Hydrodynamics, Spontaneously Broken Symmetries, Holography, and Some New Results

Based on: [Ammon, Baggioli, SG, Grieninger, Jain; '20], [Ammon, Baggioli, SG, Grieninger; JHEP '19]

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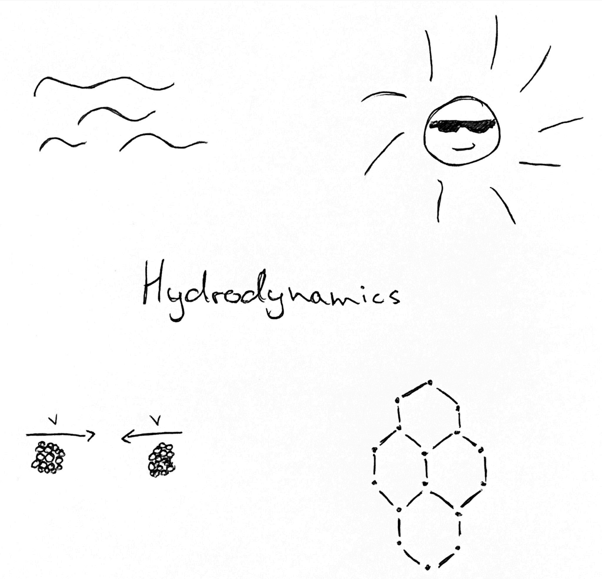
Hydrodynamics

Hydrodynamics gives insight into long-wavelength and late-time behaviour of a medium, e.g. sound modes.

The hydrodynamic equations are conservation equations.

The World of Hydrodynamics



Hydrodynamics in QFT

Relativistic hydrodynamics effectively describes small momentum and small frequency fluctuations in QFT with finite temperature.

Conservation equations

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0, \qquad \partial_\mu \langle J^\mu \rangle = 0.$$

One point functions of symmetry currents are expressed as derivative expansions, e.g.[1]

$$\langle T^{\mu\nu} \rangle = \varepsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \eta \Delta^{\mu\alpha}\Delta^{\nu\beta} \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d} \eta_{\alpha\beta} \partial_\rho u^\rho \right) + \mathcal{O}(\partial^2).$$

[1] Here we have assumed conformal invariance.

Derivative Expansion

Ideal part:

$$\langle T^{\mu\nu}\rangle = \varepsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \eta\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d}\eta_{\alpha\beta}\partial_\rho u^\rho\right) + \mathcal{O}(\partial^2),$$

where

- $u^\mu(t,\vec{x})$: velocity of the fluid
- $\varepsilon(T,\mu)$: energy density
- $p(T,\mu)$: pressure
- $\eta(T,\mu)$: shear viscocity

and $\Delta^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$ is a projector.

Thermodynamics enters through the equation of state for $p(T,\mu)$.

Derivative Expansion

Viscous part:

$$\langle T^{\mu\nu}\rangle = \varepsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \eta\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d}\eta_{\alpha\beta}\partial_\rho u^\rho\right) + \mathcal{O}(\partial^2),$$

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and $\Delta^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$ is a projector.

Determines fluid response to perturbation.

Derivative Expansion

Truncation:

$$\langle T^{\mu\nu} \rangle = \varepsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \eta \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d}\eta_{\alpha\beta}\partial_\rho u^\rho\right) + \mathcal{O}(\partial^2),$$

where

- $u^\mu(t, \vec{x})$: velocity of the fluid
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and $\Delta^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$ is a projector.

Terms beyond one derivative not completely constructed

Constraints on Derivative Expansion

Form of derivative expansion is constrained by

- Equations of motion
- Frame choice
- Onsager relations
- Positivity of local entropy production

Finding consistent derivative expansions is very challenging – terms can be, and have been, overlooked.

