

A New Phase Transition and Chiral Symmetry in 1+2D Thirring Models

PRD96 (9) 2017; PRD100 (5) 2019

Julian J. Lenz

with

D. Schmidt, B. Wellegehausen, A. Wipf

Theoretisch-Physikalisches Institut, FSU Jena

Kick-Off Meeting RTG 2522, 25./26.02.2020

Outline

1 Introduction

2 Absence Of Chiral Symmetry Breaking

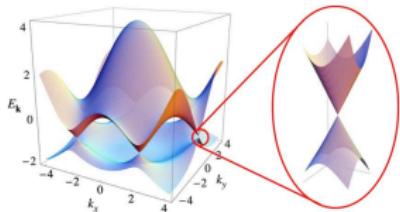
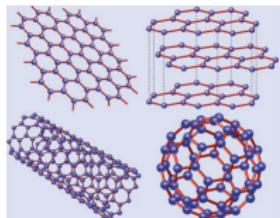
3 The New Transition

4 Conclusion

Introduction

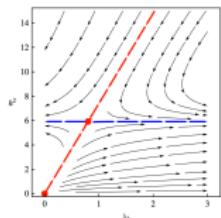
Applications

Solid state physics



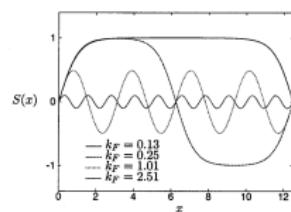
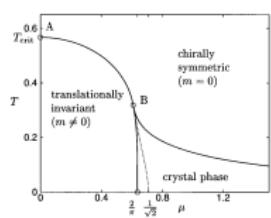
from [CASTRO NETO et al., RMP 2009],
see also [HERBUT et al., PRB 2009]

Asymp. free/save



from [BRAUN et al., PRD 2011]

QCD toy model



from [THIES et al., PRD 2003]

Four-Fermion Theories

$$\mathcal{L} = \sum_{a=1}^{N_f} \bar{\psi}_a (\mathrm{i}\not{\partial} - \mathrm{i}m) \psi_a - \frac{1}{4\lambda} \sum_{a,b=1}^{N_f} (\bar{\psi}_a M_1 \psi_a) (\bar{\psi}_b M_2 \psi_b)$$

with

$\psi, \bar{\psi}$ spinors,

a flavour index,

m mass,

N_f number of flavours

λ (inverse) coupling,

M_1, M_2 matrices

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Gross-Neveu (GN)	Thirring (Th)	Nambu-Jona-Lasinio (NJL)
$(\bar{\psi}\psi)^2$	$(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)$	$(\bar{\psi}\psi)^2 + (\bar{\psi}\mathrm{i}\gamma_5\psi)^2$

$\overline{\text{Chiral Symmetry}} \text{ U}(N_f)$

Observation

Chirality does not exist in odd space-time dimensions.

Chiral Symmetry $U(N_f)$

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But: In reducible representation (4-comp. spinors) we have γ_5 to define

$$\psi_a \mapsto \left(e^{i\gamma_5 \alpha} \right)_{ab} \psi_b, \quad \bar{\psi}_a \mapsto \left(e^{i\gamma_5 \alpha} \right)_{ab} \bar{\psi}_b.$$

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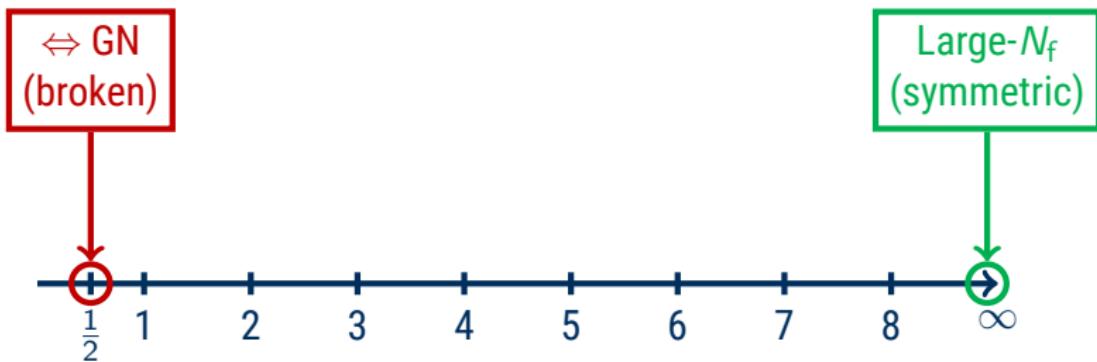
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Order parameter

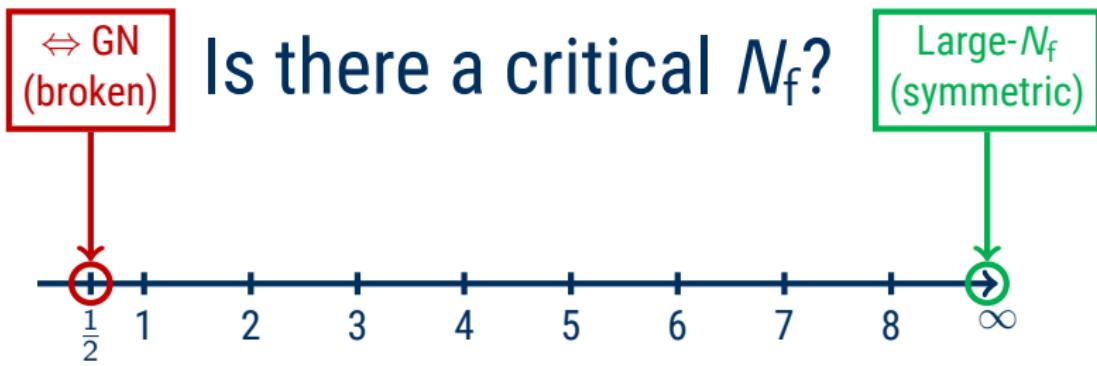
$$\Sigma = \langle \bar{\psi} \psi \rangle \neq 0$$

Absence Of Chiral Symmetry Breaking

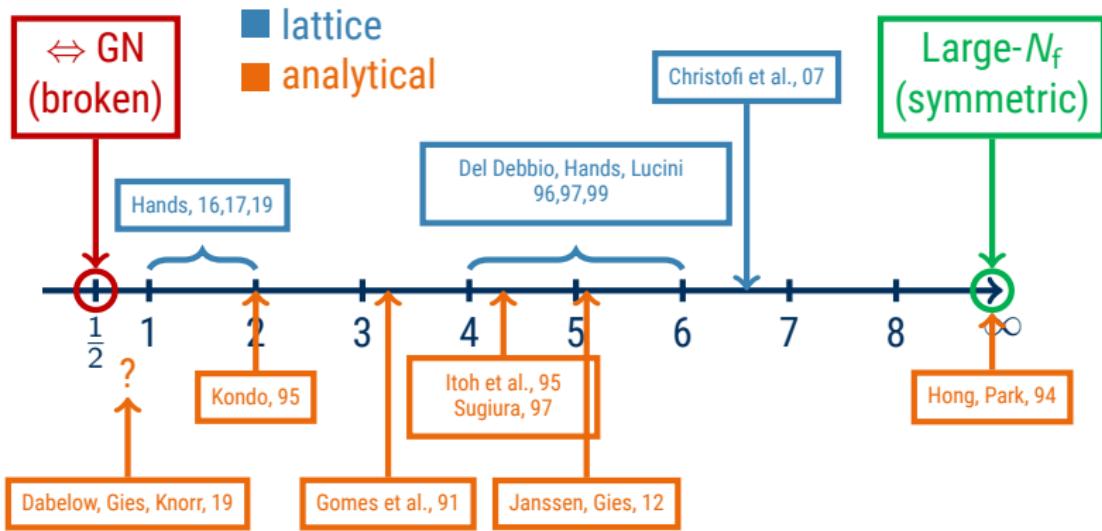
What to expect?



