QED with string-localized fields

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Abstract

Several old problems of QED can be addressed by formulating the interaction with the help of "string-localized potentials".

The latter are expected to be useful also in other theories.

(Joint work with J. Mund and B. Schroer, JHEP 01 (2020) 001)

Gauss' Law and Locality

QED has a well-known puzzle (Ferrara–Picasso–Strocchi 1974):

Gauss' Law

$$Q_R = \int_{R \cdot B^3} d^3 x \, \vec{\nabla} \vec{E}(\vec{x}) = \oint_{R \cdot S^2} d\vec{a} \cdot \vec{E}(\vec{x})$$

is in **conflict with local commutation relations** if the interacting Dirac field ψ were an (anti-)local quantum field.

Namely, the right hand side would have to commute with $\psi(y)$ if R is large enough, while in the limit $R \to \infty$, Q_R becomes the **total** charge operator satisfying

$$[Q_{\mathrm{elm}},\psi(y)]=-q\cdot\psi(y).$$

This kind of problems is hard to see in Euclidean or functional integral approaches to QED.

It is also not a feature of the scattering matrix ("on-shell").

It concerns the actual **quantum fields as algebraic objects** (operators on a Hilbert space).

Quantum fields as operators on a Hilbert space \mathcal{H} :

- Are operator-valued distributions $f \mapsto \Phi(f) = \int d^4x f(x) \Phi(x)$.
- Transform covariantly under Ad_U , U a unitary repn of the Poincaré group.
- Commute at spacelike distance (Einstein Causality).
- Only for free fields is the commutator specified otherwise.
- Only free fields are given in terms of "creation and annihilation operators".
- Only free fields satisfy well-defined a priori field equations.
- Perturbation theory: Expresses (at least in a first step) interacting fields as "power series" in free fields.
- Renormalized interacting field equations may follow.

"Free QED" is just a tensor product of Maxwell fields $F_{\mu\nu}$ (living on the photon Fock space) and Dirac fields ψ and their current $j^{\mu} = \overline{\psi}\gamma^{\mu}\psi$ (living on the spinor Fock space). No interaction.

QED couples the fields. E.g., Gauss' Law

$$\partial_{\mu}F^{\mu\nu}(x) = -q\,j^{\nu}(x)$$

requires that the first-order perturbation of the operator on the l.h.s. must equal -q times the free Dirac current.

Several ways out of the QED puzzle:

• The Dirac field is not observable anyway, so why bother about it at all?

 \rightarrow How to set up perturbation theory?

• Pass to a formulation with **indefinite metric** (negative-norm states) which has a compensating "fictitious current" (see below).

 \rightarrow Need to pass back to a physical Hilbert space (Gupta-Bleuler, BRST)

• The problem is due to the "infrared photon cloud" attached to a charged particle.

New idea: Search for the source of the problem in the Maxwell field, coupled to the Dirac field.

The photon cloud is known to be responsible for several other **IR** issues:

- Its asymptotic electric flux must commute with all observables. Hence it is a multiple of 1 in every irreducible representation. Its values distinuguish infinitely many inequivalent repns (superselection sectors). (Buchholz 1982)
- The asymptotic flux is not Lorentz invariant (it depends on the velocity of the charged particles). Therefore, it breaks Lorentz invariance in the charged sectors. (Fröhlich–Morchio–Strocchi 1979)
- It prevents the electron from having a sharp mass. (Buchholz 1982)
- This in turn explains the failure of LSZ scattering theory, which therefore requires proxies like "soft photon inclusive cross sections". (Bloch–Nordsieck 1937)

All these "infrared issues" are due to the masslessness and the photon and the long-range of electro-magnetic forces.

They are expected to make a similar appearance with gravitons. E.g., the Bondi-Misner-Sachs group should be related to infrared superselection sectors.

Strominger's **"Infrared Triangle"** addresses the same issues and their inter-relations. (Strominger, 2018)

This talk: Provide a new understanding in terms of the operator-algebraic nature of fields coupled to massless particles.

Indefinite metric approach

The Maxwell potential $A_{\mu}(x)$ can be **canonically quantized** provided one adds a gauge-fixing term $-\frac{\lambda}{2}(\partial A)^2$ to the Lagrangian. The resulting canonical commutation relations for the creation and annihilation operators yield an **indefinite inner product** for one-particle and Fock states (they involve $\eta_{\mu\nu}$ and $\delta'(p^2)$.)

