Quantum fields inside black holes

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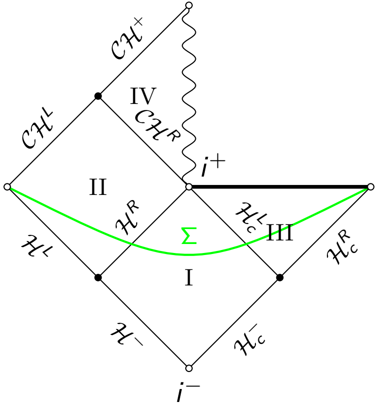
based on [2006.10991]
with Stefan Hollands, Jochen Zahn

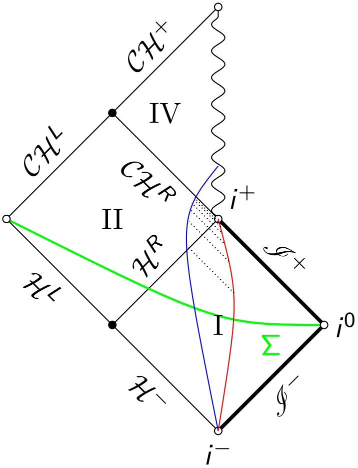
Outline

The RNdS spacetime



- $g = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2$
- $f(r) = -\frac{\Lambda}{3}r^2 + 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$
- Horizons:
  - $r_- \leftrightarrow \mathcal{CH}$
  - $r_+ \leftrightarrow \mathcal{H}$
  - $r_c \leftrightarrow \mathcal{H}_c$

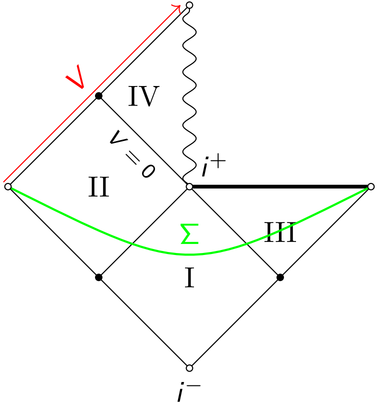
Strong cosmic censorship



- Cauchy horizon beyond which events not determined by initial data on Σ
- Signals reaching $\mathcal{CH}^R$ infinitely blueshifted [Penrose, 1974]
- Strong cosmic censorship conjecture (sCC): For generic initial data, metric is inextendible as H^1_{loc}-function across Cauchy horizon [Christodoulou, 2008]

$\Rightarrow$ sCC violated in RNdS [Cardoso et al., 2017]

Quantum effects on sCC in RNdS



- Quantum: Energy flux towards $\mathcal{CH}^R$ for state Hadamard around Σ of real quantum scalar field $\langle T_{VV} \rangle_\Psi \sim CV^{-2}$[Hollands et al., 2020]

$\Rightarrow$ sCC holds given $C \neq 0$ generically

- C computed for conformally coupled scalar for very few cases [Hollands et al., 2020]

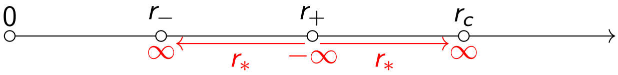
$\Rightarrow$ Can one improve method of calculation to test generic behaviour of C?

The Klein-Gordon-equation on RNdS

- Klein-Gordon-equation: $\left[\Box_g - \mu^2\right]\psi = 0$
- Mode solution: $\psi_{\omega\ell m} = (4\pi|\omega|)^{-1} r^{-1} Y_{\ell m}(\theta,\phi) e^{-i\omega t} R_{\omega\ell}(r)$
- Equation for $R_{\omega\ell}(r)$:

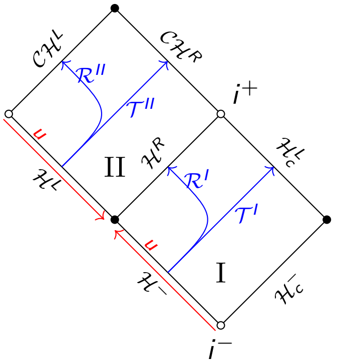
$$\left[\partial_{r_*}^2 - V(r) + \omega^2\right] R_{\omega\ell}(r) = 0$$

$\Rightarrow$ Klein-Gordon equation reduces to one-dimensional scattering problem with exponentially decaying potential