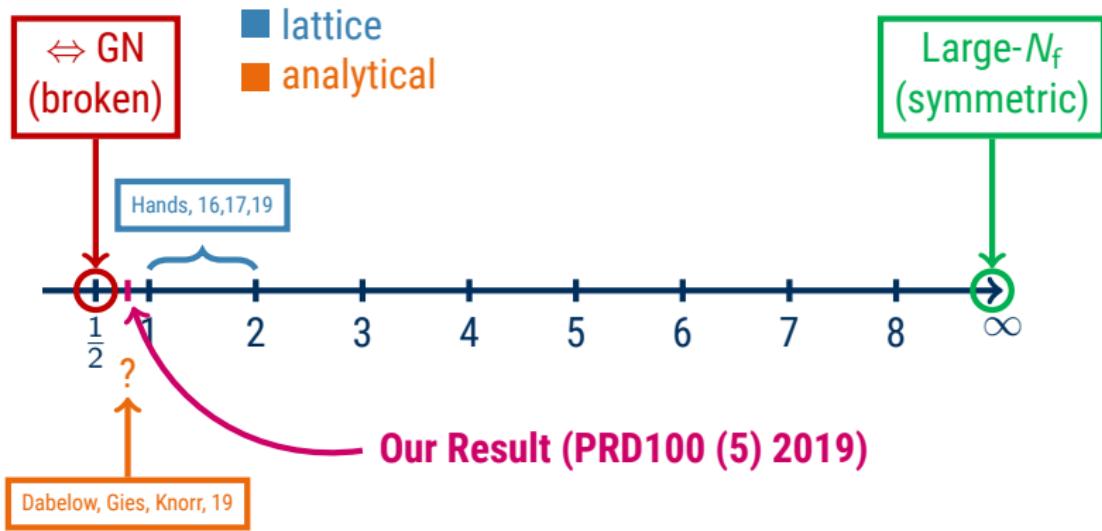
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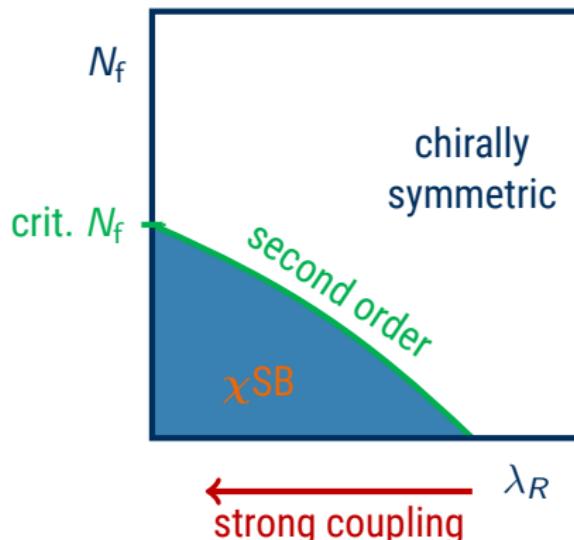
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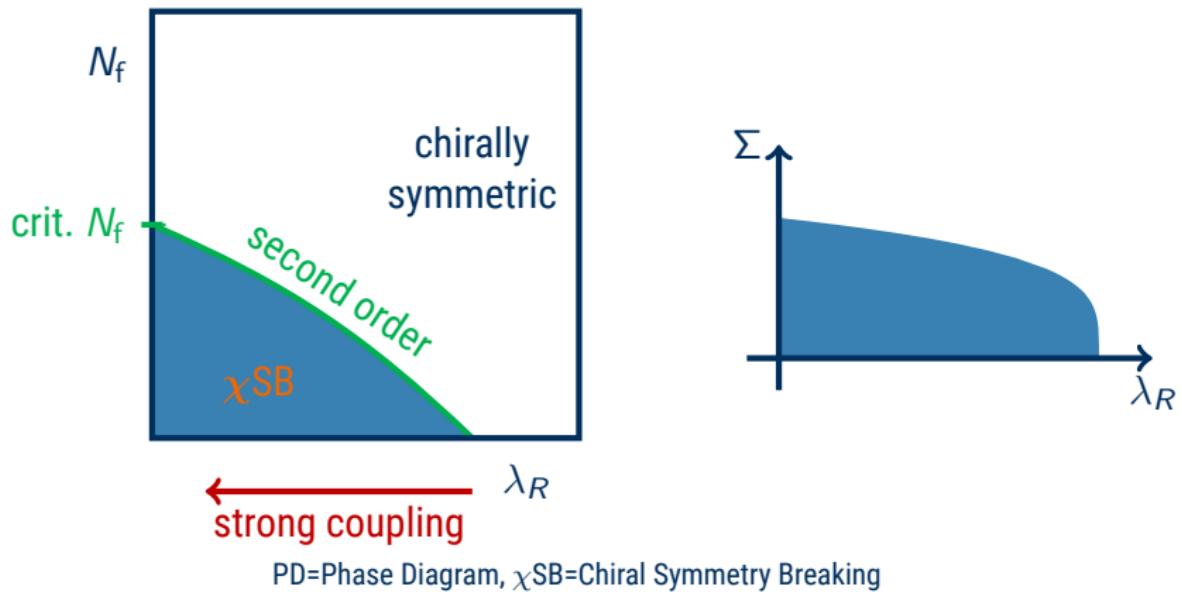


The (Putative) Continuum PD

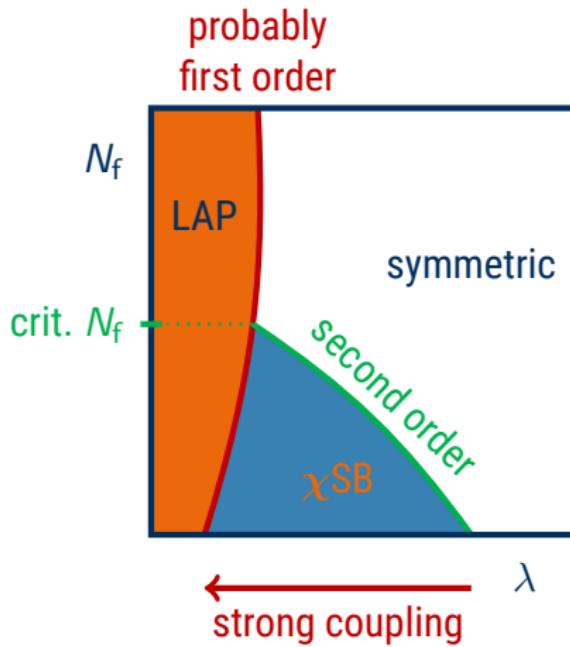


PD=Phase Diagram, χ^{SB} =Chiral Symmetry Breaking

The (Putative) Continuum PD

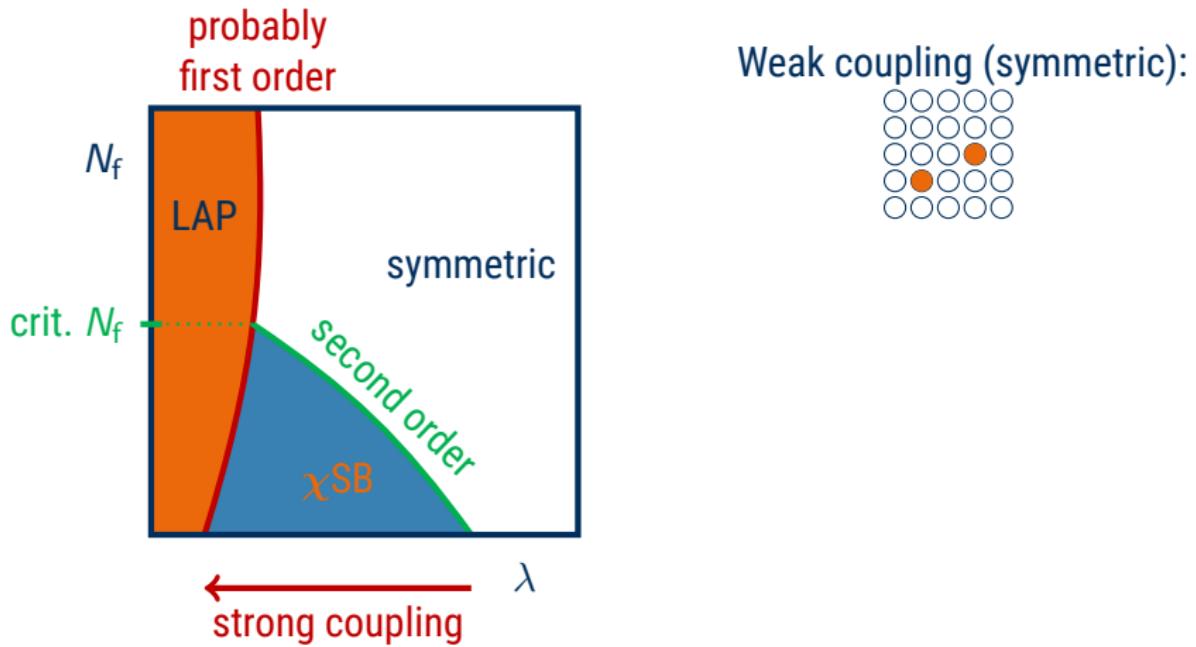


The (Putative) Lattice PD



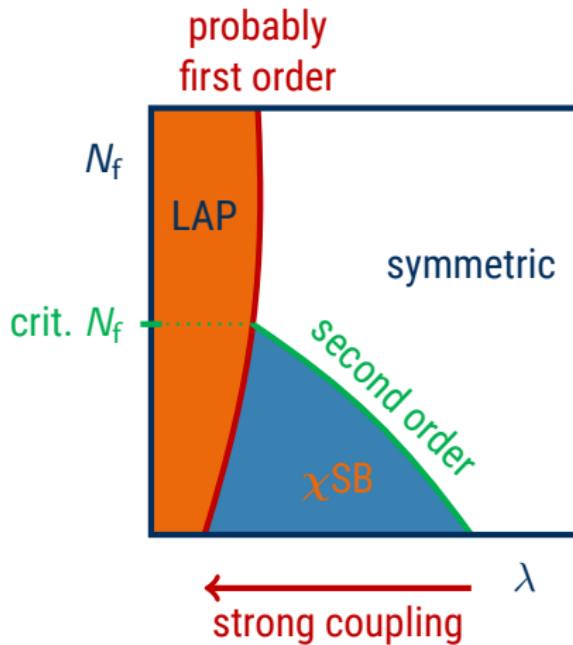
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The (Putative) Lattice PD



Weak coupling (symmetric):