Thus the theory is built on a Krein space ("Hilbert space with indefinite metric"). This is physically intolerable, and the physical Hilbert space is defined as

$$\mathcal{H} := \mathcal{H}_0/\mathcal{H}_{00}$$

where \mathcal{H}_0 is the semi-definite subspace of vectors satisfying the **Gupta-Bleuler condition** $(\partial A)^- \Psi = 0$, and $\mathcal{H}_{00} \subset \mathcal{H}_0$ is the subspace of \mathcal{H}_{00} of zero-norm states.

(BRST generalizes this to non-abelian gauge theories.)

On the Krein space, one can construct (anti-)local interacting Maxwell and Dirac fields. But charged states created by the Dirac field cannot satisfy the GB condition, and the **Dirac field just drops out** of the theory upon passage to the physical Hilbert space.

How about the **Gauss Law in the Krein space**? The asymptotic electric field certainly commutes with the Dirac field. So the total flux cannot equal the charge operator.

Indeed, before the GB condition is imposed, Maxwell's equations hold with an extra term:

$$\partial_{\mu}F^{\mu\nu} = -\boldsymbol{q}\cdot j^{\nu}_{\mathrm{Dirac}} - \lambda\cdot\partial^{\nu}(\partial A).$$

The second term is a "fictitious current": it vanishes between physical states satisying the GB condition. But the total integral Q_{fict} over its density cancels the commutator $[Q_{\text{elm}}, \psi]$. This solves the contradiction, but does not settle the physical puzzle.

String-localized approach

Covariant quantization starts with the potential A and defines the Maxwell field strength as the derivative F = dA.

String-localized ("s-loc") **quantization** starts with the field strength and constructs the potential A(e) as a primitive of F.

Autonomous quantization of $F_{\mu\nu}$:

Take the unitary one-particle representation of the Poincaré group with helicity ± 1 (Wigner 1939, Mackey 1952). Follow the prescription of Weinberg (1964) to get a local quantum field on the Fock space.

F lives directly on the physical Hilbert space. And so does A(e).

There are many primitives of F = dA. A particularly nice choice is

$$A_\mu(x,e)=\int_0^\infty ds\, F_{\mu
u}(x+se)e^
u$$

where *e* is any non-zero **directional vector** in \mathbb{R}^4 . It solves F = dA(e).

The direction e has **no physical meaning**. The operator A(e) is just a tool to write down the coupling

$$\mathcal{L}_{ ext{int}} = A_{\mu}(e) j^{\mu}$$

of the Dirac current $j^{\mu} = \overline{\psi}\gamma^{\mu}\psi$ to the electromagnetic field, avoiding negative-norm states and explicit breaking of Lorentz covariance.

Its choice will however **affect the charged states** generated by the (unobservable) interacting Dirac field.

See below

 $A_{\mu}(e)$ is **Poincaré covariant** in the sense

$$U(a,\Lambda)A_{\mu}(x,e)U(a,\Lambda)^{*}=\Lambda^{\nu}{}_{\mu}A_{\nu}(\Lambda x+a,\Lambda e).$$

It is localized along the "string"

$$S_e(x) = x + \mathbb{R}_+ \cdot e$$

in the sense that A(x, e) commutes with A(x', e') if their strings $S_e(x)$ and $S_{e'}(x')$ are spacelike separated.

In scattering theory, causal commutativity is important for the "separation of wave packets at asymptotic times". Because timelike strings can never be spacelike separated, we limit ourselves to spacelike strings, WLOG $e^2 = -1$.

S-loc vs positive quantizations

The **timelike** choice $e = (1, \vec{0}) \Rightarrow A_0(e) = 0$ and $\vec{A}(x, e) = \int_{x^0}^{\infty} dt \vec{E}(t, \vec{x})$ is identical with the usual "**Coulomb** gauge".

Indeed, the Coulomb gauge potential is highly non-local.

For every **fixed spacelike** e, A(e) is identical with the "axial **gauge**": $e^{\mu}A_{\mu}(e) = 0$. However, unlike in the axial gauge, e transforms under Lorentz transformations.