Holographic techniques discovered new transport phenomena, i.e. the chiral magnetic and vortical effects. [Erdmenger, Haack, Kaminski, Yarom; '08], [Benerjee et. al; '08] [Son, Surowka; '09], [Landsteiner, Megias, Melgar, Pena-Benitez; '11], [Gooth et. al; '17]

Symmetries

Symmetries are often idealisations – the real world breaks them!

Explicit symmetry breaking:

- Global symmetry of QFT broken by external effects
- Corresponding symmetry current no longer conserved
- Beyond the regime of hydrodynamics[2]

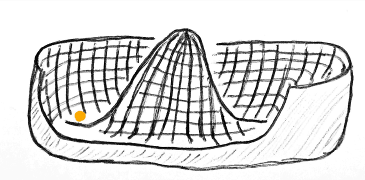
Spontaneous symmetry breaking (SSB):
Ground state of QFT no longer invariant under global symmetry.

[2] If the source of explicit breaking is small enough, hydrodynamics can still be applied (with modification)

Spontaneous Symmetry Breaking: a Simple Example

SSB of continuous global symmetry:

- Vacuum state lives in ‘sombrero’ potential $V((\phi^*\phi)^2)$



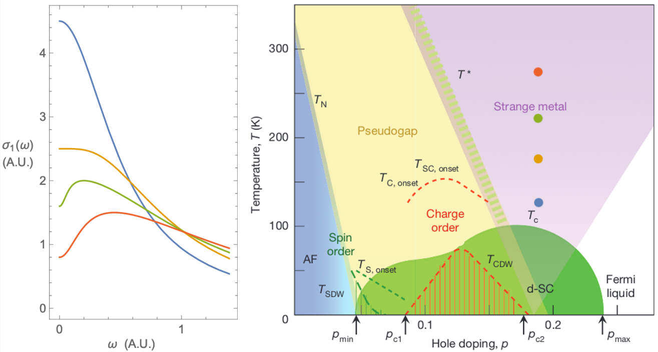
- New massless degrees of freedom: Goldstone bosons
- Symmetry currents still conserved; hence suitable for hydrodynamics.

SSB of Translational Invariance

Consider breaking of spatial translational invariance.

Why?

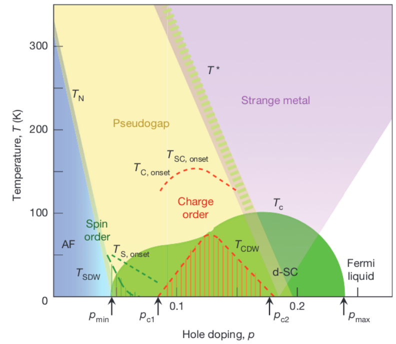
SSB of Translational Invariance: Motivation



[Delacrétaz, Goutéraux, Hartnoll, Karlsson; '16], [Keimer, Kivelson, Norman, Uchida, Zaanen; '15]

Shift and broadening of peaks in optical conductivity of 'strange metals' potentially explained using hydrodynamics with pseudo-SSB of translations. [Delacrétaz, Goutéraux, Hartnoll, Karlsson; '16]

SSB of Translational Invariance: Motivation



[Keimer, Kivelson, Norman, Uchida, Zaanen; '15]

Strange metals believed to arise from quantum critical point (QCP).

Pseudo-SSB may be imprint of symmetry breaking of QCP, which could also affect other phases.

SSB of Translational Invariance: Motivation

In QFT:

[Zippelius, Halperin, Nelson; ’80], [Chaikin, Lubenski; ’95], ..., [Delacrétaz, Goutéraux, Hartnoll, Karlsson; ’16, ’17], [Davison, Delacrétaz, Goutéraux, Hartnoll; ’16]...

Can holography shed light on these questions?

[Alberte, Ammon, Baggioli, Jimenéz-Alba; ’17, ’17], [Ammon, Baggioli, SG, Grieninger, (Jain); ’19, (’20)],

[Amoretti, Areán, Goutéraux, Musso; ’17, ’17, ’18, ’19, ’19],

[Donos, Martin, Pantelidou, Ziogas; ’19, ’19, ’19], [Armas, Jain; ’19, ’20]...

