

Asymptotically safe QED

Non-trivial UV fixed points

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Outline

- ① QED Basics
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- ④ Beta Functions
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- ⑦ Summary and outlook

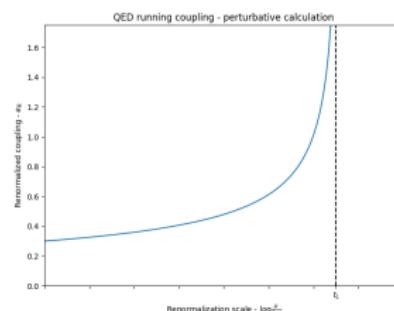
QED, its Success

$$\mathcal{L}_{QED} = \bar{\psi} (i\cancel{\partial} - e\gamma^\mu A_\mu - m) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (1)$$

- ▶ Quantization of classical theory of electromagnetism
- ▶ Perturbatively renormalizable QFT
- ▶ Most accurate QFT prediction in the world:
 - ▶ $a_{\text{experiment}} = 0.00115965218085(76)$ [Odo+06]
 $\Rightarrow \alpha_{\text{theory}}^{-1} = 137.035999710(96)$ [Gab+06]
 - ▶ $\alpha_{\text{experiment}}^{-1} = 137.03599878(91)$ [Cla+06]
- ▶ Comparatively simple theory

QED, its Issues

- ▶ Only a part of the SM
- ▶ Existence of a Landau Pole
[LAK54]
⇒ Perturbatively UV-incomplete
- ▶ Embedding into the SM does not remove Landau pole
⇐ Generic issue of gauge theories with U(1) factor



Asymptotically safe QED?

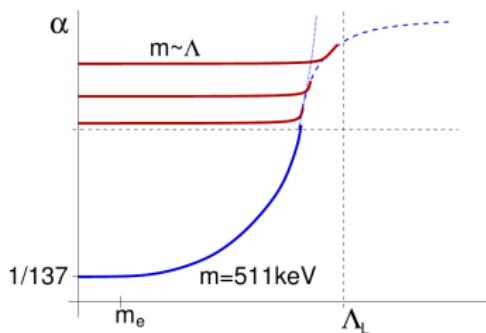
Open questions

- ▶ Can QED, being so simple and successful, be a fundamental theory?
- ▶ Is the Landau pole an artifact of perturbative calculations?

Chiral symmetry breaking prevents strong coupling completion

Assuming the presence of a UV fixed point beyond the Landau pole as well as no explicit chirality-breaking terms:

- ▶ Large values of e lead to dynamical chiral symmetry breaking
- ▶ Dynamical chiral symmetry breaking leads to heavy fermions for large values of e [Mir85]



- ▶ QED with large values of e cannot be connected to perturbative QED [Goc+98; GJ04]

What about explicit Chiral symmetry breaking?

Recent EFT study suggests a Pauli spin coupling term could screen the Landau pole [DGM18]

$$\kappa \bar{\psi} \sigma_{\mu\nu} F^{\mu\nu} \psi \quad (2)$$

Pauli Spin coupling term

- ▶ Chiral symmetry breaking
- ▶ U(1) invariant
- ▶ perturbatively non-renormalizable

Our adapted FRG approach

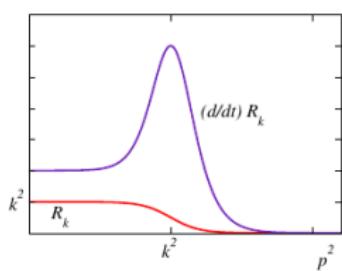
- ▶ Include relevant, marginal parameters as well as irrelevant Pauli term $\kappa \bar{\psi} \sigma_{\mu\nu} F^{\mu\nu} \psi$
⇐ unique $U(1)$ invariant dimension 5 operator
- ▶ Use the truncation (note the Euclidean signature)

$$\Gamma_k = \bar{\psi} (i\cancel{D} - i\bar{m} + i\bar{\kappa} \sigma_{\mu\nu} F^{\mu\nu} + \bar{e} \gamma_\mu A^\mu) \psi + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (3)$$