Quantum effects on sCC in RNdS

- $C \sim \sum_{\ell}(2\ell+1)\int_0^\infty \mathrm{d}\omega\,\omega\, n_\ell(\omega)$
- $n_\ell(\omega)$ depends on scattering coefficients $\mathcal{T}^{I}_{\omega\ell}$, $\mathcal{R}^{I}_{\omega\ell}$, $\mathcal{T}^{II}_{\omega\ell}$ and $\mathcal{R}^{II}_{\omega\ell}$ of "up"-modes $\psi^{I/II}_{\omega\ell m} \sim e^{-i\omega u}$ on $\mathcal{H}^{-/L}$ [Hollands et al., 2020]



$\Rightarrow$ Solve radial part of Klein-Gordon equation locally

$\Rightarrow$ Compute $\mathcal{T}^{I/II}_{\omega\ell}$, $\mathcal{R}^{I/II}_{\omega\ell}$

$\Rightarrow$ Verify $C \neq 0$

The Klein-Gordon-equation on RNdS

- Alternative radial function: $F(r) = r^{-1} R_{\omega\ell}(r)$
- "up"-Boundary conditions:

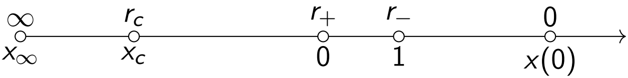
  Outside black hole:

$$F^{I}_{up}(r) \to \begin{cases} r_c^{-1} \mathcal{T}^{I}_{\omega\ell} e^{i\omega r_*} & r_* \to \infty \\ r_+^{-1} (e^{i\omega r_*} + \mathcal{R}^{I}_{\omega\ell} e^{-i\omega r_*}) & r_* \to -\infty \end{cases}$$

  Inside black hole:

$$F^{II}_{up}(r) \to \begin{cases} r_+^{-1} e^{i\omega r_*} & r_* \to -\infty \\ r_-^{-1} (\mathcal{T}^{II}_{\omega\ell} e^{i\omega r_*} + \mathcal{R}^{II}_{\omega\ell} e^{-i\omega r_*}) & r_* \to \infty \end{cases}$$

Converting radial KGE into (almost) Heun equation

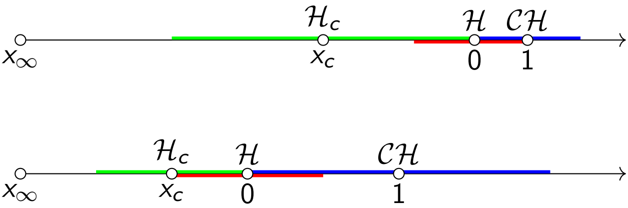


- Adapted variable for $\mathcal{H}$: $x = x_\infty \frac{r-r_+}{r-r_o}$ [Suzuki et al.,1999]
- Ansatz: $F(x) \sim e^{i\omega r_* \frac{x-x_\infty}{1-x_\infty}} h(x)$
- Equation for $h(x)$: Heun equation + extra term for non-conformally coupled case
- Boundary conditions: $h(x)$ regular on the horizon and $h(0) = 1$

Solving the equation for $h(x)$ around $\mathcal{H}$

- Power-series ansatz: $h(x) = \sum_{n=0}^{\infty} h_n x^n$, $h_0 = 1$
- Solve for h_n: 5-term recurrence relation for h_n

  $a_n h_{n+2} + b_n h_{n+1} + c_n h_n + d_n h_{n-1} + e_n h_{n-2} = 0$
- Repeat for other horizons



Solving for the scattering coefficients

$\Rightarrow$ Normalized solution $R_{i,\omega\ell}(x)$ around each r_i

- Take into account boundary conditions:

  Outside BH:

  $F^{I}_{up} = \mathcal{T}^{I}_{\omega\ell} R_{c,\omega\ell}(x)$, and
  $F^{I}_{up} = R_{+,\omega\ell}(x) + \mathcal{R}^{I}_{\omega\ell} \overline{R_{+,\omega\ell}}(x)$;

  Inside BH:

  $F^{II}_{up} = R_{+,\omega\ell}(x)$, and
  $F^{II}_{up} = \mathcal{T}^{II}_{\omega\ell} R_{-,\omega\ell}(x) + \mathcal{R}^{II}_{\omega\ell} \overline{R_{-,\omega\ell}}(x)$

- Equate definitions for F^{I}_{up} and F^{II}_{up} to solve for scattering coefficients