The main difference is that the potentials A(e) for all e "live **together**" on the same **physical Hilbert space** of the field strength $F_{\mu\nu}$. They are all "made of" the same creation and annihilation operators for the **physical two photon states**. No longitudinal photons and no ghosts occur.

S-loc vs covariant quantizations

The s-loc potential

$$A_{\mu}(x,e) := \int_0^\infty ds \, F_{\mu
u}(x+se)e^
u \equiv I_e(Fe)_{\mu}(x)$$

is also defined on the Krein space, where $F = dA^{K}$ is the derivative of a covariant indefinite-metric potential A^{K} ("K" standing for "Krein"). In this case, the difference is

$$A_{\mu}(x,e) = A_{\mu}^{K}(x) + \partial_{\mu}\phi(x,e)$$

where the "escort field" $\phi(x, e) = I_e(A^K e)(x)$ is another string-localized field.

Like $A_{\mu}^{K}(x)$, the escort field $\phi(x, e)$ is defined only on the Krein space. Only F and A(e) descend to the physical quotient space.

This co-existence permits interesting contrasts mediated by the escort field ("hybrid approach").

Comparison of the QED coupling

The coupling to the Dirac current $j^{\mu} = \overline{\psi} \gamma^{\mu} \psi$ satisfies

$$\mathcal{L}_{ ext{int}}(e)\equiv {\sf A}_{\mu}(e)j^{\mu}={\sf A}_{\mu}^{K}j^{\mu}+\partial_{\mu}ig[\phi(e)j^{\mu}ig].$$

In classical theory, this would mean that the action $\int d^4x \mathcal{L}_{int}(x)$ is the same, and the field equations are the same.

It is by no means obvious that this survives the quantization.

Aims: Show that string-localized ("s-loc") QED

- yields the same cross sections (no "new QED"),
- gives a better description of the properties of its interacting quantum fields (including Gauss' Law, the behaviour at spacelike infinity, and infra-particles).

The scattering matrix

The scattering matrix is defined in terms of the interaction density ("Lagrangian" $\mathcal{L}_{\rm int})$ as

$$S = \lim_{g(y) \to q} T \exp\left(i \int d^4 y \, g(y) \mathcal{L}_{int}(y)\right)$$

The limit $g(x) \rightarrow q$ of removing the IR cutoff of the coupling constant is called "adiabatic limit".

In PT, one defines the time-ordered exponential via the Wick expansion as a power series. The problem is that expressions like

$$T\mathcal{L}_{int}(y_1)\ldots\mathcal{L}_{int}(y_n)$$

are **not** a **priori** defined (multiplication of distributions).

Renormalization

Microlocal analysis shows that $T\mathcal{L}_{int}(y_1) \dots \mathcal{L}_{int}(y_n)$ can be defined in a Lorentz covariant way, and such that they fulfill the **causality** features suggested by the time-ordering symbol T. However, the definition is in general not unique, and the freedom is some derivative of the "total delta fn" supported at $y_1 = \dots = y_n$. (Epstein–Glaser 1973)

In Fourier space, derivatives of the total delta fn are polynomials in the momenta, and the freedom is the usual **freedom of renormalization** by "counter terms" (that also produce polynomials in momentum space). The freedom has to be fixed.

In the indefinite-metric approach, the most prominent requirement is gauge invariance (so that the S-matrix descends to the physical state space).

String-independence

In the s-loc approach, the **principle of string-independence (PSI)** is imposed: The scattering matrix must not depend on the arbitrary direction *e* in the adiabatic limit.

Recall: e was only introduced in order to be able to write down $\mathcal{L}_{int}(e) = A_{\mu}(e)j^{\mu}$ as an operator on the Hilbert space.

PSI requires $\partial_e[\mathcal{TL}_{int} \dots \mathcal{L}_{int}] \stackrel{!}{=}$ total derivative.

"Time-ordering" in the s-loc approach is w.r.t. strings rather than points. The "total delta fn" is supported on configurations where strings intersect. This makes renormalization a **much more complicated task**. It can be done in lowest orders by hand, and the PSI can be satisfied.