- ▶ Search for UV fixed points of dimensionless quantities e , κ and m
- ▶ Try to connect them to physical IR parameters

The exact (nonperturbative) Renormalization Group Flow

- ▶ Employ cutoff function R_k to suppress fluctuations with high momentum modes $p^2 \gg k^2$.
- ▶ Obtain the Wetterich equation for the *effective average action* Γ_k [RW94]



$$\partial_k \Gamma_k = \frac{1}{2} S \text{Tr} \left[(\partial_k R) \left(\Gamma_k^{(2)} + R_k \right)^{-1} \right] \quad (4)$$

- ▶ $\lim_{k \rightarrow 0} R_k = 0$ and $\Gamma = \Gamma_0$
- ▶ Expansion in operator dimensions result in (regulator-dependent) beta functions of the model parameters.

Modified QED Beta Functions

The beta functions of e, κ, m in the Landau gauge and with the Litim regulator [Lit01] read

$$\partial_t e = \frac{-1 + 7m^2 + 3m^4}{10(1+m^2)^3\pi^2} e\kappa^2 + \frac{5 + 9m^2}{10(1+m^2)^3\pi^2} e^2\kappa m + \frac{8 + 47m^2 + 56m^4 + 21m^6}{96(1+m^2)^4\pi^2} e^3 + \dots \quad (5)$$

$$\begin{aligned} \partial_t \kappa = \kappa &+ \frac{1 - m^2}{96(1+m^2)^3\pi^2} m e^3 + \frac{8 + 11m^2 + 5m^4 + 3m^6}{24(1+m^2)^4\pi^2} e^2 \kappa + \frac{-20 + 11m^2}{20(1+m^2)^3\pi^2} e\kappa^2 m \\ &+ \frac{(3 + m^2)(-3 + 7m^2)}{15(1+m^2)^3\pi^2} \kappa^3 + \dots \end{aligned} \quad (6)$$

$$\partial_t m = -m + \frac{2 + 4m^2}{5(1+m^2)^2\pi^2} \kappa^2 m + \frac{9(-2 + m^2)}{20(1+m^2)^2\pi^2} e\kappa - \frac{6 + m^2}{16(1+m^2)^2\pi^2} e^2 m + \dots \quad (7)$$

where ... represent NLO terms.

Modified QED Beta Functions

A sanity check

For $\kappa = 0$, we recover the perturbative result for the QED beta function [PS95]

$$\partial_t e = \frac{e^3}{12\pi^2} \quad (8)$$

in the limit $m \rightarrow 0$.

Reminder

- ▶ e, κ, m are dimensionless and renormalized

UV Fixed Points

Including the NLO terms, we find several sets of UV fixed points with different discrete symmetries

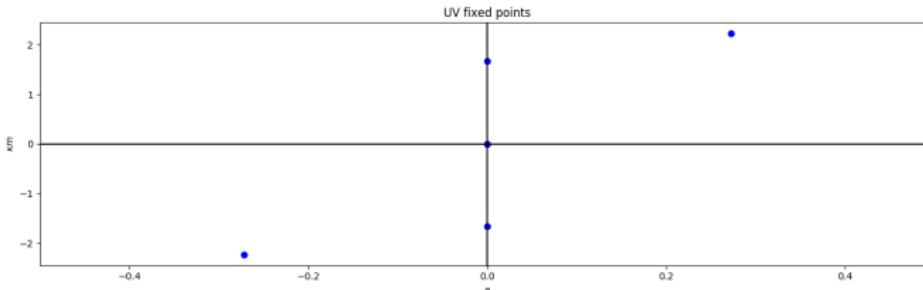
e	κ	m	symmetry group	n_{phys}	θ_{\max}
0	5.09	0.328	$\mathbb{Z}_2 \times \mathbb{Z}_2$	2	3.10
15.6	0	0	\mathbb{Z}_2	2	13.7
0	3.82	0	\mathbb{Z}_2	3	2.25
0	0	0	—	1	1.00

where n_{phys} and θ_{\max} denote the number of physical parameters and the largest critical exponent respectively.