A first step is to study pure spontaneous symmetry breaking...

SSB of Translational Invariance: Goldstone Mode

Consider spontaneous breaking of spatial translational invariance in a $(2+1)$-dimensional QFT.

Goldstone bosons associated with this spontaneous breaking are the phonons

$$\Phi = \begin{pmatrix} \Phi_x \\ \Phi_y \end{pmatrix}.$$

Since the phonons are massless fields, they will contribute to the hydrodynamics.

Phonon Contribution: Derivative Expansion

Derivative expansion of spatial components of conserved current becomes[3]

$$\langle T_{ij}\rangle = \delta_{ij}\left[p-(B+G)\partial\cdot\langle\Phi\rangle\right] - 2G\left[\partial_{(i}\langle\Phi_{j)}\rangle - \delta_{ij}\partial\cdot\langle\Phi\rangle\right] - \eta\sigma_{ij} + \mathcal{O}(\partial^2),$$

where $\partial\cdot\langle\Phi\rangle$ is the divergence of the expectation value of Φ.

[3] We have chosen a frame where $u^\mu = (1,0,0)$.

Phonon Contribution: Derivative Expansion

Derivative expansion of spatial components of conserved current becomes[3]

$$\langle T_{ij} \rangle = \delta_{ij} \left[p - (B+G)\partial \cdot \langle \Phi \rangle \right] - 2G \left[\partial_{(i} \langle \Phi_{j)} \rangle - \delta_{ij} \partial \cdot \langle \Phi \rangle \right] - \eta \sigma_{ij} + \mathcal{O}(\partial^2),$$

where $\partial \cdot \langle \Phi \rangle$ is the divergence of the expectation value of Φ.

Terms which appeared without spontaneous breaking; $\eta \sigma_{ij}(t, \vec{x})$ is the viscous contribution.

[3] We have chosen a frame where $u^\mu = (1, 0, 0)$.

Phonon Contribution: Derivative Expansion

Derivative expansion of spatial components of conserved current becomes[3]

$$\langle T_{ij}\rangle = \delta_{ij}\left[p - (B+G)\partial\cdot\langle\Phi\rangle\right] - 2G\left[\partial_{(i}\langle\Phi_{j)}\rangle - \delta_{ij}\partial\cdot\langle\Phi\rangle\right] - \eta\sigma_{ij} + \mathcal{O}(\partial^2),$$

where $\partial\cdot\langle\Phi\rangle$ is the divergence of the expectation value of Φ.

Additional contributions due to presence of Goldstone bosons Φ_i.

New coefficients:

- $G(T,\mu)$: shear elastic modulus
- $B(T,\mu)$: bulk elastic modulus

[3] We have chosen a frame where $u^\mu = (1,0,0)$.

Derivative Expansion: Configuration Pressure

New derivative expansion of spatial components of conserved current becomes[3]

$$\langle T_{ij}\rangle = (p+\mathcal{P})\delta_{ij} + (\partial_T p + \partial_T \mathcal{P})\delta_{ij}T + (\mathcal{P} - B + G)\,\delta_{ij}\partial\cdot\langle\Phi\rangle \\ - 2G\partial_{(i}\langle\Phi_{j)}\rangle - \eta\sigma_{ij} + \mathcal{O}(\partial^2)$$

where $\partial\cdot\langle\Phi\rangle$ is the divergence of the expectation value of Φ.

Previously overlooked terms: Configuration pressure $\mathcal{P}(T,\mu,\dots)$, and $\partial_T\mathcal{P}$.

Configuration pressure accounts for a strained equilibrium state.

[Armas, Jain; '19]

$\mathcal{P}$ and $\partial_T\mathcal{P}$ also enter in other components of $T^{\mu\nu}$.

[3] We have chosen a frame where $u^\mu = (1,0,0)$.

