Transmission Amplitude through a Coulomb-blockaded Majorana Wire

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Outline

- Majorana zero modes
- Aharonov-Bohm interferometer setup
- Scattering matrix description of Majorana wires
- Transmission amplitude

Majorana fermions

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

- Are there real solutions of the Dirac equation?
- Yes, if the particle is its own anti-particle

$$\gamma = \gamma^\dagger$$

- Neutral fermion
- In this talk: emergent quasi-particles, zero energy bound states

Majorana zero modes

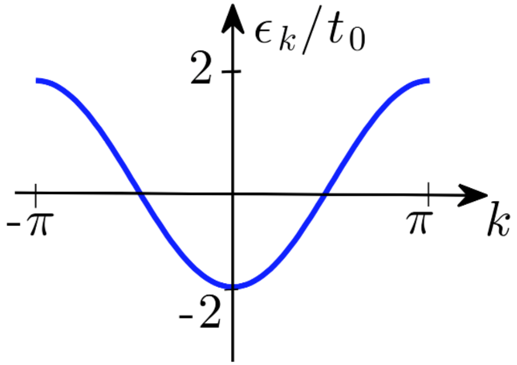
- Usual fermions $[H\,,\,c^\dagger] = Ec^\dagger,\;\; [H\,,\,c] = -Ec$
- Majorana fermions $[H\,,\,\gamma^\dagger] = [H\,,\,\gamma] \Rightarrow E = 0$

- Charge neutral fermion: equal superposition of particle and hole
- Quasi-particles in superconductors $\gamma_E^\dagger = \gamma_{-E} \Rightarrow E = 0$
- S-wave superconductor $\gamma^\dagger = uc_\uparrow^\dagger + vc_\downarrow \neq \gamma$
- Look for spin-less superconductor: p-wave pairing symmetry

$$\boxed{\gamma = uc^\dagger + vc \Rightarrow \gamma^\dagger = \gamma \quad \text{for} \quad v = u^*}$$

Kitaev model

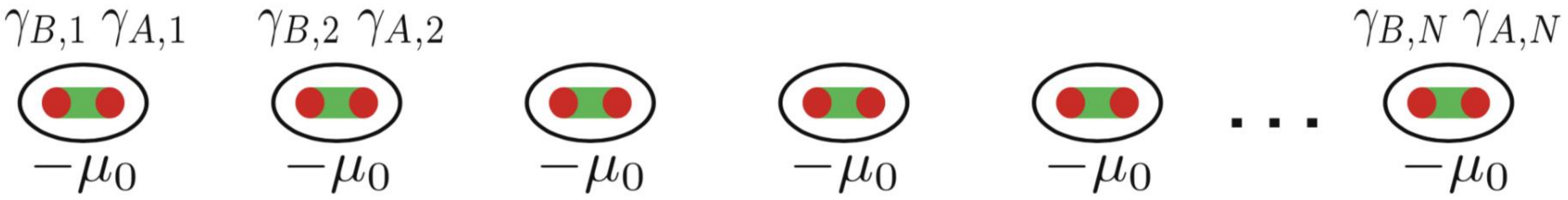
$$H_K = -\sum_{i=1}^{N}\left(t_0 c_i^\dagger c_{i+1} + \Delta_0 c_i^\dagger c_{i+1}^\dagger + \text{h.c.}\right) - \mu_0 \sum_{i=1}^{N} c_i^\dagger c_i$$



- Introduce Majorana fermions $\quad \gamma_{\mathrm{A},j} = c_j^\dagger + c_j \;,\;\; \gamma_{\mathrm{B},j} = (c_j^\dagger - c_j)/i$
- Fermion number $\quad i\gamma_{\mathrm{A},j}\gamma_{\mathrm{B},j} = 1 - 2c_j^\dagger c_j$
- Chemical potential term $\quad -\mu_0 \sum_{j=1}^{N} c_j^\dagger c_j = \frac{1}{2}\mu_0 \sum_{j=1}^{N} i\gamma_{\mathrm{A},j}\gamma_{\mathrm{B},j} - \frac{\mu_0 N}{2}$

Kitaev model – trivial phase

- Consider limit $t_0 = 0 = \Delta_0$



Majorana operators at the same lattice site are coupled by the Hamiltonian

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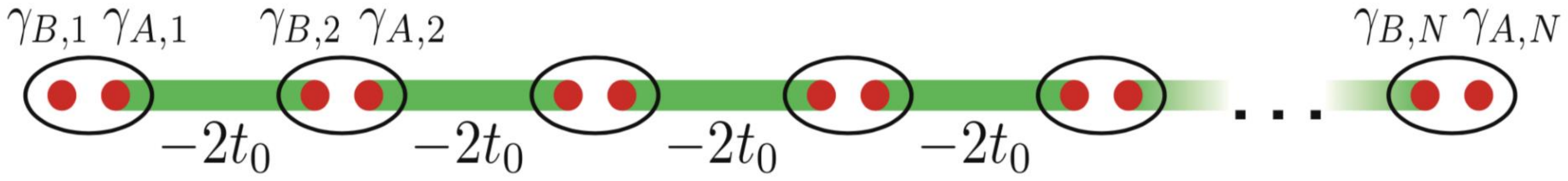
Kitaev model – topological phase

- Consider limit $t_0 = \Delta_0,\ \mu_0 = 0$



- Majorana operators at neighboring lattice sites are coupled by Hamiltonian
- End Majoranas $\gamma_{\mathrm{B},1}$ and $\gamma_{\mathrm{A},N}$ are uncoupled: zero energy quasi-particles

Ground state degeneracy described by operator $i\gamma_{\mathrm{B},1}\gamma_{\mathrm{A},N}$

- Introduce Majorana fermions $\gamma_{\mathrm{A},j} = c_j^\dagger + c_j\ ,\ \ \gamma_{\mathrm{B},j} = (c_j^\dagger - c_j)/i$
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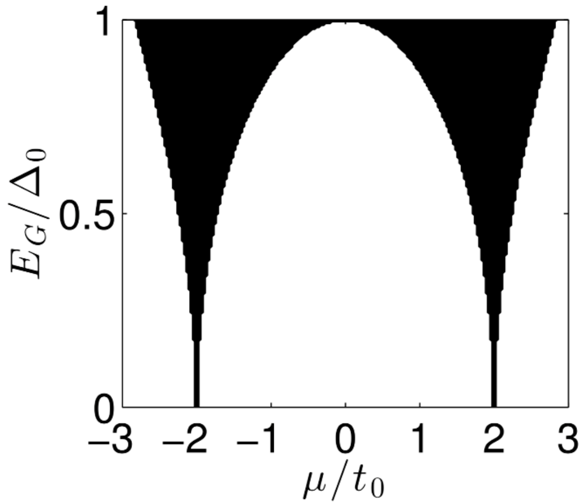
Kitaev model – topological phase transition

$$\mathcal{H}_K = \begin{pmatrix} \varepsilon_k & 0 & 0 & \Delta_0 \sin(k) \\ 0 & \varepsilon_{-k} & -\Delta_0 \sin(k) & 0 \\ 0 & -\Delta_0^* \sin(k) & -\varepsilon_k & 0 \\ \Delta_0^* \sin(k) & 0 & 0 & -\varepsilon_{-k} \end{pmatrix}$$ in basis $\left(c_k^\dagger, c_{-k}^\dagger, c_k, c_{-k}\right)$