 \rightarrow Some interesting mathematics in differential renormalization of loop diagrams (Gass, PhD project)

Interacting fields

Interacting fields are constructed via "Bogoliubov's formula"

$$\Phi_{\rm int}(x) = \lim_{g(y)\to q} \frac{-i\delta}{\delta f(x)} \left(T e^{i\mathcal{L}_{\rm int}(g)} \right)^* T e^{i\mathcal{L}_{\rm int}(g) + i\Phi(f)}.$$

As a formal expansion, the interacting field is a series in iterated retarded commutators

$$\begin{aligned} R(\mathcal{L}_{\text{int}}(y_1),\ldots,\mathcal{L}_{\text{int}}(y_n);\Phi(x)) &= \\ &= i^n \theta(x^0 > y_1^0 > \cdots > y_n^0) [[\ldots [\Phi(x),\mathcal{L}_{\text{int}}(y_1)],\ldots],\mathcal{L}_{\text{int}}(y_n)], \end{aligned}$$

expanded by Wick's theorem into operators and propagators, and integrated over $g(y_1) \dots g(y_n)$.

The retarded commutators contributing to $\Phi_{int}(x)$ involve free fields localized in the past of x. Clearly, $\Phi_{int}(x)$ does not satisfy causal commutativity with the free fields.

What matters is, however, that **(observable) interacting fields causally commute among each other**. The Bogoliubov map $\Phi \mapsto \Phi_{int}$ is known to secure this feature, without being an algebraic isomorphism (i.e., the non-vanishing commutators change.)

Again, to define Φ_{int} , the retarded commutators = products of distributions **need a definition**, which may introduce some **renormalization freedom**. For observable fields, the freedom must be fixed by gauge invariance, or PSI, resp.

String-localized propagators have a more involved structure as distributions in x (or p) and e.

First order QED

The vector potential in first order is

$$F_{\rm int}^{\mu\nu}(x) = F^{\mu\nu}(x) + i \int d^4 y \, g(y) \underbrace{\theta(x^0 - y^0)[F^{\mu\nu}(x), A_\kappa(y, e)]}_{i\left(\partial_x^{[\mu}\delta_\kappa^{\nu]} + \partial_x^{[\mu}e^{\nu]}\partial_\kappa^{\nu}I_e^{\nu}\right)G_0^{\rm ret}(x-y)} j^\kappa(y).$$

The Dirac field in first order is

$$\psi_{\mathrm{int}}(x,e) = \psi(x) + \int d^4 y \, g(y) (i \, \partial_x + m) G_m^{\mathrm{ret}}(x-y) \gamma^{\kappa} \psi(y) \mathbf{A}_{\kappa}(\mathbf{y},\mathbf{e}).$$

Both are string-localized expressions. But in $F_{\text{int}}^{\mu\nu}$, the term involving I_e can be partially integrated and vanishes in the adiabatic limit because $\partial_{\kappa} j^{\kappa} = 0$. The result coincides with "ordinary" QED.

In contrast, the **interacting Dirac field inherits the string-localization** from the vector potential.

From this,

$$\partial_{\mu}F^{\mu\nu}_{\rm int}(x) = -g(x)j^{\nu}(x) + e^{\nu}\int_{0}^{\infty} ds \,\partial_{\kappa}g(x-se)j^{\kappa}(x-se) + O(g^{2})$$

 $\stackrel{g(x) \to q}{\longrightarrow} -qj^{\nu}(x)$

because the region where $\partial_{\kappa}g \neq 0$ "moves outside" where matrix elements of j^{κ} decay rapidly.

Thus, Gauss' Law is satisfied (in first order in the adiabatic limit).

The initial **conflict** between Gauss' Law and causal commutativity is **resolved** because $\psi_{int}(y, e)$, being localized along a string extending to ∞ , does not commute with the asymptotic electric flux.

The finding that **unobservable charged fields may become string-localized** upon interaction, is a feature that was not anticipated by the celebrated Wightman axiomatics (Streater–Wightman 1963).

It rather confirms a finding (Buchholz–Fredenhagen 1982) in the more general Algebraic QFT setting, that charged states may differ from the vacuum in spacelike cones extending to ∞ (and thus require s-loc fields to "create" them from the vacuum).

The next task is to compute **asymptotic fluxes**, i.e., the expectation value of $r^2 \vec{E}_{int}(r\vec{n})$ for large r in states $\Psi = \psi_{int}^*(f, e)\Omega$, $\|\Psi\| = 1$.