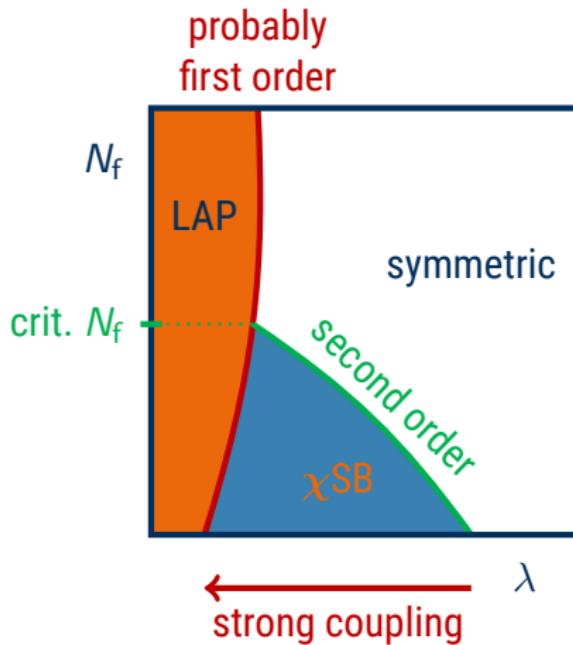


Intermediate (potentially χ SB):

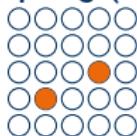


PD=Phase Diagram, LAP=Lattice Artifact Phase, χ SB=Chiral Symmetry Breaking

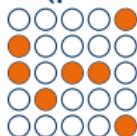
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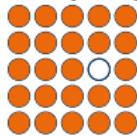
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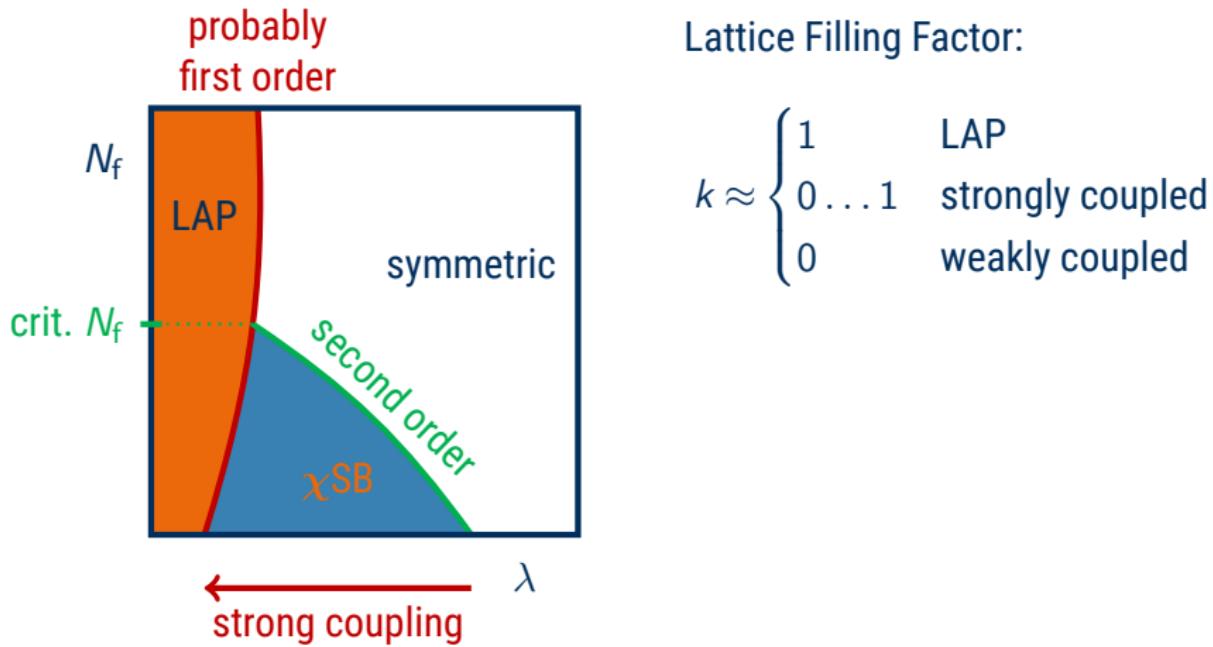


Strong coupling (LAP):



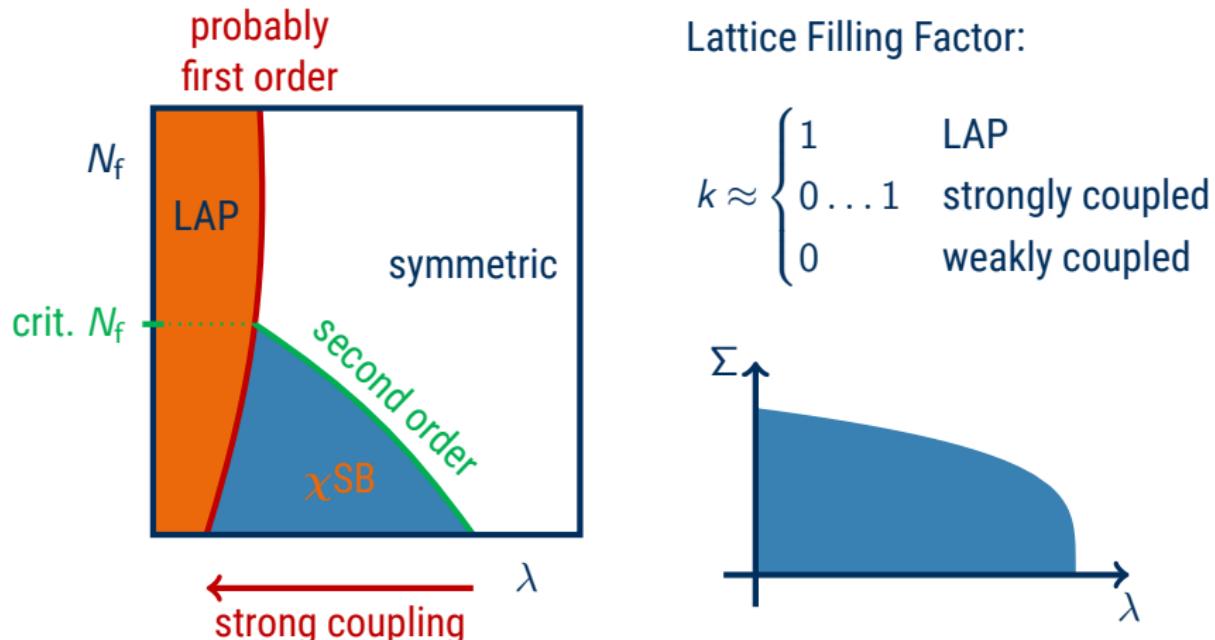
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The (Putative) Lattice PD



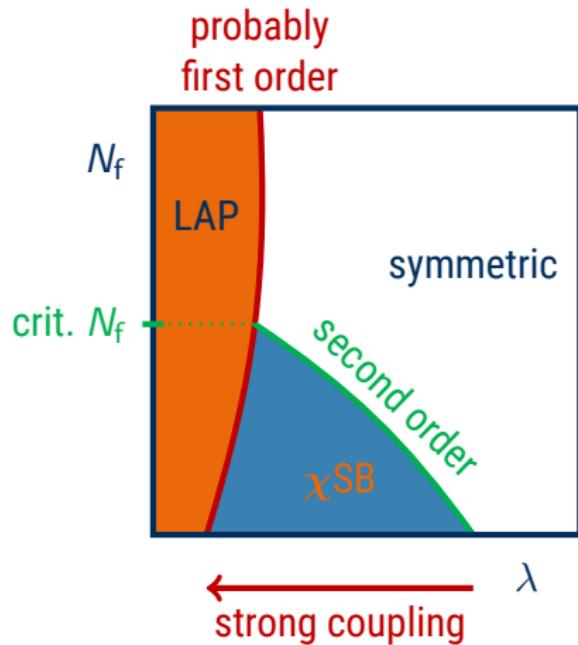
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The (Putative) Lattice PD



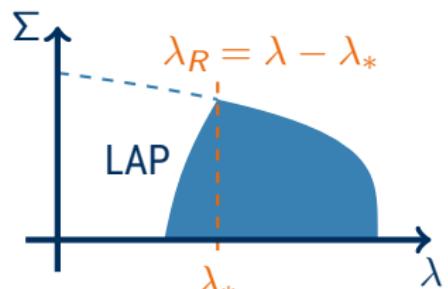
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The (Putative) Lattice PD



Lattice Filling Factor:

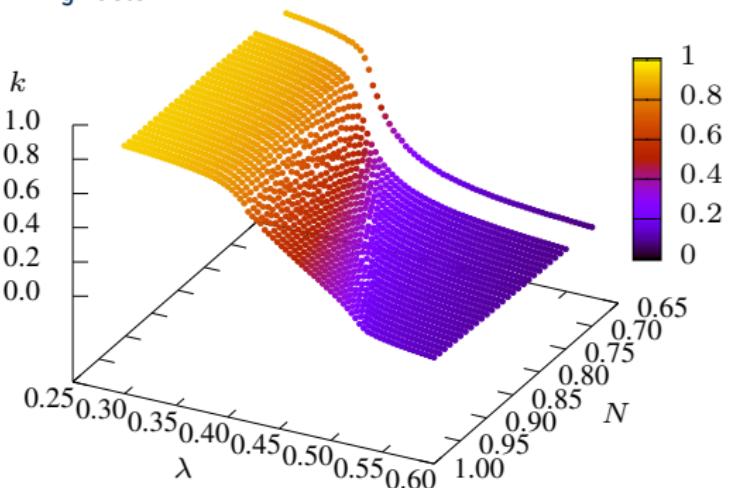
$$k \approx \begin{cases} 1 & \text{LAP} \\ 0 \dots 1 & \text{strongly coupled} \\ 0 & \text{weakly coupled} \end{cases}$$



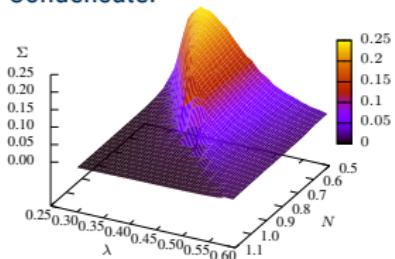
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Overview (16×15^2)

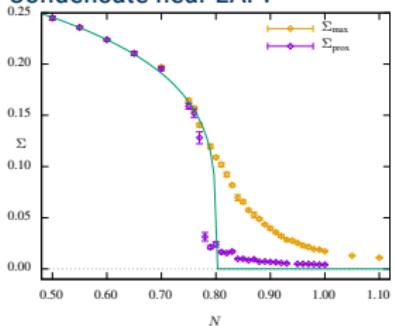
Filling Factor:



Condensate:

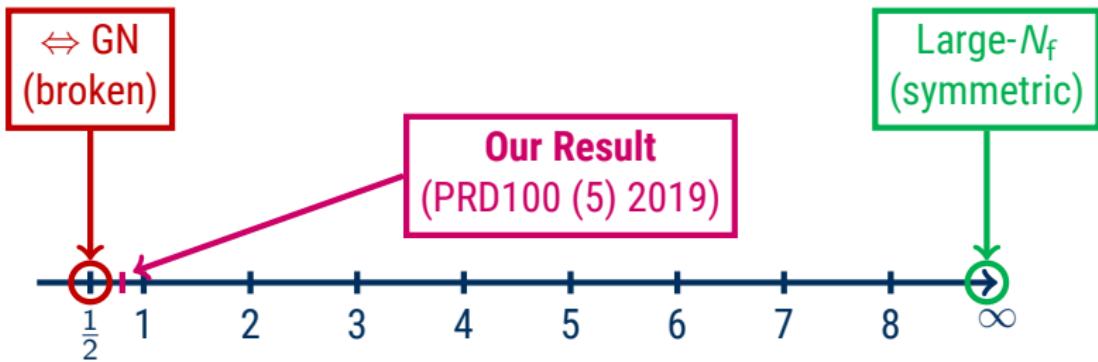


Condensate near LAP:

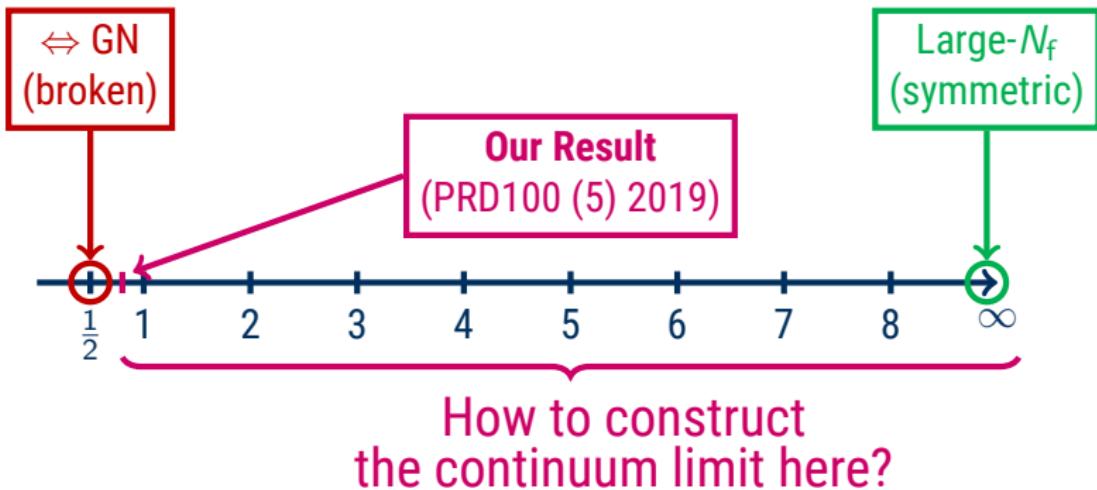


The New Transition

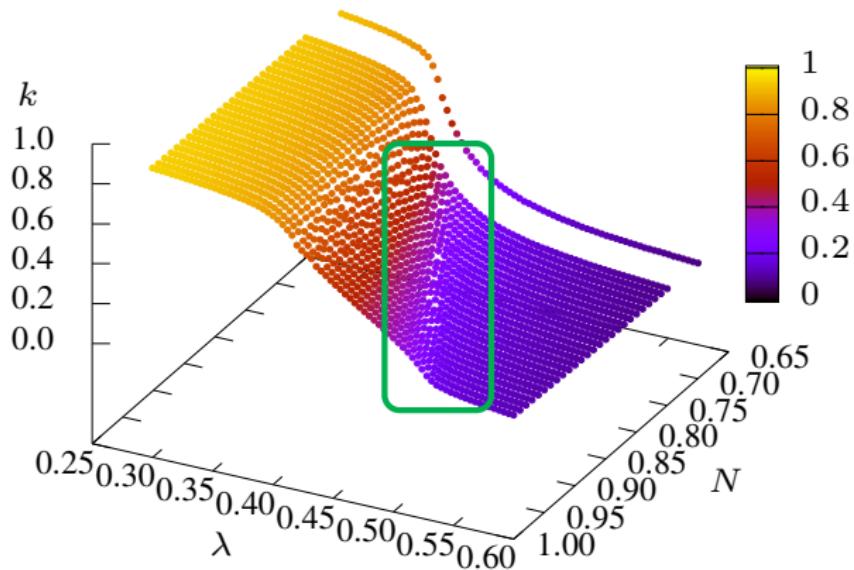
But...



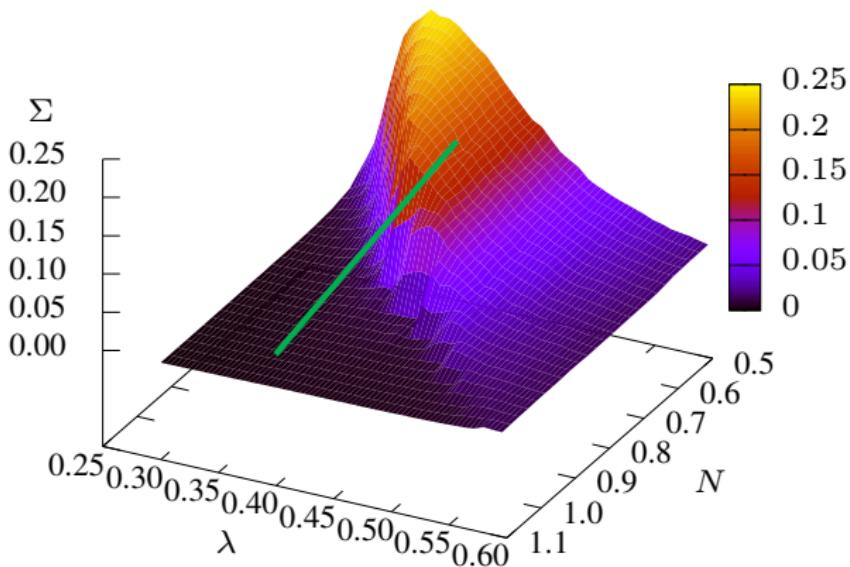
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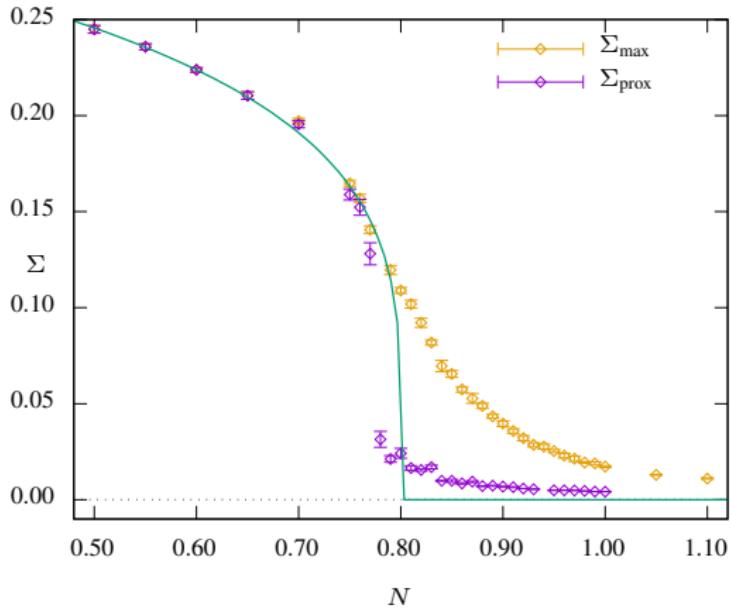
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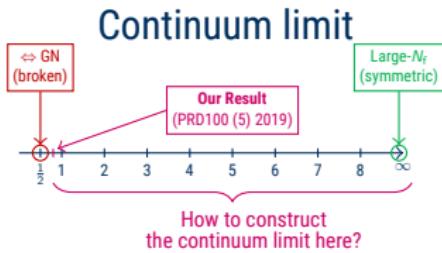
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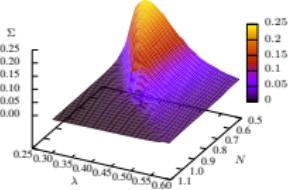
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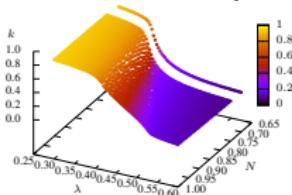
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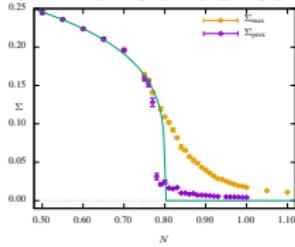
Maximum's Position