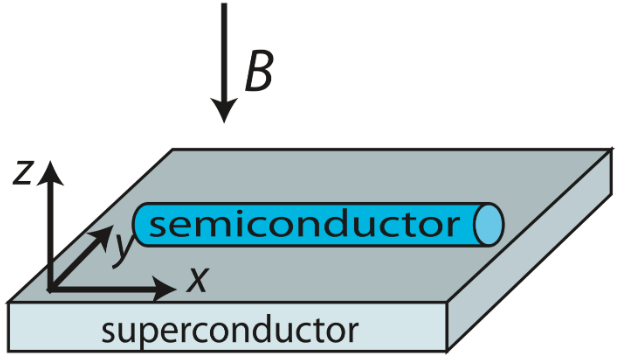
- Quasi-particle energies $E_k = \sqrt{(-2t_0 \cos(k) - \mu_0)^2 + |\Delta_0|^2 \sin^2(k)}$
- Pairing term vanishes for $k = 0, k = \pi \Rightarrow$ energy gap closes for $\mu_0 = \pm 2t_0$

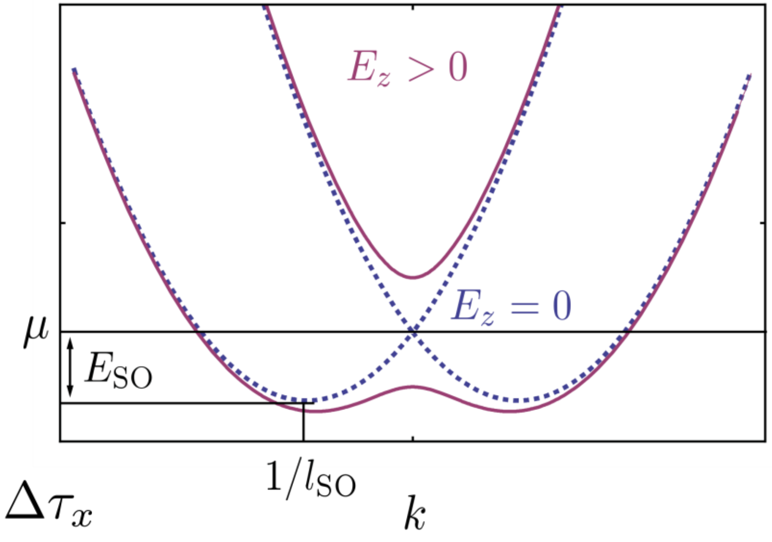


- Topological phase transition at $\mu_0 = \pm 2t_0$
- We know the system is topological for $\mu_0 = 0$

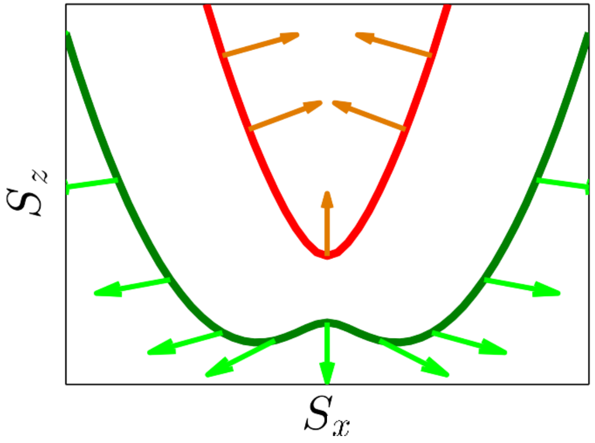
$\boxed{\text{Topological phase for } |\mu| < 2t_0}$

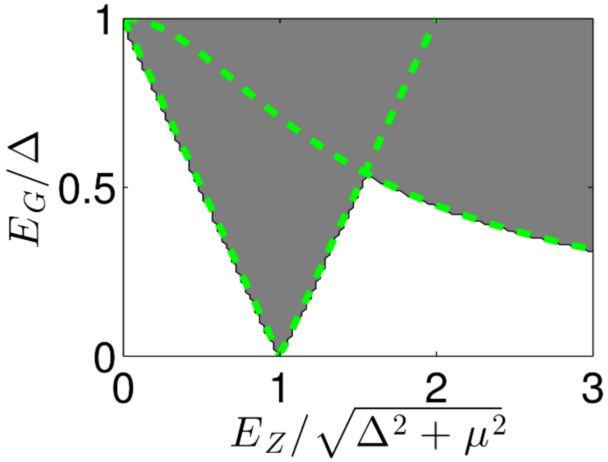
More realistic model – proximitized Rashba wire





$$\mathcal{H} = \left[-\frac{\hbar^2}{2m}\partial_y^2 - \mu - i\alpha_{\mathrm{R}}\sigma_x\partial_y \right]\tau_z - E_z\sigma_z + \Delta\tau_x$$



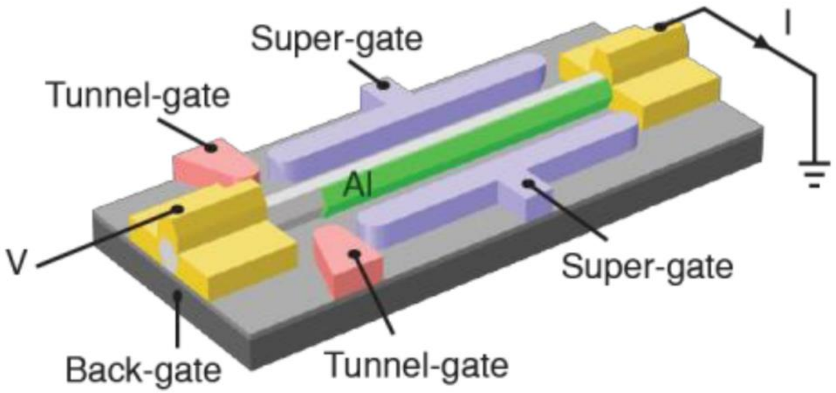


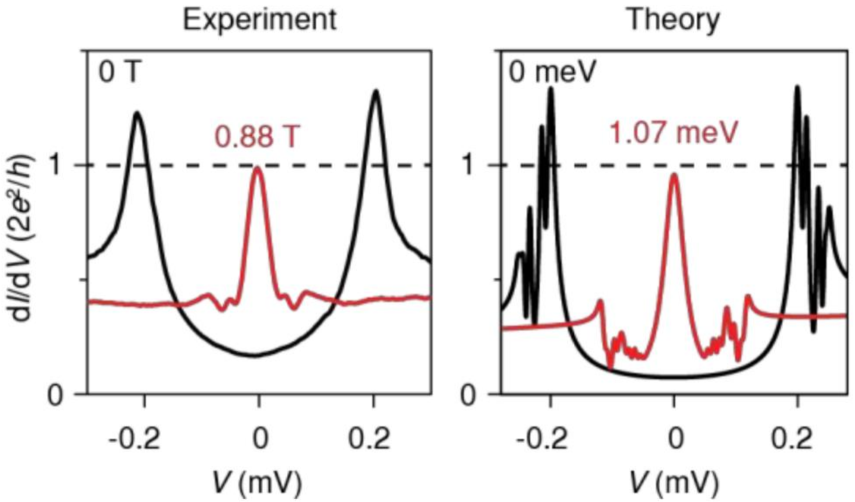
R. M. Lutchyn, J. D. Sau, and S. Das Sarma, Phys. Rev. Lett. 105, 077001 (2010);