For the interaction, we allow to average $\mathcal{L}_{int}(e)$ over $e = (0, \vec{e})$ with some smearing function $h(\vec{e})$.

The computation is lengthy. It reveals several cancellations of terms that would correspond to the "fictitious current". The final result is (in first order in the asymptotic limit)

 $\lim_{r\to\infty}r^2(\Psi(h),\vec{E}_{\rm int}(r\vec{n})\Psi(h))=-q\cdot h(\vec{n})\vec{n},$

i.e., the smearing function h determines the directional profile of the asymptotic electric flux of the photon cloud.

The asymptotic flux is **not string-independent**. This does not conflict with PSI, because ψ_{int} is not an observable. Instead, while the electric field as an operator is independent of *e*, its expectation value in the **string-dependent state** $\Psi(h) = \psi_{int}^*(f, h)\Omega$ is not.

By choosing e (or the smearing $h(\vec{e})$), one can "engineer" the shape of the photon clouds of charged states at will.

Moreover, the non-vanishing of the asymptotic flux is a perturbative confirmation of the abstract expectation (necessary for Gauss' Law), and it entails, by Buchholz' argument, that the mass operator $M^2 = P_{\mu}P^{\mu}$ cannot have a discrete spectrum in charged states.
Electrons are infra-particles.

Photon cloud superselection

We have given (Mund, KHR, Schroer 2020) a simplified (and "semi-nonperturbative") argument in favour of the conclusion that normalized charged states $\Psi(h) = \psi_{int}^*(f, h)\Omega$ prepared with different photon clouds (asymptotic flux profiles = averaging functions h in $\mathcal{L}_{int}(h)$) are mutually orthogonal. These states yield a continuum of inequivalent representations of the algebra of observables.

This is a perturbative confirmation of the structural conclusions of Fröhlich–Morchio–Strocchi (1979) and Buchholz (1982) in the axiomatic setting.

Speculation: The asymptotic symmetry underlying this superselection structure seems to be a Heisenberg group with asymptotic electric fluxes as "phase operators" e^{iaP} and asymptotic magnetic fluxes as "shift operators" e^{iaQ} .

More applications of s-loc QFT

The same construction

$${\sf P}_\mu(x,e):=\int_0^\infty ds\,{\sf F}_{\mu
u}(x+se)e^
u$$

can also be made with the **Proca field** ("massive photons") of spin 1, where $F_{\mu\nu}(x) = \partial_{\mu}P_{\nu}(x) - \partial_{\nu}P_{\mu}(x)$ and $\partial_{\mu}F^{\mu\nu} = -m^2P^{\nu}$. It holds again

$$P_{\mu}(x) = P_{\mu}(x, e) - \partial_{\mu}\phi(x, e).$$

The Proca field P has dimension 2 (which forbids its coupling of massive photons to a Dirac current of dimension 3) while P(e) has dimension 1. P does not have a massless limit, while P(e) does.

Thus the escort field $\phi(x, e)$ "carries away" both the undesired bad UV behaviour of the Proca field, and its contribution obstructing the massless limit (Mund-KHR-Schroer 2017).

Similar with the massive **Rarita-Schwinger field** ψ_{μ} of spin $\frac{3}{2}$: It

has dimension $\frac{5}{2}$, hence its current $j^{\mu} = \overline{\psi}_{\nu}\gamma^{\mu}\psi^{\nu}$ has dimension 5 and cannot be coupled to anything. In contrast, the s-loc RS field $\psi_{\mu}(x, e)$ has dimension $\frac{3}{2}$, and yields a current of dimension 3 that may couple electromagnetically.

While ψ_{μ} has no massless limit, the massless limit of $\psi_{\mu}(e)$ exists and is the helicity $\frac{3}{2}$ field ("gravitino").

Likewise for fields in all other representations of the Poincaré group.

The **"infinite-spin" representations are exceptional**: They do not admit any point-localized fields (Yngvason 1970). But a string-localized field can be constructed directly from the Wigner representation (Mund-Schroer-Yngvason 2005), or as a massless limit of s-loc tensor fields of increasing spin (KHR 2017).

String-localized versions of massive fields have **better UV behaviour** (dimension 1 rather than s + 1 for integer spin, dimension $\frac{3}{2}$ rather than s + 1 for half-integer spin) and therefore **admit couplings that are otherwise non-renormalizable**.