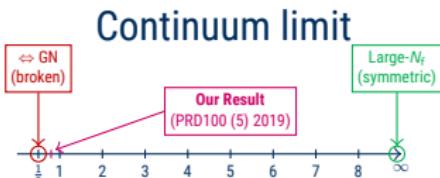
Second Drop



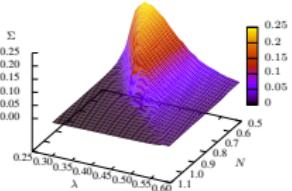
Maximal Condensate



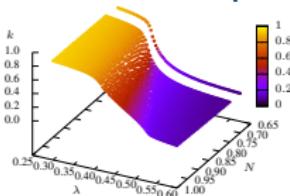
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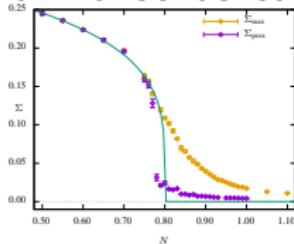
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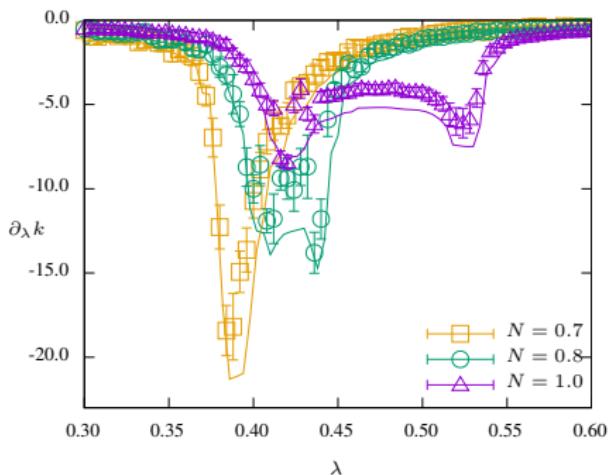
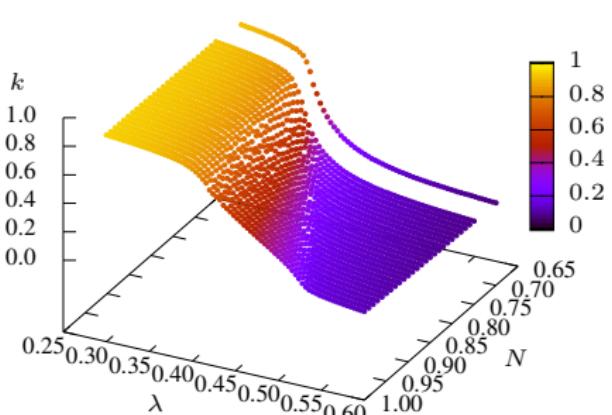
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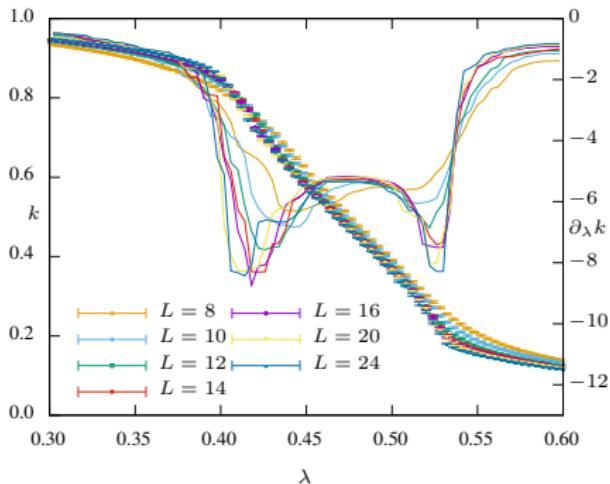
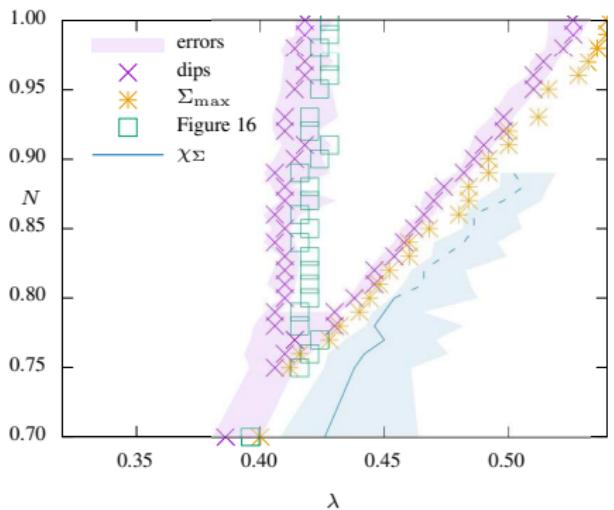
The Answer is...

... a **new second order phase transition** not related to chiral symmetry.

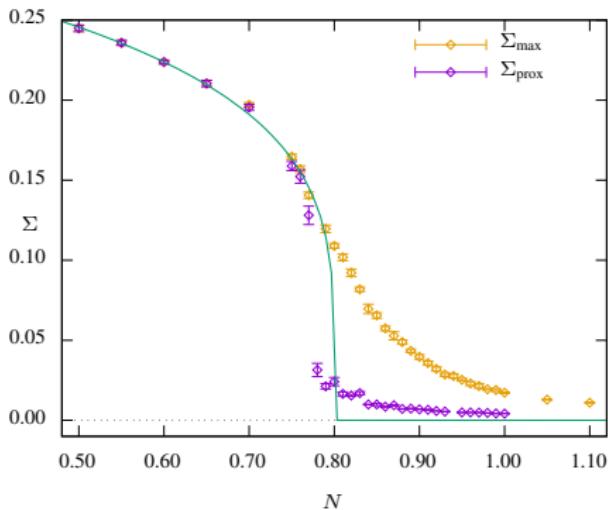
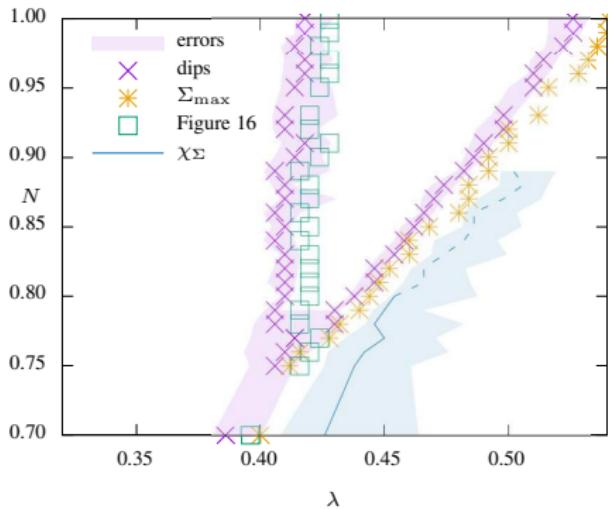
Filling Factor k



