetry.
5.	Uses absence of GW stochastic background (from ergoregion instability).
	Assumes: hard surface (perfect reflection); exterior Kerr; all objects are horizonless.

Cardoso and Pani, Living Reviews in Relativity *(to appear)*

Open questions

Why are photosphere modes not in spectrum?

What is amplitude of QNM excitation,
are power-law tails excited to observable levels?

What are EM signals of ultracompact objects?

Can we build “reasonable” ultracompact objects?

Nonlinear evolution os ultracompact objects

What are generic consequences of resolving singularities?
Is any of this affecting horizons?

Conclusions: exciting times!

Gravitational wave astronomy *will* become a precision discipline, mapping compact objects throughout the entire visible universe.

Black holes remain the most outstanding object in the universe. BH spectroscopy will allow to test GR and provide strong evidence for the presence of horizons… improved sensitivity pushes putative surface closer to horizon, like probing short-distance structure with accelerators. BHs can play the role of perfect laboratories for particle physics, or high energy physics.

"But a confirmation of the metric of the Kerr spacetime (or some aspect of it) cannot even be contemplated in the foreseeable future."

S. Chandrasekhar, The Karl Schwarzschild Lecture, Astronomischen Gesellschaft, Hamburg, 18 Sept. 1986

Thank you