L-*V*-pairs and *L*-*Q* pairs

The principle of string-independence (PSI) for the scattering matrix in first order imposes **conditions on the admissible interactions**: It requires that the *e*-dependence of the interaction density is a total derivative:

$$\partial_e L_{\mathrm{int}}(e) = \partial_\mu Q^\mu(e).$$

A stronger version is that there exists another point-localized interaction density ${\it L}_{\rm int}$ (without any s-loc fields in it) such that

 $L_{\mathrm{int}}(e) = L_{\mathrm{int}} + \partial_{\mu} V^{\mu}(e).$

QED in Krein space is of this type, with $V^{\mu} = \phi(e) j^{\mu}$ (see above).

Unlike gauge-invariance of the full Lagrangian, these are conditions on the interaction density separately.

They are quite restrictive. E.g., for **interactions involving massive vector fields**, they cannot be fulfilled without another massive scalar field present.

PSI in higher perturbative order imposes more constraints. For self-interacting vector bosons, PSI can be fulfilled only if the cubic coupling coefficients are **anti-symmetric and satisfy the Jacobi identity** ("gauge symmetry without gauge principle").

For a model with the field content of the weak interactions, the coupling must be **chiral**:

$$W_{\mu} \cdot \overline{\psi} \gamma^{\mu} (1 \pm \gamma^5) \psi.$$

The sign is undetermined. (Gracia-Bondia–Mund–Varilly 2018) It happens to be – in nature.

In second order, **PSI** of $T\mathcal{L}_{int}(y_1)\mathcal{L}_{int}(y_2)$ might be violated by terms of the form $\partial_e[\delta(y_1 - y_2)X(y_1, y_2, e_1, e_2)]$. Such obstructions can be cancelled by adding the "induced" Lagrangian $\mathcal{L}_{ind}(y, e_1, e_2) = -X(y, y, e_1, e_2)$ to the first-order interaction density.

E.g., in a theory with massive vector bosons, a **quartic self-potential** for the scalar field is forced by PSI to have the form

$$\Phi^2(\Phi-\Phi_0)^2.$$

PSI thus "predicts" the Higgs potential "after the shift" that is usually assumed to trigger spontaneous symmetry breaking.

This is "Higgs physics without Higgs mechanism" (and in fact, without gauge symmetry). (Mund-KHR-Schroer, work in progress)

Helicity 2

Massless fields of helicity 2 should describe gravitons, propagating in a Minkowski background ("linearized quantum gravity").

Canonical quantization requires indefinite metric, as for helicity 1.

Approximation by massive fields of spin 2 (positive metric) reveals a discontinuity at m = 0 (van Dam-Veltman-Zakharov 1970). In fact, the massive spin-2 field does not even have massless limit.

The simplest string-localized massive spin-2 field has a massless limit, but it still has the DVZ discontinuity. Indeed, its limit is a mixture of helicity 2 and helicity 0. A better s-loc massive spin-2 field was found (Mund-KHR-Schroer 2017) that has no discontinuity and decouples from helicity 0 in the limit.

Finding L-Q pairs for self-interactions of helicity-2 fields is not easy.

With s-loc fields, the lowest possible dimension is 5. This is beyond the renormalizability bound 4, but still better than p-loc self-couplings without indefinite metric (dimension at least 9, in fact 11 for anything similar to massive gravity).

The dimension-5 self-interaction has the structure

$$\mathcal{L}_{ ext{int}} = \mathbf{g} \cdot \left(h_{\mu
u} T[h]^{\mu
u} - h_{\mu\kappa} h_{\nu\lambda} R[h]^{\mu
u\kappa\lambda} \right)$$

where T[h] is an s-loc graviton stress-energy tensor quadratic in ∂h , and R[h] is the linearized curvature $\sim \partial \partial h$.

With $g = \frac{G}{3}$, \mathcal{L}_{int} coincides with Einstein-Hilbert gravity in cubic order if all its constraints could be eliminated "by hand".

Do E-H higher-order terms arise by "induction"? (Gass, PhD project)





Universität Paderborn -> Fakultäten -> Fakultät für Elektrotechnik, informatik und Mathematik -> Mathematik -> Arbeitsgruppe Mathematische Physik -> Research -> LQP 45 Workshop

Workshop LQP 45

We invite you to participate in the 45th Workshop on Foundations and Constructive Aspects of Quantum Field Theory which is going to take place on June 26 and 27, 2020 at Paderborn University.