Y. Oreg, G. Refael, and F. von Oppen, Phys. Rev. Lett. 105, 177002 (2010)

Zero bias conductance peak

- Tunneling conductance as function of gate voltage
- Zero-bias conductance peak at finite Zeeman field as Majorana signature

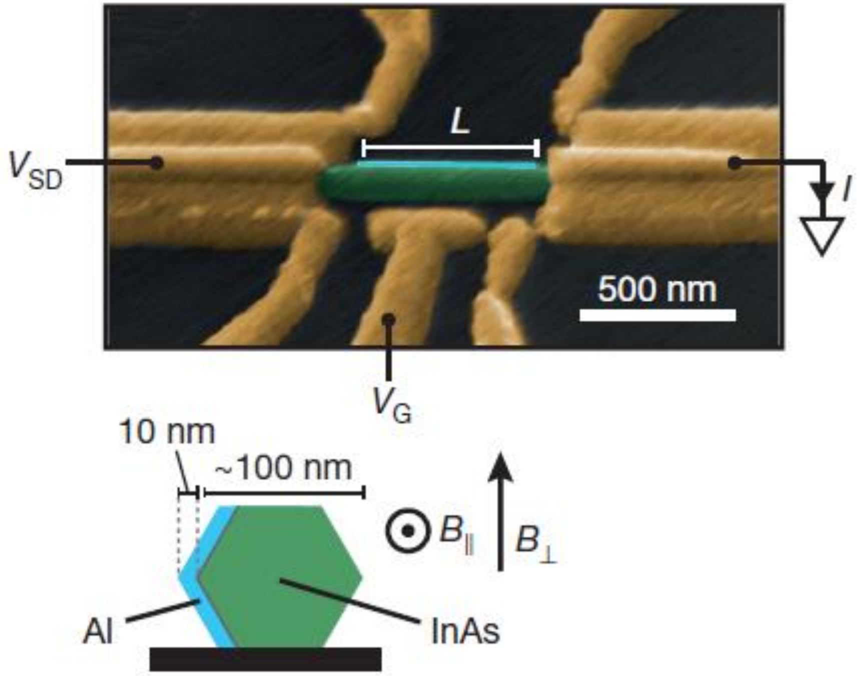




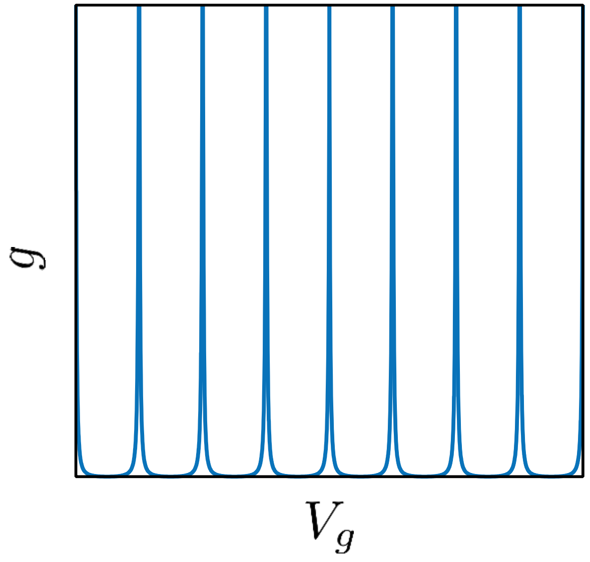
Zhang et al., Nature 556, 206 (2018)

Tunneling in Coulomb blockade

- Semiconductor-superconductor heterostructure
- Coulomb blockade
- Conductance measurement as function of the gate voltage



Albrecht et al., Nature 531, 206 (2016)



conductance resonances

Theoretical idea: Liang Fu, PRL 104, 056402 (2010)

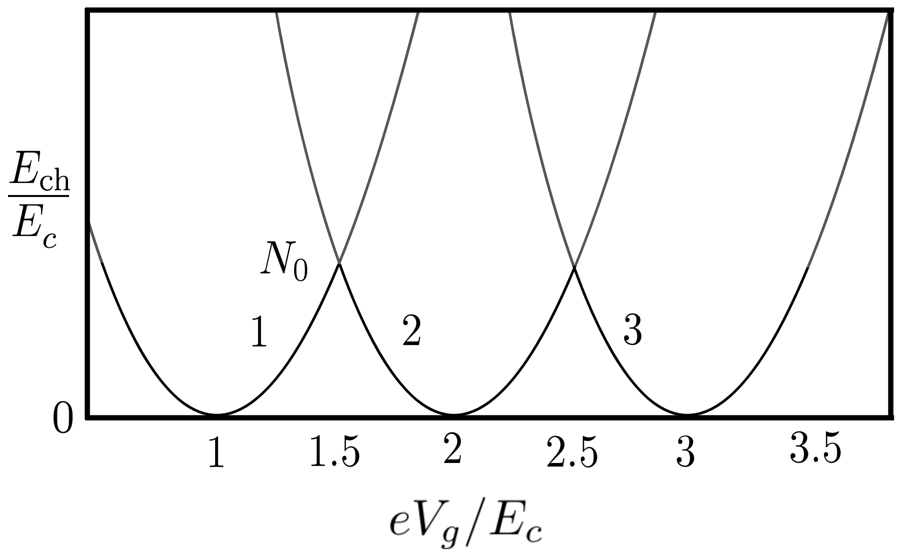
Coulomb blockade

- Spatial confinement of electrons enhances Coulomb repulsion

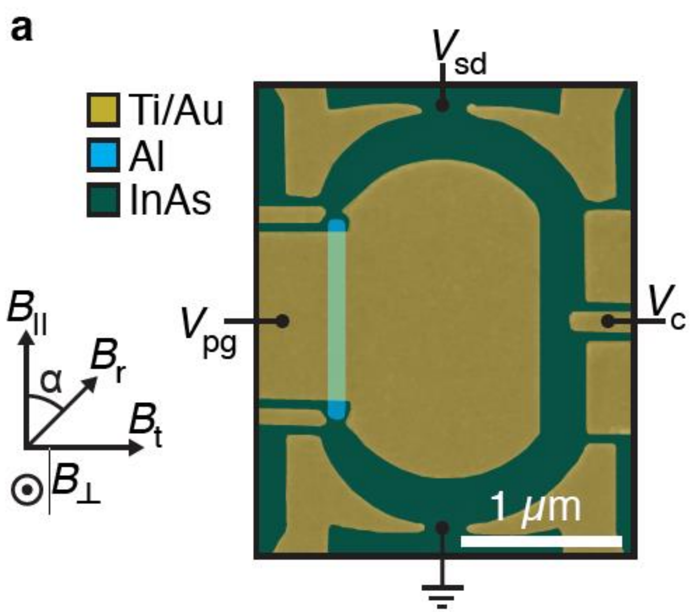
- Charging energy $E_{\mathrm{ch}} = \frac{E_c}{2}\left[N_0 - \frac{eV_g}{E_c}\right]^2$