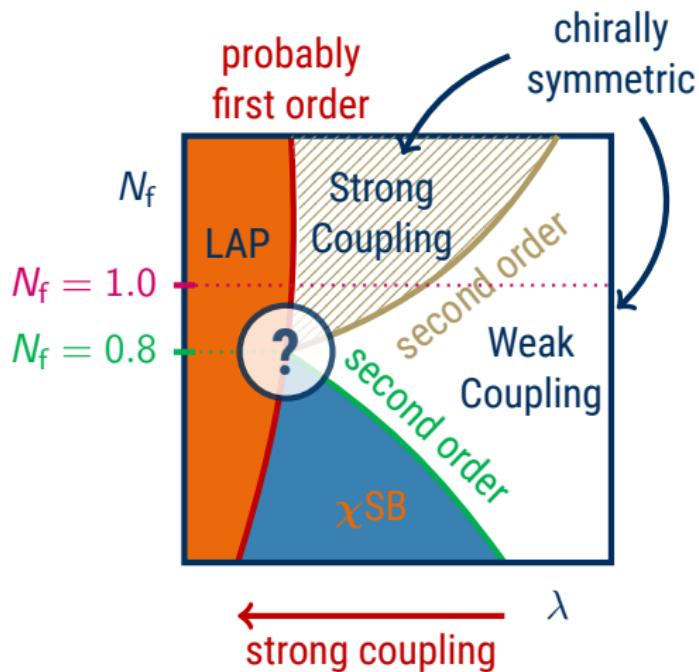
The Resolution



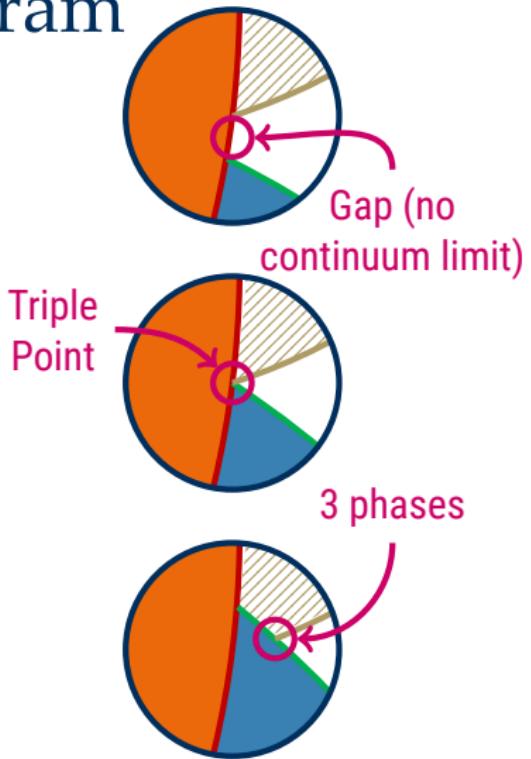
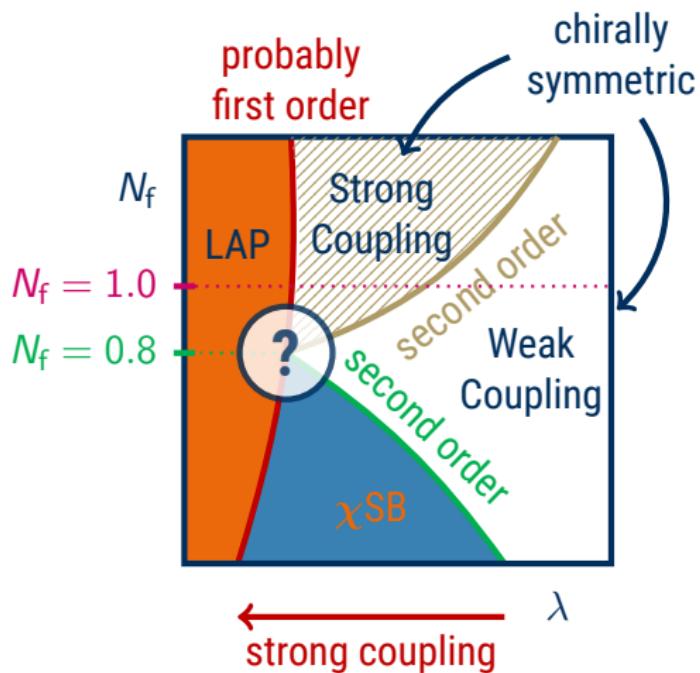
The Resolution



The New Phase Diagram



The New Phase Diagram

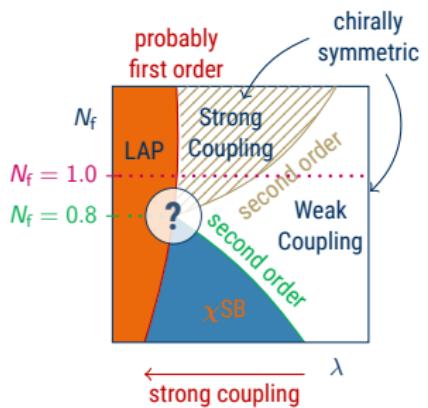


Conclusion

Conclusions and Outlook

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- **no chiral symmetry breaking** in 1+2D Thirring models (with $N_f \in \mathbb{N}$).
- a **new phase transition** not related to order parameters we know of.



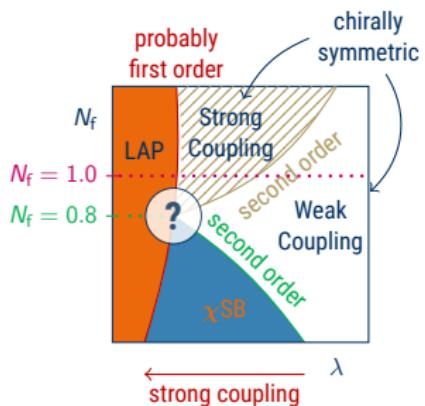
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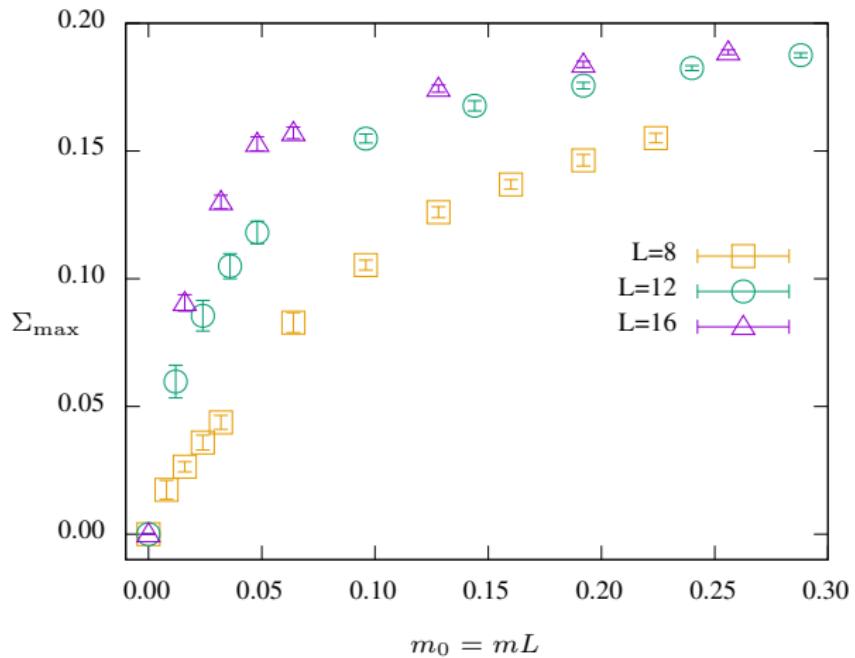
Outlook:

- Detailed FSS analysis
- Nature of the new phase
- Order parameter (?)
- Critical exponents

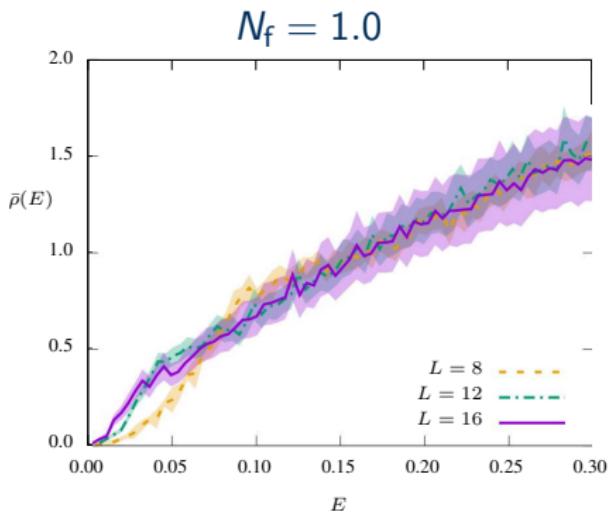
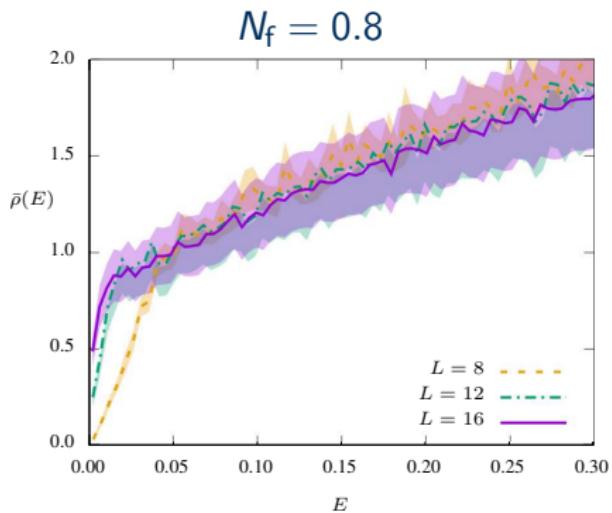