Contact

If you have any questions concerning the workshop, please feel free to drop me an e-mail via lqp45...math...upb...de. Please replace "..." by "@", by "." and by "." (in that order), respectively.

Location

The workshop will take place in lecture hall D2 at Paderborn University. It is located at the ground floor of building D. Please check the campus map for details.

Important Dates

April 5	Deadline: Application for Financial Support
April 24	Hotel reservation expires
May 18	Deadline: Registration

SIE INTERESSIEREN SICH FÜR:

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Backup

S-loc propagators

The Feynman propagator for the field strength F

$$i(\Omega, T[F_{\mu\nu}(x)F_{\kappa\lambda}(y)]\Omega) = (\eta_{\mu\kappa}\partial_{\nu}\partial_{\lambda} \pm \text{three terms})G_0(x-y)$$

implies the propagator for A(e)

$$e^{
u}e^{\prime\lambda}(\eta_{\mu\kappa}\partial_{
u}\partial_{\lambda}\pm {
m three \ terms})I^{x}_{e}I^{y}_{e^{\prime}}G_{0}(x-y).$$

(Recall $I_e f(x) := \int_0^\infty f(x + se)$. In momentum space: $I_e = \frac{-i}{(pe)-i0}$, $I_{e'} = \frac{i}{(pe')+i0}$.)

The propagator for *F* would admit a renormalization by $c(\eta_{\mu\kappa}\eta_{\nu\lambda} - \eta_{\nu\kappa}\eta_{\mu\lambda})\delta(x - y)$. But PSI requires c = 0.

(Similar for the retarded commutators)

A "magic formula"

In the "hybrid approach" (A(e) and A^{K} coexist on the Krein space and differ by the derivative $\partial \phi(e)$ of the "escort") one can prove

$$\psi\Big|_{\mathcal{L}(e)=A(e)j} = \left(e^{iq\phi(e)}\psi\right)\Big|_{\mathcal{L}=A^{K_{j}}}$$

- L.h.s. descends to the physical Hilbert space, but is a priori badly de-localized.
- R.h.s. is string-localized, but a priori only defined on Krein space.
- \bullet By equality, $\psi_{\rm int}$ is string-localized on the Hilbert space.

There are subtle issues.

Magic formula

- The magic formula is "semi-perturbative": the exponential $e^{iq\phi(x,e)}$ should be taken serious and considered as a Weyl operators $W(f,h) = e^{i\phi(f,h)}$ with smearing functions f(x) and h(e) such that $\int f = q$ and $\int h = 1$.
- Because of the string-integration involved in $\phi(e)$, these Weyl operators are highly singular and require several regularizations (mass, multiplicative renormalization).

 \rightarrow Simpler analogy in 2D (Schroer 1962).

 $\bullet\,$ In the limit, one obtains a positive state ω on the Weyl algebra in which

 $\omega(W(f,h)^*(\text{neutral operators})W(f,h')) \sim \delta_{hh'}.$

• The Weyl operators live in the GNS Hilbert space of this state, which splits into a direct integral of Hilbert spaces describing states with different asymptotic fluxes.

BCRV

The analysis of the paper JHEP 01(2020)001 was triggered by Buchholz, Ciolli, Ruzzi, Vasselli: LMP 109 (2019) 2601–2610.

The authors point out that s-loc potentials do not contain the unphysical "longitudinal component" of the vector potential, which is, however, needed to create from the vacuum states in which local electric fluxes are non-zero (states with local charge distributions). Thus, QED cannot be constructed with the s-loc potentials only.

Their conclusion is correct as long as QED is constructed from the electromagnetic sector only, and the current DEFINED by $j^{\nu} = \partial_{\mu}F^{\mu\nu}$, without reference to Dirac fields. This is in the spirit of Buchholz, Ciolli, Ruzzi, Vasselli: LMP 107 (2017) 201–222 and LMP 109 (2019) 829–842.

In contrast, we DO use Dirac fields, and analyse their algebraic properties, which – in spite of them being unobservable – reflect localization properties of physical photon clouds.