- Conduction resonance at $E_{\mathrm{ch}}(N_0) = E_{\mathrm{ch}}(N_0 + 1)$

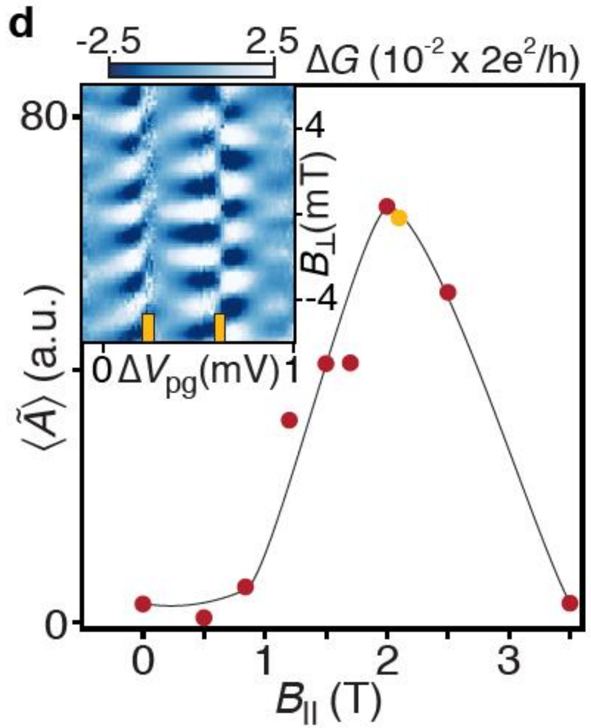
- Consider tunneling in between resonances



Coherent transport through Coulomb-blockaded Majorana wire



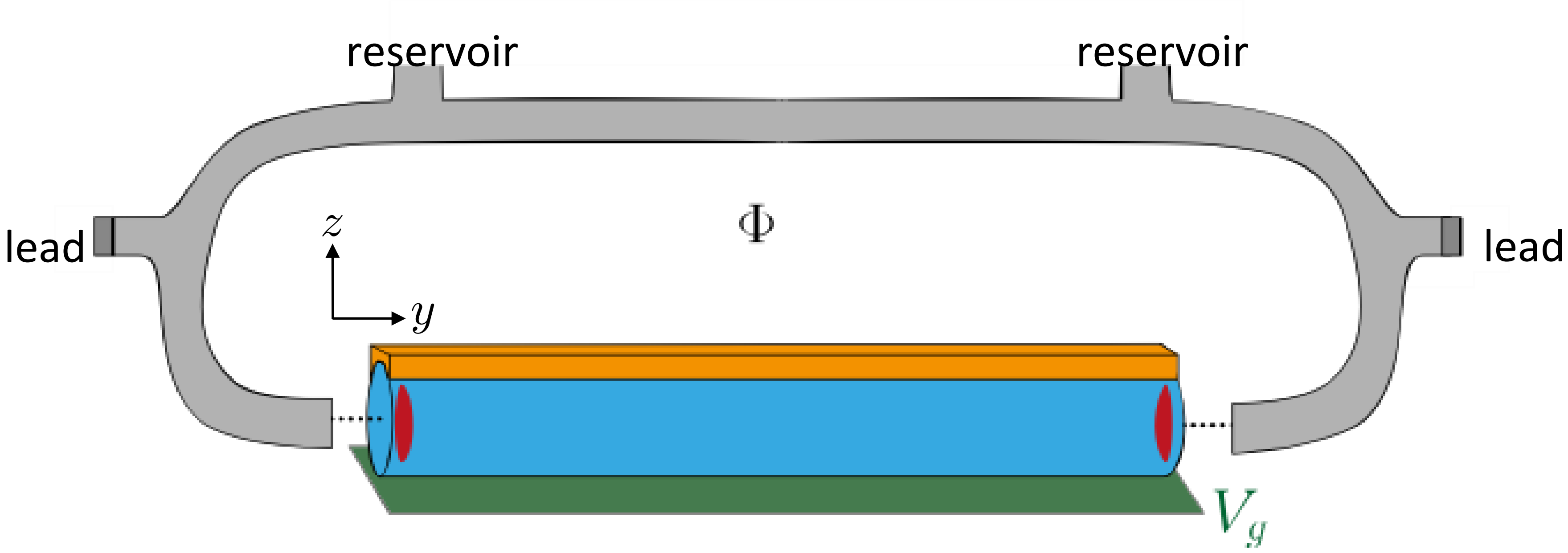
- Interferometer setup



- Transmission amplitude peak

Whiticar et al., preprint arXiv:1902.07085 (2019)

Aharonov-Bohm interferometer



- Rashba wire with s-wave superconductor in proximity
- Aharonov-Bohm interferometer to measure transmission amplitude and phase
- Transmission amplitude $|T_{\uparrow\uparrow} + T_{\downarrow\downarrow}|$ can be obtained from current oscillations $I(\Phi)$

Scattering matrix

- Scattering matrix formalism

$$S = 1 - 2\pi i W \frac{1}{\varepsilon - H_{\text{eff}} + i\pi W^\dagger W} W^\dagger,$$

- In the Coulomb-blockade regime, Weidenmüller formula can be used with BdG wave functions

- Bogolubov - de Gennes Hamiltonian

$$\mathcal{H} = \begin{pmatrix} H_w & \Delta\sigma_0 \\ \Delta^*\sigma_0 & -H_w \end{pmatrix}$$