UV Fixed Points (2)

The fixed points ($\pm 15.6, 0, 0$)

- ▶ Very large anomalous dimensions ($\eta_\psi \approx -4.18; \eta_A \approx 8.38$)
 - ▶ Not present in LO approximation
- ⇒ Unreliable



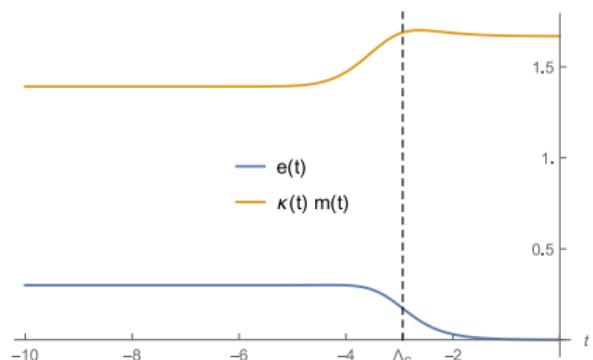
The Massive Fixed Points

- ▶ Two relevant directions
 - ⇒ one free IR parameter
- ▶ IR boundary condition: $e \approx 0.3$
 - ⇒ no free IR parameter

UV fixed point	$\bar{\kappa} \bar{m} = \kappa m$ in the IR	$\bar{\kappa} \Lambda_c$ in the IR
(0, 5.09, 0.328)	1.39	5.17
(0, -5.09, 0.328)	-1.05	-4.92
(0, 5.09, -0.328)	-1.05	4.92
(0, -5.09, -0.328)	1.39	-5.17

Table: The first group of fixed points and their corresponding IR values of $\bar{\kappa}$

The Massive Fixed Points (2)



- ▶ IR value of $\kappa m = \mathcal{O}(1)$
⇒ large correction to anomalous magnetic moment
- ▶ Currently $|\kappa| \leq 2.5 * 10^{-11}$

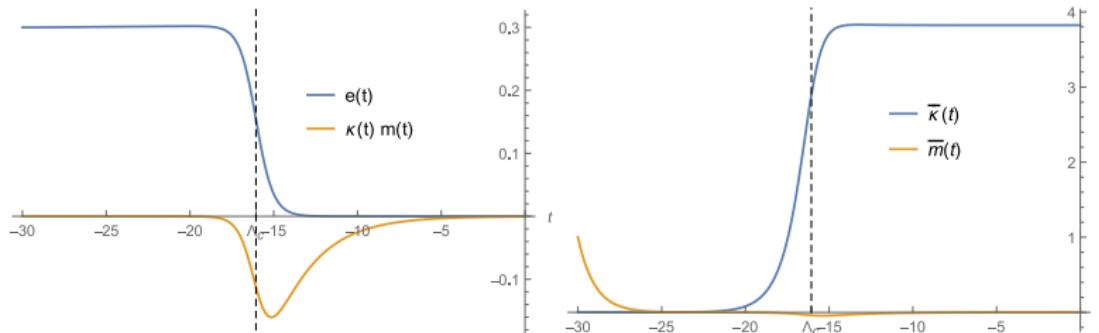
⇒ These IR trajectories are unphysical

The Massless Fixed Points

- ▶ Three relevant directions
 - ⇒ two free IR parameters
- ▶ IR boundary condition: $e \approx 0.3$
- ▶ IR boundary condition: $\bar{\kappa} \approx 0$
 - ⇒ no free IR parameter

Such flows are candidates for UV completions of QED!

The Massless Fixed Points (2)



⇒ Correct IR physics!

Summary and outlook

- ▶ Evidence for UV completion of QED!
- ▶ UV Fixed points within and beyond deep Euclidean region
- ▶ Different fixed points corresponding to different universality classes
- ▶ Qualitative behaviour of fixed points is the same in LO and NLO
- ▶ Are there other physical RG flows?
- ▶ Can an asymptotically safe flow be embedded into the SM?

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