Convergence in ℓ

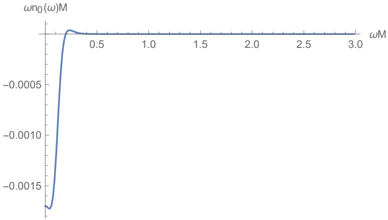


Figure: $\omega n_0(\omega)$ for $\mu^2 = 0$, $\Lambda = 0.02M^{-2}$, $Q = 0.9917M$

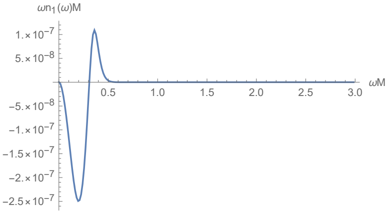


Figure: $\omega n_1(\omega)$ for $\mu^2 = 0$, $\Lambda = 0.02M^{-2}$, $Q = 0.9917M$

μ^2-dependence

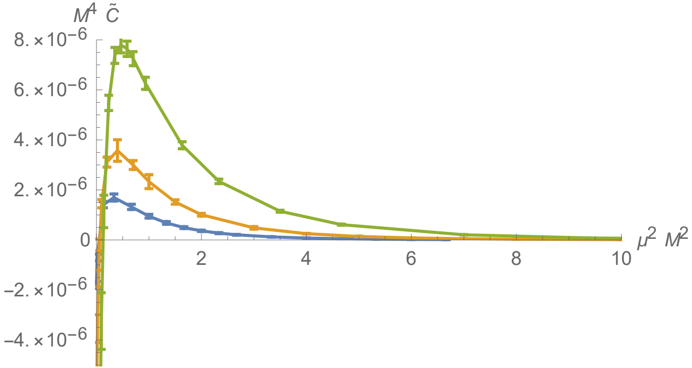


Figure: C for $\Lambda = 0.02M^{-2}$, $Q = 0.9917M$ (blue), $\Lambda = 0.06M^{-2}$, $Q = 0.992M$ (orange) and $\Lambda = 0.14M^{-2}$, $Q = 0.9945M$ (green), as a function of μ^2

Q/M-dependence

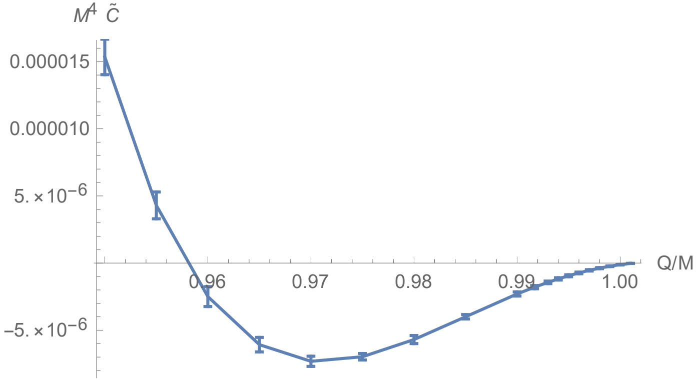
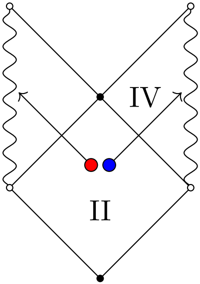


Figure: C for the massless real scalar, $\Lambda = 0.02M^{-2}$, as a function of Q/M

Summary and Interpretation

- Power-series ansatz for the scattering problem in RNdS $\Rightarrow$ quicker than numerical integration, but works only if cosmological horizon not too far away from event horizon
- Calculation of divergence coefficient C as series in ℓ $\Rightarrow$ sum over ℓ seems to converge for small μ^2
- C non-zero in general $\Rightarrow$ sCC holds
- C can be positive and negative $\Rightarrow$ whether observer is compressed or stretched to death depends on spacetime parameters and scalar field mass

Motivation



- Charged black hole ⇒ some charged matter
- Charged scalar ⇒ electromagnetic flux inside the black hole

⇒ Development of field strength inside black hole

⇒ Electromagnetic flux across $\mathcal{CH}^R$ and stress-energy tensor near $\mathcal{CH}^R$

Finding the expression for the current

- Background electromagentic potential: $A = -\frac{Q}{r}\mathrm{d}t$
- v-component of the current: $i\langle\psi^*(x)\partial_v\psi(x) - \psi(x)\partial_v\psi^*(x)\rangle$
- State: Unruh-vacuum (no incoming particles from past horizons)
- Calculation of current: point-split renormalisation

$\Rightarrow$ Mode-sum expression for the current on $\mathcal{CH}$

- Potential problems:
  - IR divergences
  - Bound states