Phonon Contribution: Josephson Relation

Additional equations for phonons: 'Josephson relation',

$$\partial_t \langle \Phi_i \rangle = u_i.$$

Derivation:

- Hamiltonian density at finite velocity: $\mathcal{H} = \mathcal{H}_0 + u^i T^0{}_i$
- From Goldstone theorem: $[\Phi_i(x), T^0{}_j(y)] = i\delta_{ij}\delta(x-y) + \dots$
- Hence $\partial_t \langle \Phi_i \rangle = i \langle [\Phi_i, \hat{H}] \rangle = \dots = u_i.$

As a consequence $\partial_\mu \langle \Phi_i \rangle$ is zeroth order in derivatives.

Phonon Contribution: Josephson Relation

Derivative expansion of Josephson relation

$$\partial_t \langle \Phi_i \rangle = u_i - \frac{1}{\sigma}\mathcal{P}\partial_t^2 \langle \Phi_i \rangle - \frac{1}{\sigma}\partial_T \mathcal{P} \partial_i T - \frac{1}{\sigma}\left(\mathcal{P} - B + G\right)\partial_i \partial \cdot \langle \Phi \rangle + \frac{2G}{\sigma}\partial^j \partial_{(i}\Phi_{j)} + \mathcal{O}(\partial^2),$$

where σ is a coefficient which relates to Goldstone diffusion.

Note: Contributions due to configuration pressure.

Hydrodynamic Modes

Without loss of generality we set the momentum in x-direction, and choose spacetime-dependence $e^{-i\omega t+ikx}$.

The hydrodynamic equations decompose into two sectors, transverse and parallel to momentum.

Solving set of equations in frequency space leads to hydrodynamic modes:

$$M\vec{V} = 0, \quad \text{and} \quad \det(M) \stackrel{!}{=} 0.$$

Modes of Longitudinal Sector I

Longitudinal sector

Modes of Longitudinal Sector I

[Armas, Jain; '19]

1. Pair of sound modes with dispersion relation

$$\omega = \pm v_{\parallel} k - \frac{i}{2}\Gamma_{\parallel} k^2,$$

and coefficients

- Speed of longitudinal sound:

$$v_{\parallel}^2 = \frac{1}{2} + \frac{G}{\chi_{\pi\pi}},$$

- Diffusion constant:

$$\Gamma_{\parallel} = \frac{\eta}{\chi_{\pi\pi}} + \frac{T^2 s^2 G^2}{\sigma \chi_{\pi\pi}^3 v_{\parallel}^2},$$

where $\chi_{\pi\pi}$ is the momentum susceptibility, which relates velocity to momentum via $T^0{}_i = \chi_{\pi\pi} u_i$.

[Armas, Jain; '19]

2. Diffusive mode with dispersion relation

$$\omega = -iD_{\|}k^2,$$

and diffusion constant

$$D_{\|} = \frac{s^2}{\sigma s'} \frac{B + G - \mathcal{P}}{\chi_{\pi\pi}/2 + G}.$$

In conformal theories the configuration pressure explicitly enters in the diffusive mode!

Special case: $\mathcal{P} = 0$ in equilibrium, but $\partial_T \mathcal{P} \neq 0$

$$D_{\|} = \frac{Ts^2/\sigma}{s + \partial_T \mathcal{P}} \frac{B + G}{Ts + 2G}\,.$$

[Ammon, Baggioli, SG, Grieninger, Jain; '20]

Holographic Duality

Holographic Duality

Conjectured duality between $(d+1)$-dimensional gravitational theories in asymptotically Anti-de Sitter (AdS) spacetimes and d-dimensional QFTs living on flat conformal boundary.

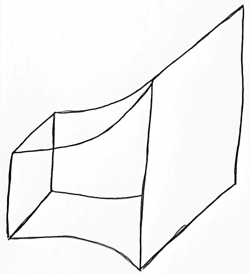
When QFT is strongly coupled, dual gravity description is weakly coupled; and vice-versa.