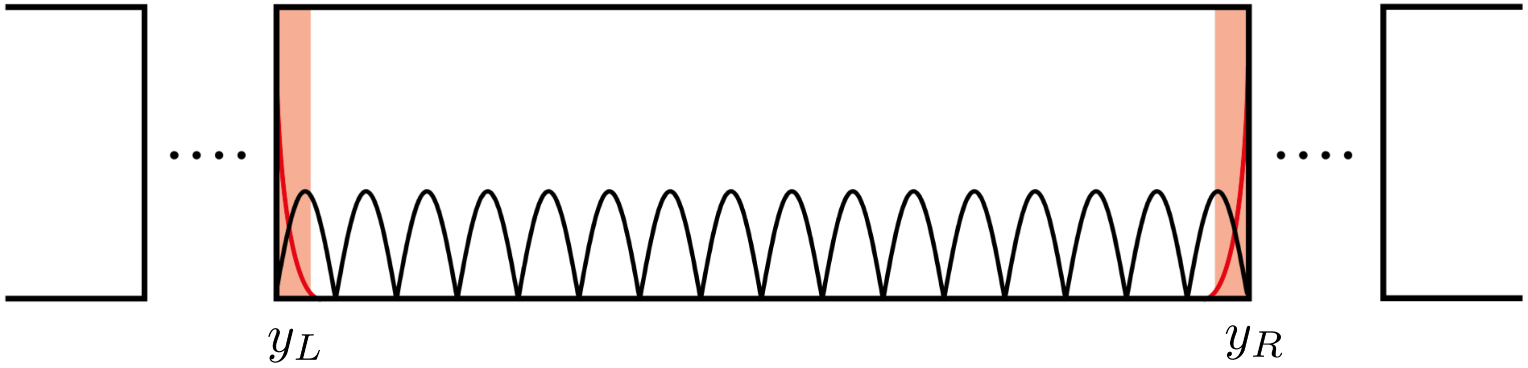
- Electron wave functions $u_{n\sigma}$ and hole wave functions $v_{n\sigma}$ satisfy the BdG equation

$$\mathcal{H}\left(u_{n\uparrow},\ u_{n\downarrow},\ v_{n\downarrow},\ -v_{n\uparrow}\right)^T = \mathcal{E}_n\left(u_{n\uparrow},\ u_{n\downarrow},\ v_{n\downarrow},\ -v_{n\uparrow}\right)^T$$

Transmission through the wire – normal case

- Scattering matrix from Weidenmüller formula $S = 1 - 2\pi i W \dfrac{1}{\varepsilon - H_{\text{eff}} + i\pi W^\dagger W} W^\dagger = \begin{pmatrix} R_{\text{RR}} & T_{\text{LR}} \\ T_{\text{RL}} & R_{\text{LL}} \end{pmatrix}$
- Approximation for single resonant level ε_n

$$T \propto \frac{\varphi_n(y_L)\,\varphi_n^*(y_R)}{E_c}$$



— φ_n eigenfunction of resonant level

— Ψ_α lead wave function

Transmission through the wire - Majorana case

- Majorana wave functions χ_α exponentially localized at wire ends
- Coulomb blockade expels additional charges from the island
- BdG wave functions $\chi_L \pm \chi_R \;\; \Rightarrow T \propto \dfrac{\chi_L(y_L)\,\chi_R^*(y_R)}{E_c}$

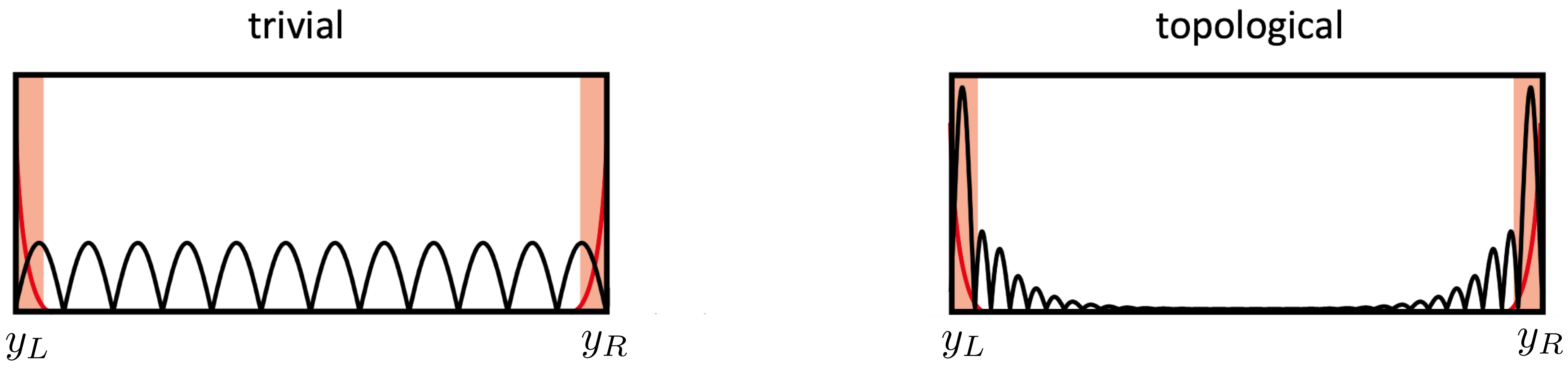
— u_n BdG wave function of the MZMs

— Ψ_α lead wave function

Transmission amplitude

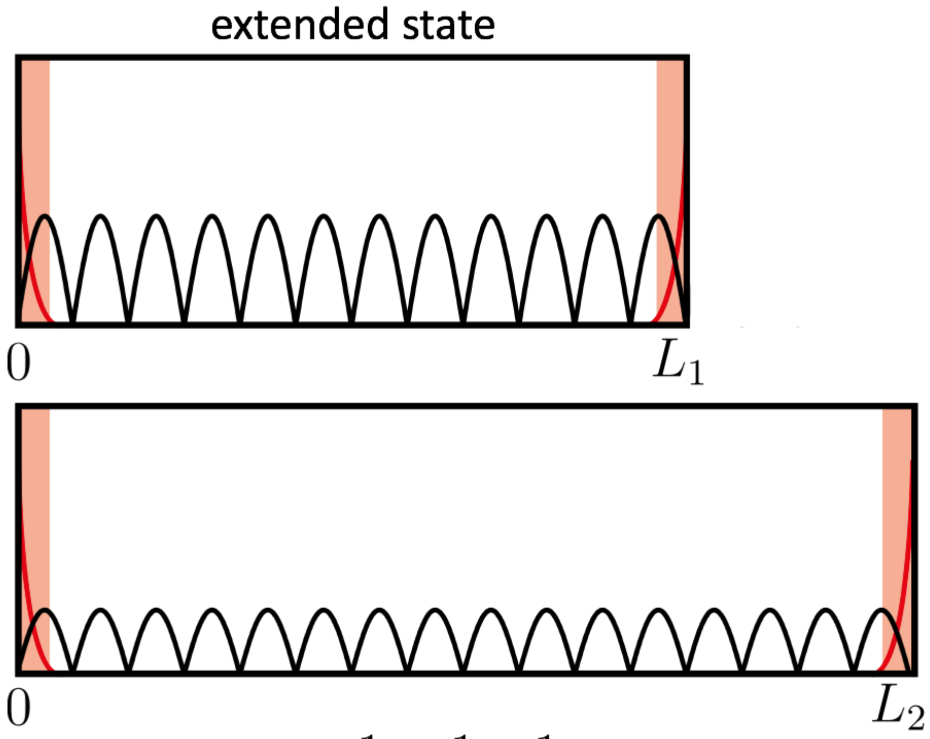
- Vary Zeeman field and measure transmission amplitude
- Large transmission for large wave function weight at wire ends
- Maximum in amplitude when entering topological regime with localize Majorana wave function

$$\chi_L \sim \frac{1}{\sqrt{\xi}} e^{-x/\xi}$$ depends on $\xi = v_F/\Delta_{\text{ind}}$ and hence Δ_{ind}