Appendix

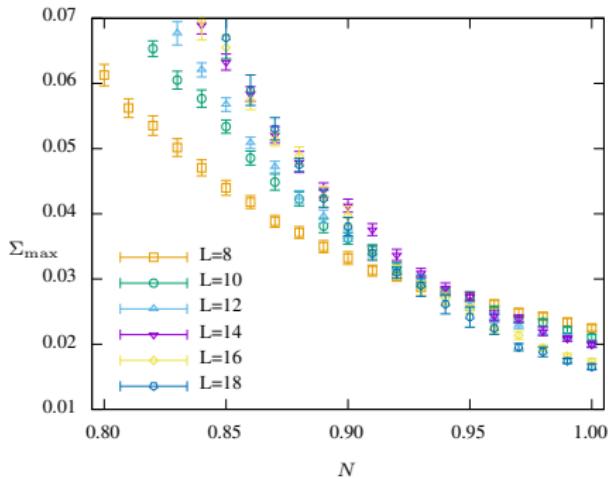
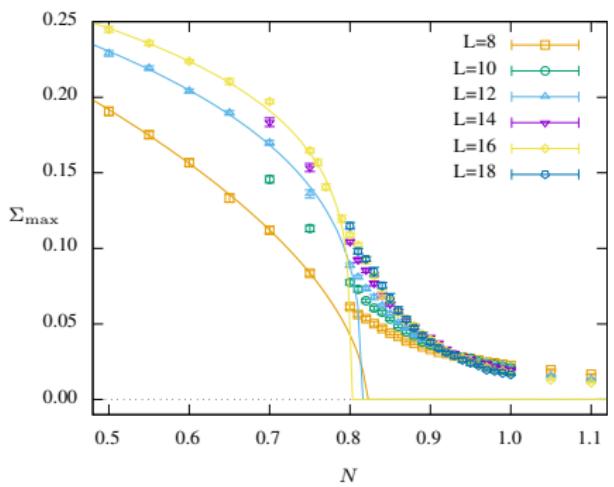
Mass Dependence



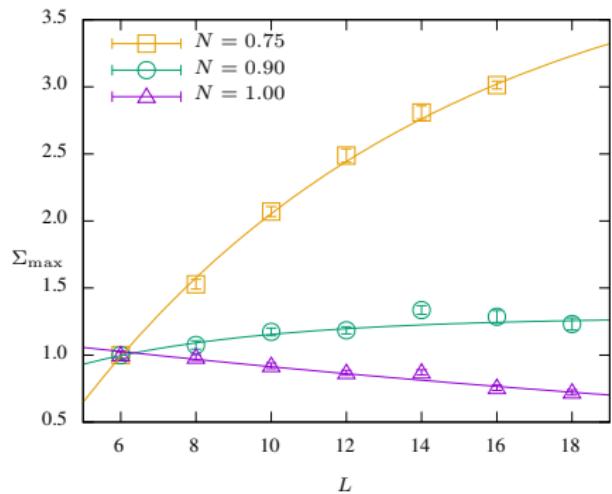
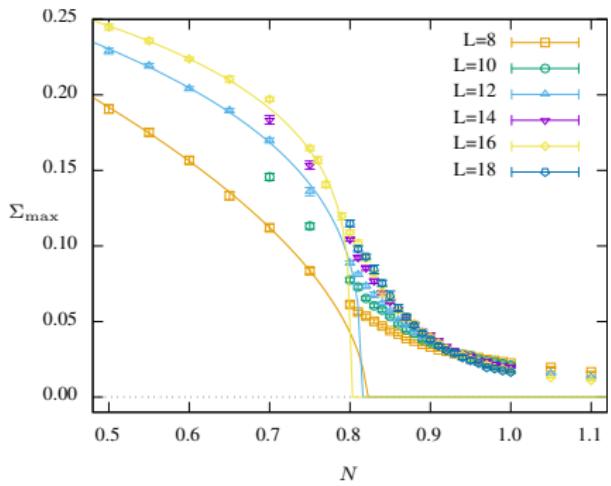
Spectral Density



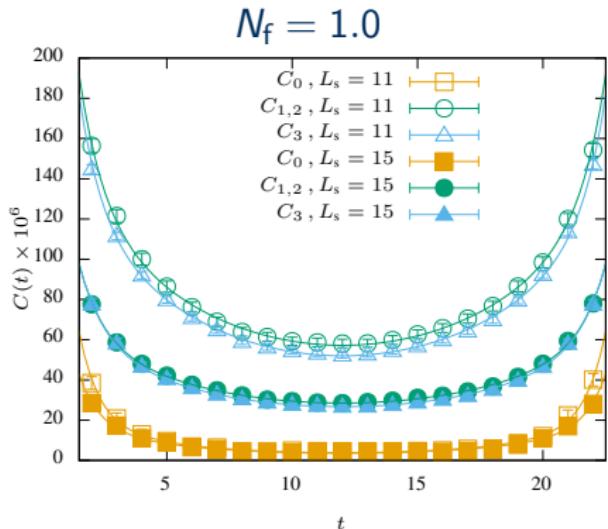
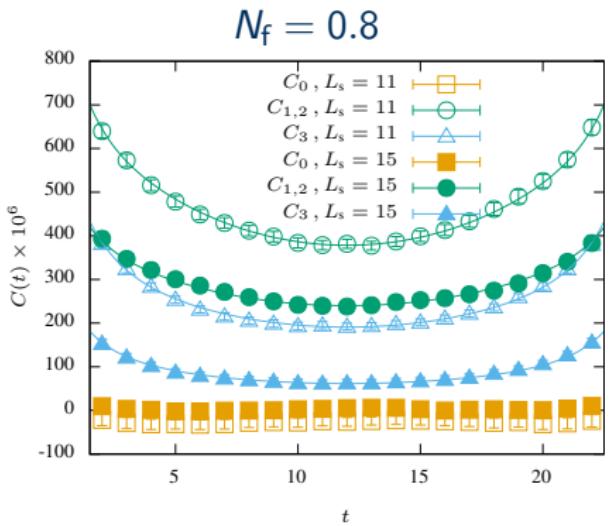
Maximal Condensate



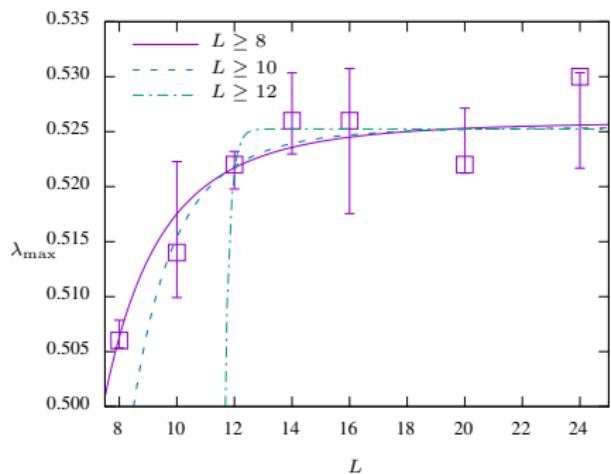
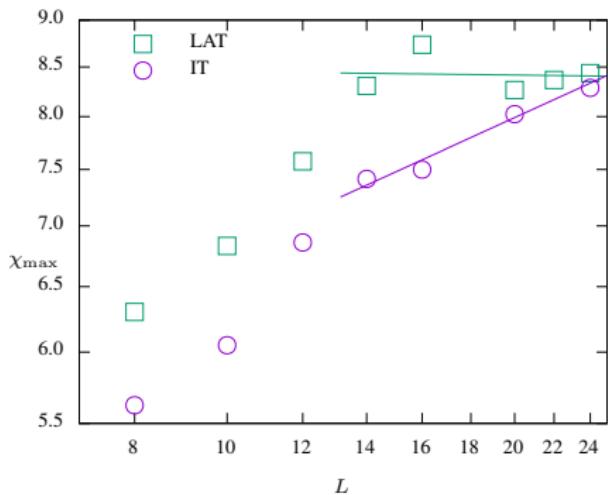
Maximal Condensate



Spectrum



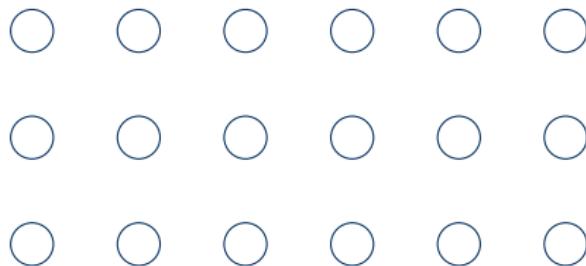
Susceptibility Scaling



Dual Formulation

The (Euclidean) Partition Function

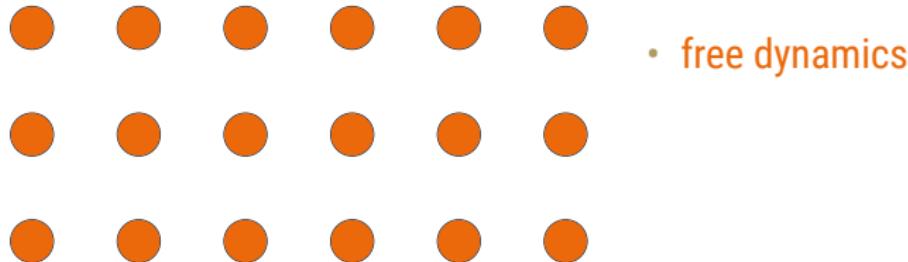
$$Z = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-\int \mathcal{L}} = C \sum_{\{k\}} w_\lambda(k) \det D[\{k\}] = \int d\Sigma e^{-U(\Sigma)}$$