Holography provides tool-set to study strongly coupled quantum phenomena.

Levels of Holography

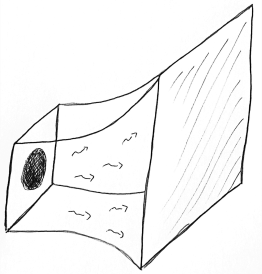
Simplest case:

Pure AdS $\Longleftrightarrow$ Vacuum state of QFT in flat-space.



Finite temperature:

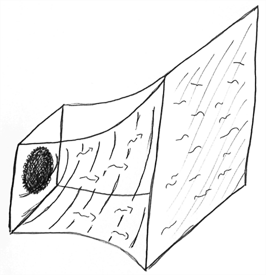
Black brane in asymptotic AdS $\Longleftrightarrow$ QFT at finite temperature.



Levels of Holography

Dynamics at finite temperature:

Pertubations of black brane (QNMs) $\Longleftrightarrow$ Poles of Green's functions.



Poles of retarded Green's functions are frequency modes of QFT.
Modes with zero frequency in limit of zero momentum correspond to hydrodynamic modes!

Hydrodynamic Coefficients

Hydrodynamic transport coefficients are given by limits of retarded Green's functions, for example

$$\eta = -\lim_{\omega \to 0} \frac{1}{\omega} \lim_{k \to 0} \mathrm{Im}\left[\mathcal{G}^R_{T_{xy}T_{xy}}(\omega, k)\right],$$
$$G = -\lim_{\omega \to 0} \lim_{k \to 0} \mathrm{Re}\left[\mathcal{G}^R_{T_{xy}T_{xy}}(\omega, k)\right].$$

Holography provides methods for calculating such coefficients using correlation functions of dual quantities in the gravitational theory.

Translational Breaking in Holography: Our Model

Construct gravity model such that dual QFT has desired properties.

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \frac{3}{\ell^2} - m^2\,V(X)\right],$$

where R is the Ricci scalar; ℓ is the radius of curvature of AdS; and

$$X = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^I,$$

with scalar field $\phi^I = \alpha x^I$. The geometry is given by

$$\mathrm{d}s^2 = \frac{\ell}{u^2}\left[\frac{\mathrm{d}u^2}{f(u)} - f(u)\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2\right],$$

where $u \in [0, u_h]$, with $u = 0$ at the conformal boundary, and $f(u_h) = 0$.

Gives spacetime of Schwarzschild black brane in asymptotic AdS, with massive graviton.

[Andrade, Withers; '13], ..., [Alberte, Ammon, Jiménez-Alba, Baggioli, Pujolas; JHEP '17]

Translational Breaking in Holography: Choice of Potentials

Consider two different potentials such that the dual QFTs display SSB of translations:

- 'Strained' models with

$$V(X) = X^N \quad \text{with} \quad N > 5/2,$$

  which do not minimise the free energy $\rightarrow \mathcal{P} \neq 0$.

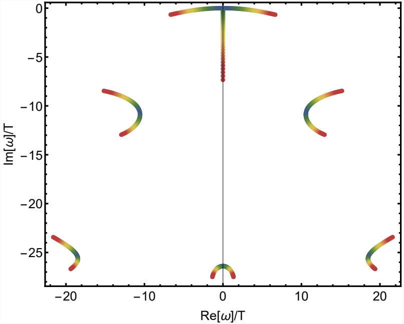
- 'Unstrained' model with

$$V(X) = X + \frac{1}{2}X^2,$$

  which minimises the free energy $\rightarrow \mathcal{P} = 0$ (but: $\partial_T \mathcal{P} \neq 0$).