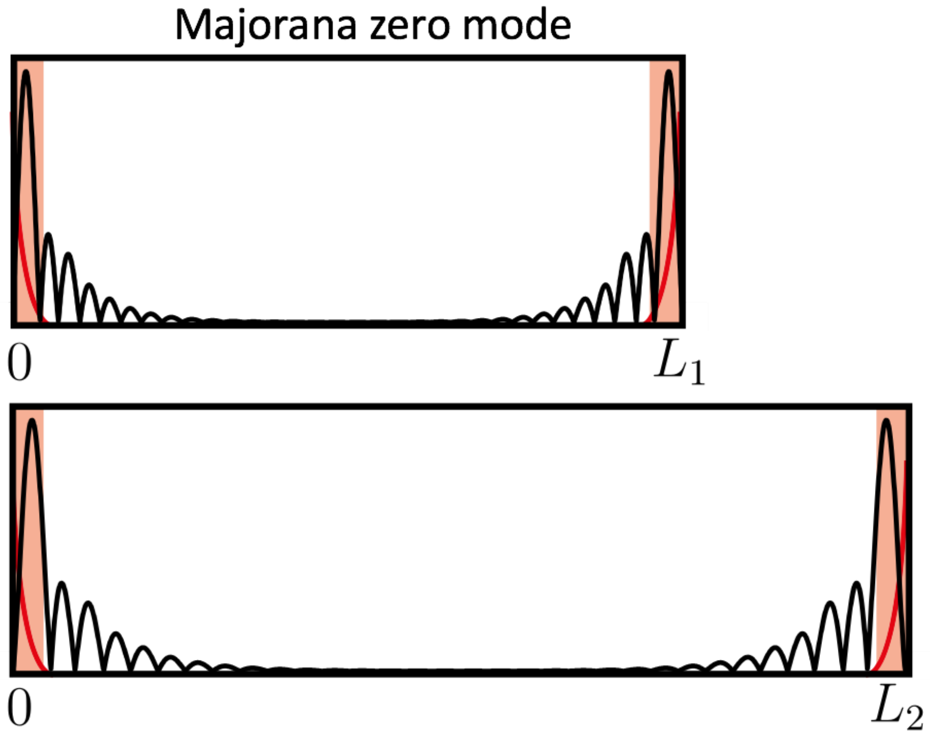


Transmission amplitude

- Charging energy $E_c \propto L^{-1}$
- Distinguish Majorana from extended states by wire length dependence of transmission amplitude