Dual Formulation

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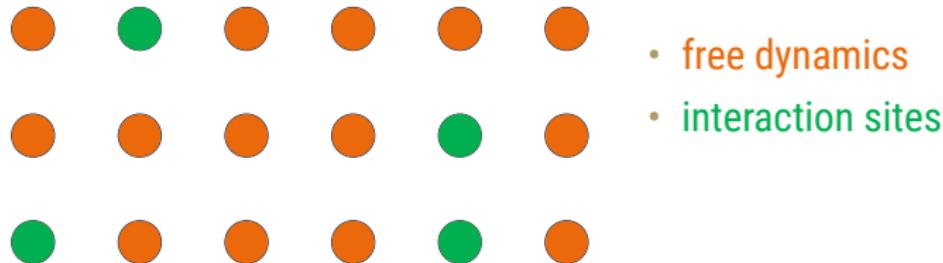
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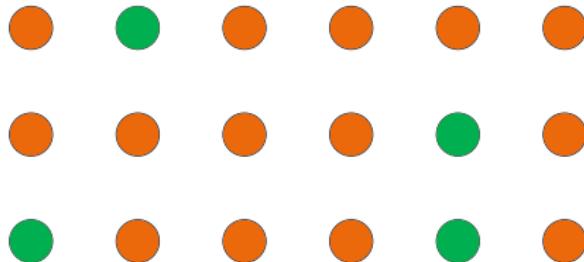
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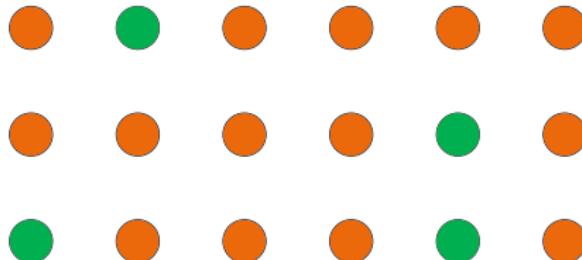


- free dynamics
- interaction sites
- Pauli: exclude interaction sites

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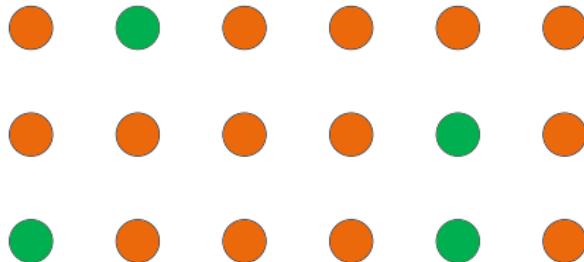


- free dynamics
- interaction sites
- Pauli: exclude interaction sites
- one coupling per interaction

Dual Formulation

The (Euclidean) Partition Function

$$Z = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-\int \mathcal{L}} = C \sum_{\{k\}} w_\lambda(k) \det D[\{k\}] = \int d\Sigma e^{-U(\Sigma)}$$

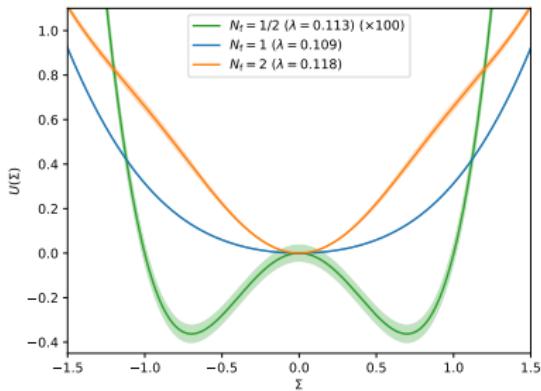


- free dynamics
- interaction sites
- Pauli: exclude interaction sites
- one coupling per interaction

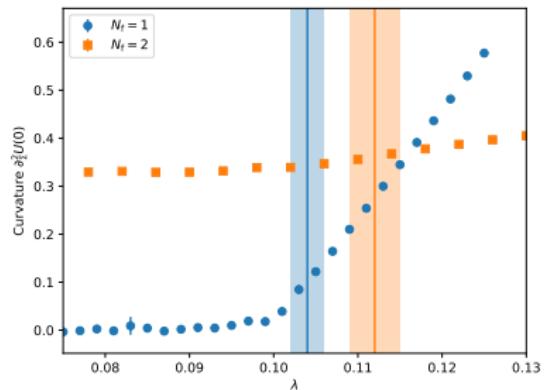
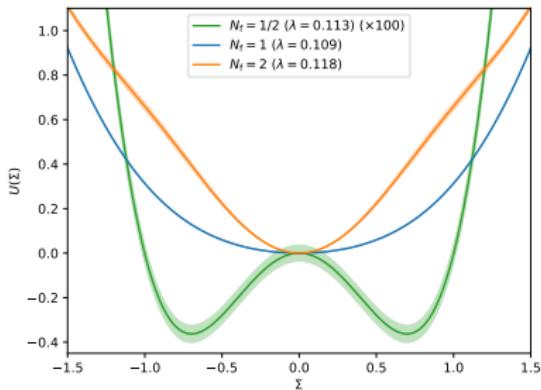
The effective potential $U(\Sigma)$...

... can be calculated from expectation values measurable on the lattice.

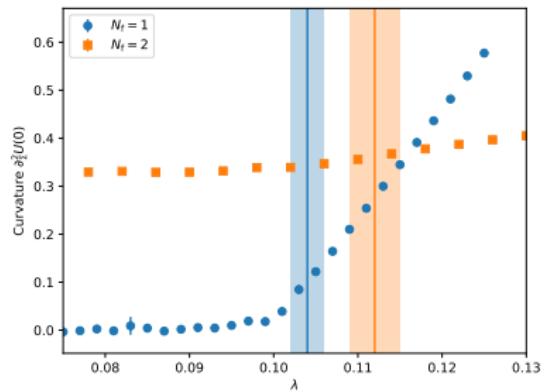
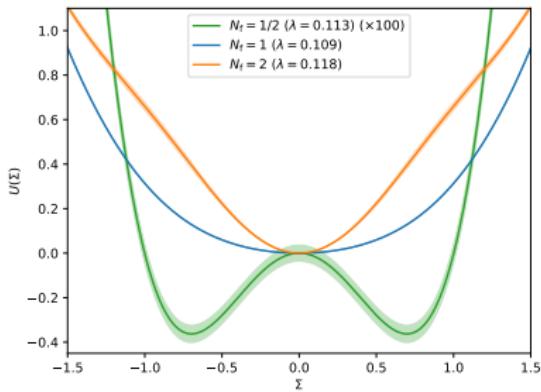
The Effective Potential (16×15^2)



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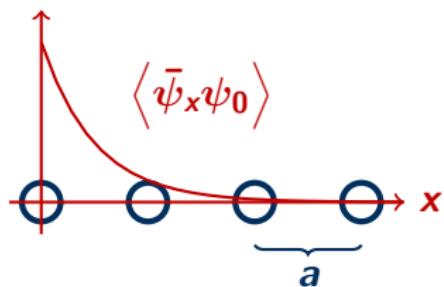
Conclusion:

No chiral symmetry breaking in red. 1+2D Thirring models (with $N_f \in \mathbb{N}$).

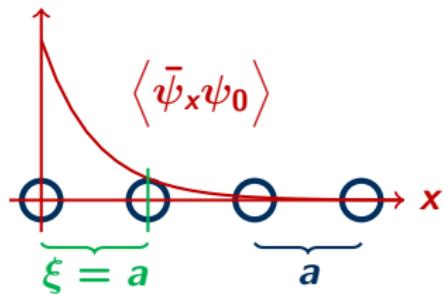
Continuum Limit



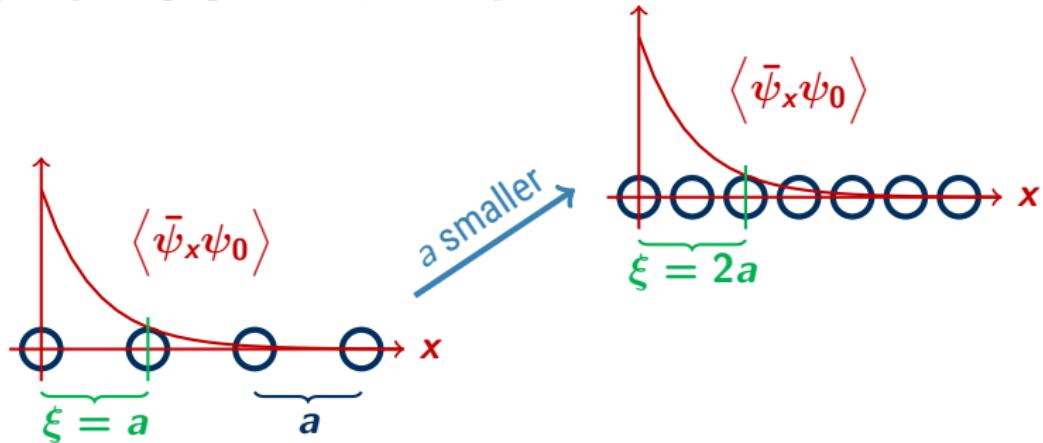
$\overline{\text{Continuum Limit}}$



$\overline{\text{Continuum Limit}}$



Continuum Limit



Continuum Limit