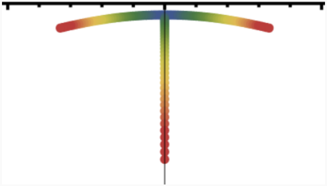
Method

1. Compute (numerically) QNMs of gravity theory



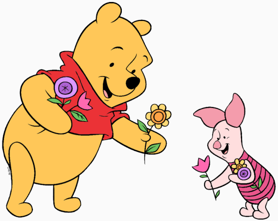
Method

2. Identify hydrodynamic modes



Method

3. Extract coefficients



Method

4. Compare QNM results to hydrodynamic formulae

Results: Strained Models

Longitudinal sector:

Strained models

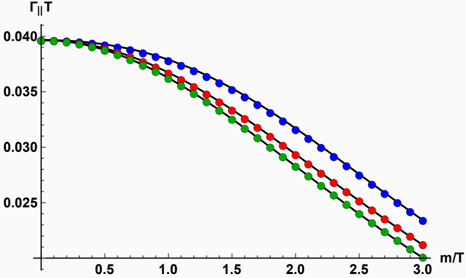
$$V(X) = X^N$$

Results: Strained Models (Sound Attenuation)

[Ammon, Baggioli, SG, Grieninger, Jain; '20]

Propagating mode of longitudinal sector:

$$\omega = \pm c_L k - \frac{i}{2}\Gamma_{\parallel} k^2$$



$$N = 3, 4, 5$$

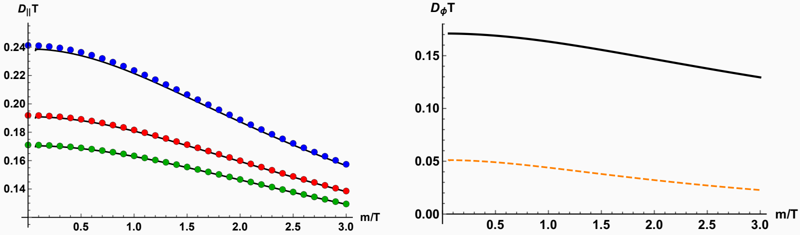
Plotted as a function of dimensionless SSB scale m/T.

Results: Strained Models (Diffusive Mode)

[Ammon, Baggioli, SG, Grieninger, Jain; '20]

Diffusive mode of longitudinal sector

$$\omega = -i D_{\parallel} k^2$$



Left: Comparison to hydrodynamics including $\mathcal{P}$, $N = 3, 4, 5$.

Right: Discrepancy between QNMs and hydrodynamics without $\mathcal{P}$, $N = 5$. [Ammon, Baggioli, SG, Grieninger; '19]

Results: Unstrained Model

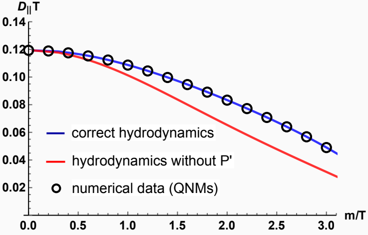
Longitudinal Sector:

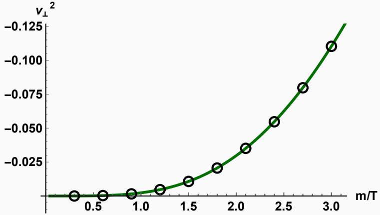
Unstrained model

$$V(X) = X + \frac{1}{2}X^2$$

Results: Unstrained Model

[Ammon, Baggioli, SG, Grieninger, Jain; '20]





Matching to hydrodynamics requires $\partial_T \mathcal{P} \neq 0$ even when $\mathcal{P} = 0$ (left).

Model is dynamically unstable; imaginary speed of sound (right), and negative diffusion constant for large m/T.

Other holographic models with $\mathcal{P} = 0$ have similar issues. Why?

Conclusion and Outlook

Hydrodynamics is interesting!
Symmetry breaking is hard!
Holography provides novel approach!

Future directions:

- Find stable unstrained models
- Fluid/Gravity approach to derivative expansion
- Investigate pseudo-spontaneous symmetry breaking from first-principles
- Move on with our lives

Thank you for your attention!