$$T \propto \frac{1}{\sqrt{L}} \frac{1}{\sqrt{L}} \frac{1}{E_c} \propto 1$$

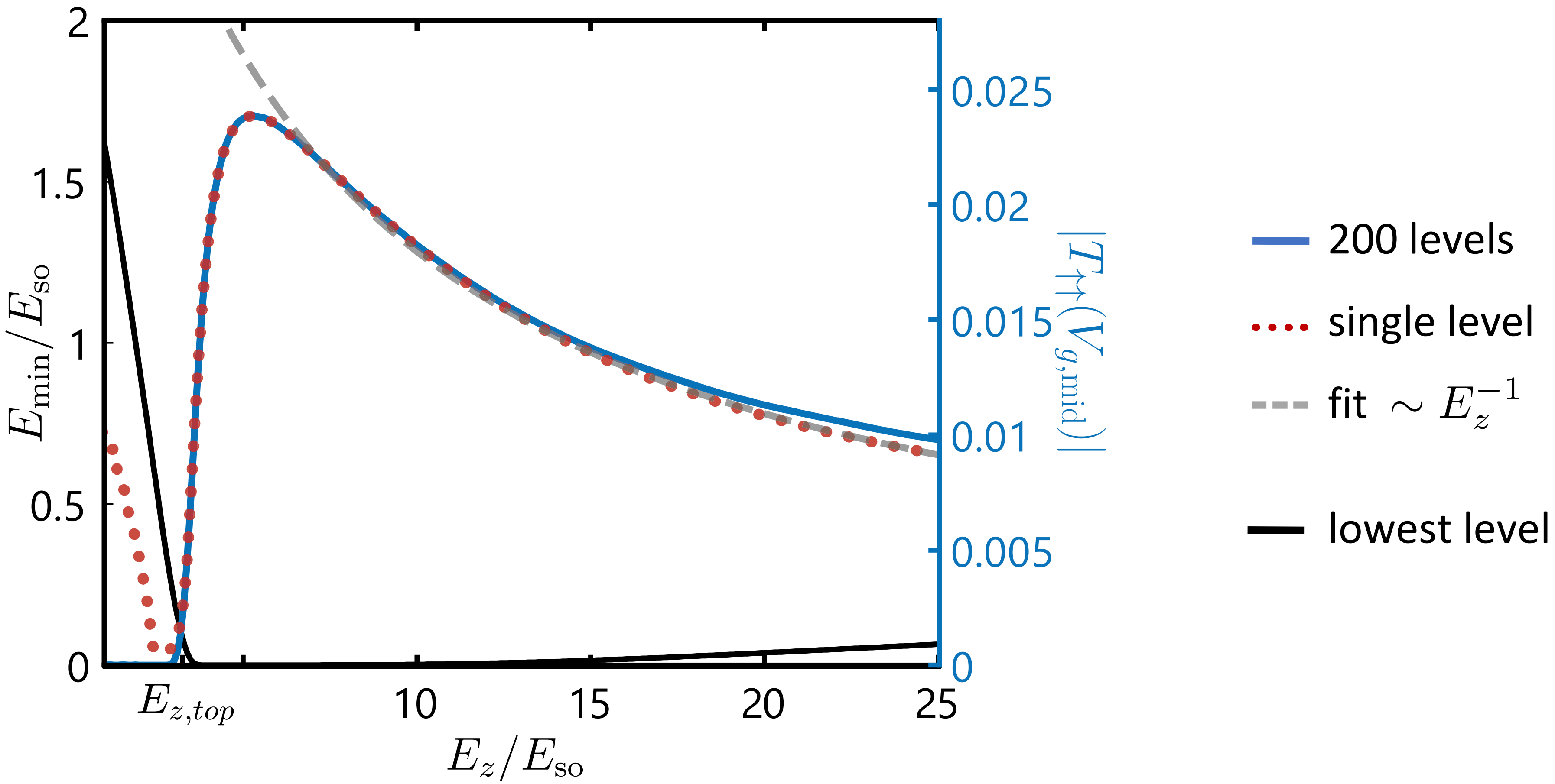


$$T \propto \frac{1}{\sqrt{\xi}} \frac{1}{\sqrt{\xi}} \frac{1}{E_c} \propto L$$

Transmission amplitude

Transmission amplitude – constant order parameter

- Transmission amplitude $|T_{\uparrow\uparrow}(V_{g,\mathrm{mid}})|$



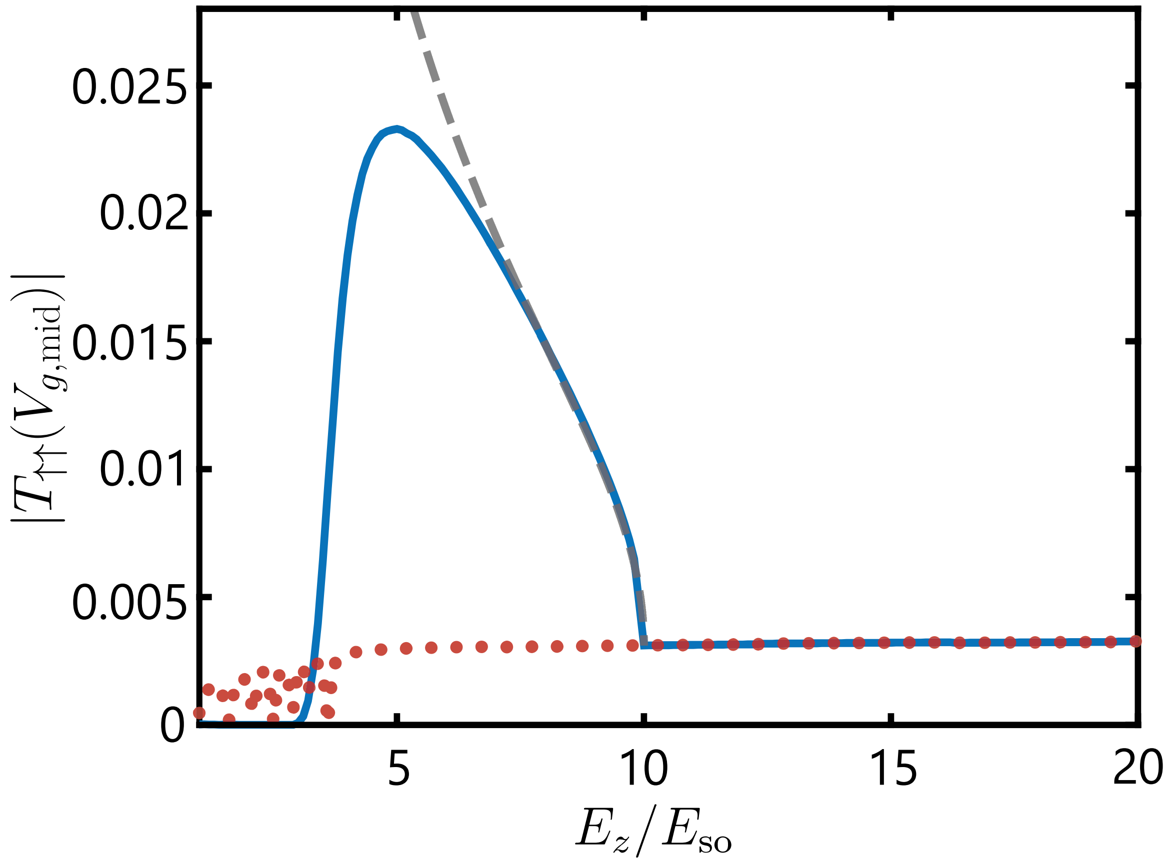
- Maximum in amplitude when entering topological regime
- Maximum exclusively determined by MZMs, robust against considering more levels

Transmission amplitude – self-consistent order parameter

- Order parameter

$$\Delta(E_z) = \Delta(0)\left[1-\left(\frac{E_z}{E_{z,c}}\right)^2\right]^{1/2}$$

$$\Delta(4.5\,E_{\mathrm{so}}) = 2\,E_{\mathrm{so}}, \quad E_{z,c} = 10E_{\mathrm{so}}$$



200 levels

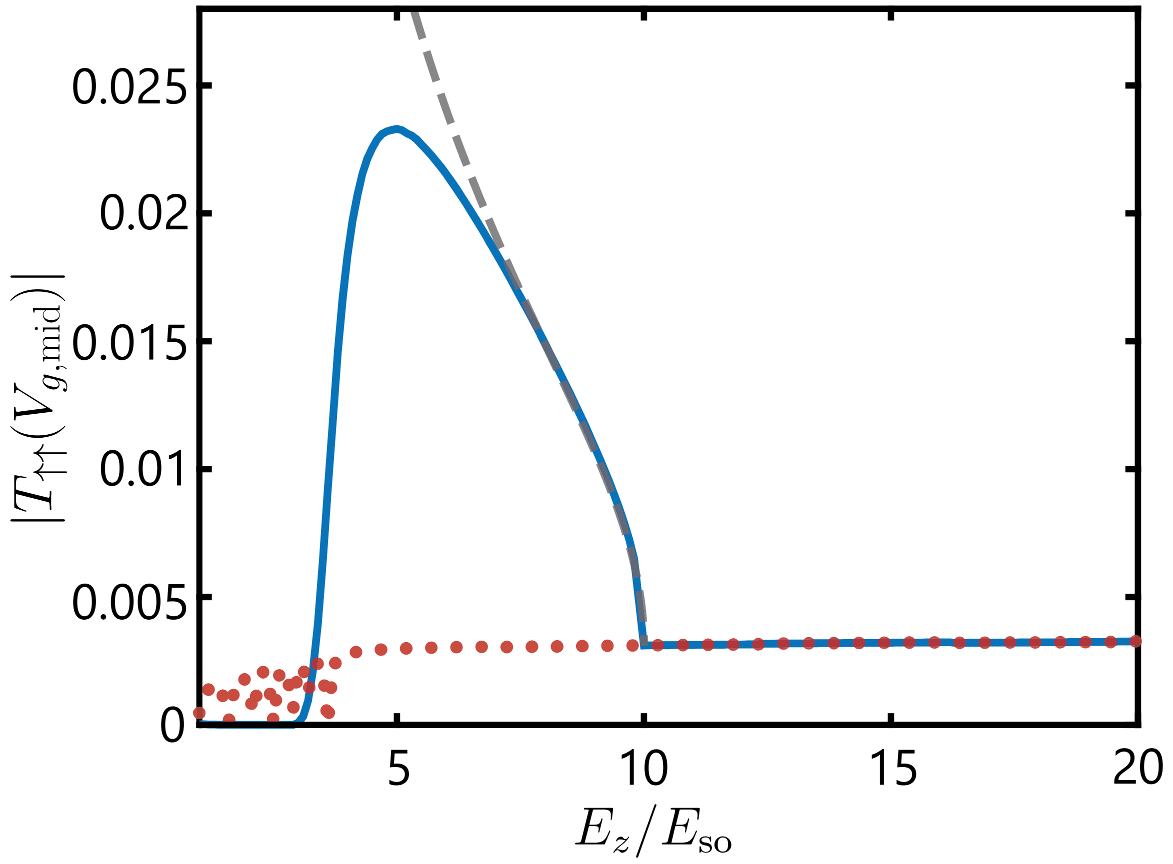
normal conducting

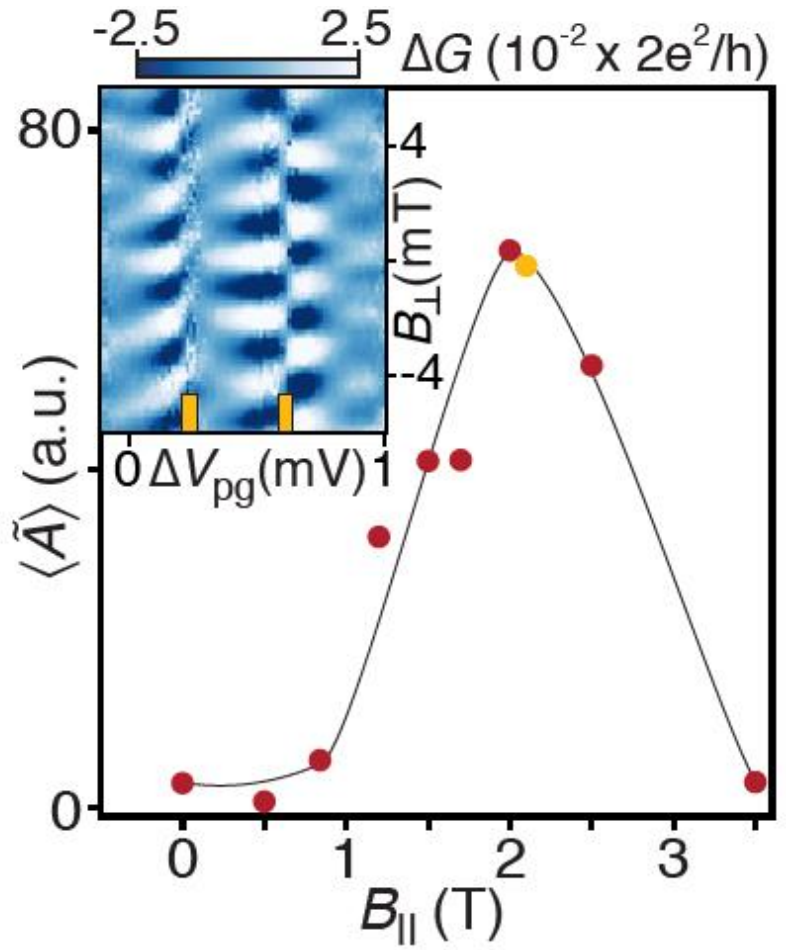
fit $\overline{A}(E_z) \sim \Delta(E_z)/E_z$

- Amplitude decays when MZMs are destroyed

Transmission amplitude – self-consistent order parameter

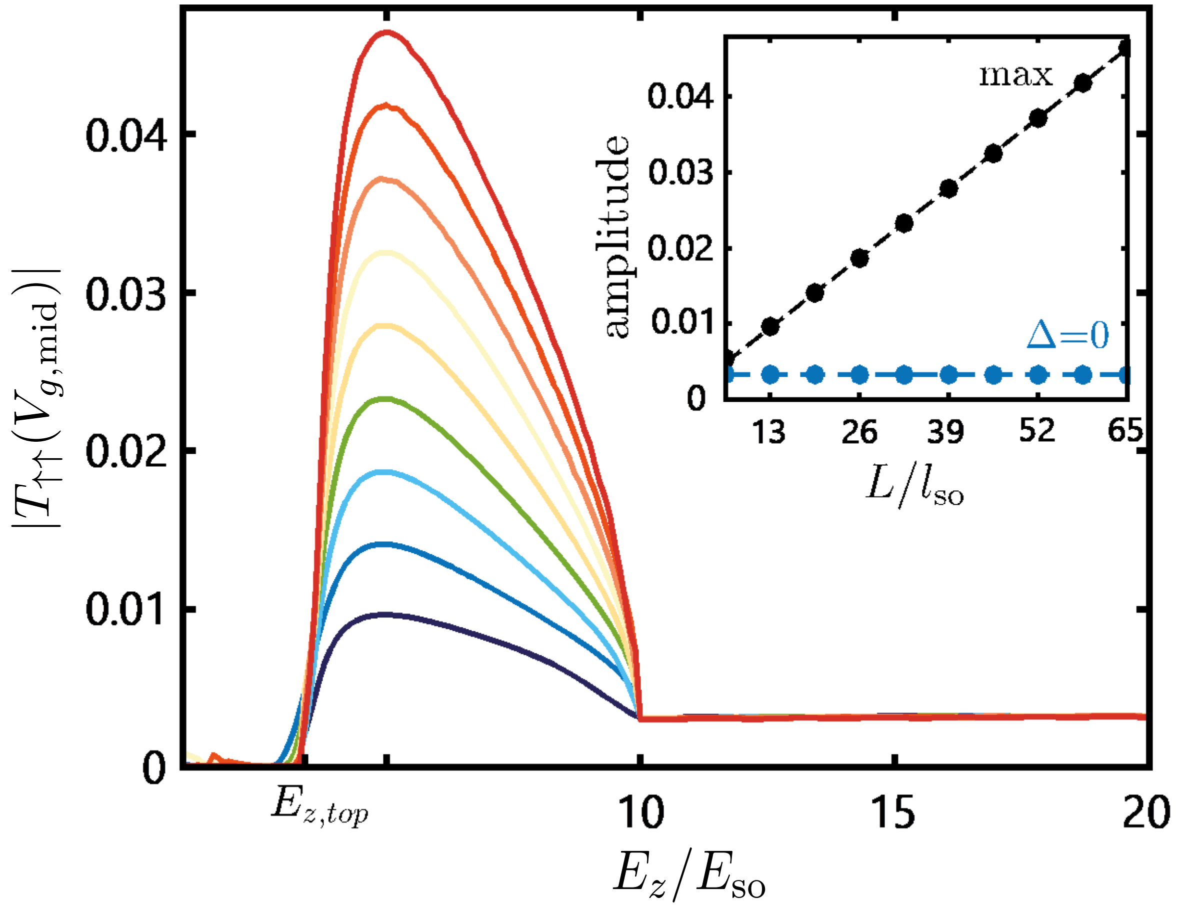
- Agrees with recent experiment: Whiticar et al., arXiv:1902.07085 (2019)





Transmission amplitude – wire length dependence

- Wires of various lengths $L = 13\,l_{\rm so}, 19.5\,l_{\rm so}, 26\,l_{\rm so}, 32.5\,l_{\rm so}, 39\,l_{\rm so}, 45.5\,l_{\rm so}$, and $58.5\,l_{\rm so}$



- Amplitude maximum increases with L
- Amplitude in normal conducting region independent of L
- Other zero modes like Andreev bound states do not contribute for long wires

MT and Bernd Rosenow, in preparation

Mid-gap amplitude peak increasing with wire length due to non-locality of MZMs
$\Rightarrow$ provides clear evidence for MZMs

Conclusion

- Coherent transport through Coulomb blockaded wire hosting MZMs
- Can determine Majorana localization length via transmission amplitude measurement
- Transmission amplitude maximum, whose height is proportional to the wire length as unique signatures for the presence of Majorana zero modes

Thank you for your attention!